EFFECT OF NUCLEAR MAGNETIC DISTRIBUTION ON THE PHOTO PRODUCTION OF LONGITUDINALLY POLARIZED LEPTON-PAIRS IN THE FIELD OF Na_{11}^{23} AND Al_{13}^{27} NUCLEI

by

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In this work we treat the influence of the nuclear magnetic moment on the photo production of e^- e+, μ^- μ^+, τ^- τ^+ i. e. l^- l^+, lepton pairs. We examine the process: $\gamma + N(Ze, \mu_j) \to N(Ze, \mu_j) + l^-(p_-) + l^+(p_+)$ of production of $(l^- l^+)$ – pairs by circularly polarized photons $\gamma(p_\gamma)$ in the field of nucleus, N, having electric (coulomb) charge Ze, and magnetic dipole moment μ_j, where j is the nuclear spin. Taking into account the longitudinal polarizations of the produced pairs, we calculate the differential cross-sections for the angular and energy distributions of the pair particles. We investigate the effect of the nuclear magnetic moment on the angular and energy distribution of the antilepton $l^+(e^+,\mu^+,\tau^+)$, showing that the contribution the cross-section from the nuclear magnetic dipole moment rises considerably on going to high energies of incident photon and large emission angles of the pair particles. The results of calculations for Na_{11}^{23} (j =1/2) and Al_{13}^{27} – target nuclei, and for the three lepton pair types (e^- e^+, μ^- μ^+, τ^- τ^+) are presented graphically [1, 2].

Key words: *pair production, photon interaction, Coulomb field of nuclei, leptons*

Introduction

In the past years, the production of lepton pairs in the relative heavy collision of ions has gained a lot of attention, which is mainly related to the operation of relativistic heavy ion collider (RHIC), especially in Born approximation the cross-sections is occupied random from photon energy [3]. On the other hand, in heavy ion collisions, the αz parameter is order of unity, and the problem arises in order to take into account the effect of CC. It is worth noting that the total yield of lepton pairs in heavy ion collisions must be measured, because the pair's production consists of a huge phone in experiments with relative heavy ions. For example, the total cross-section of process in RHIC energies is tens of kilos for heavy ion collisions. In large hadron collider (LHC) energies, this amount becomes hundreds of kilos if one is valued according to the Racah equation [4]. Moreover, the production of the intense pair can destroy ion beams circulating in the accelerator as a result of electron capture by heavy ions, see for example [5].

As known that the polarization of production due to the photon interaction has attracted a lot of attention. In this regard, Jost *et al.* [6] had been calculated the angular and momentum distribution of the recoil nucleus in pair production by a photon. The results show that the disagreement with a recent experiment, particularly for small angles and high momentum transfers. Also, the polarization dependence of the leptonic-pair production due to the photon interaction has been discussed in the case of coulomb field of light nuclei [7]. Recently, the

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strong field QED in lepton colliders and electron/laser interactions has been reviewed [8]. More recently, the photon emission and electron-positron pair productions by Coulomb processes in particle-in-cell simulations based on wide range of physical conditions has been studied [9]. Also, the influence of magnetic fields on pairs of oppositely charged particles in ultra-relativistic heavy-ion collisions has been outlined [10]. It has been found that the particles gain an intensity-dependent rest mass shift that accounts for their presence in the dispersive vacuum while Multi-photon events involving more than one external field photon occur at each vertex.

The differential cross-section of the pair production process.

The Fourier expansion of the operator of the interaction energy of the photon with the coulomb and magnetic field of a nucleus with charge Ze and magnetic moment, μ_j, has the form [11, 12]:

$$U = U^E + U^M = -\frac{4\pi Ze^3}{L^3}\sum_q\left[1-\frac{i}{Ze}\left(\vec{\alpha}\cdot\vec{d}\right)\right]\frac{e^{-iqr}}{q^2} \tag{1}$$

where

$$\vec{d} = \left(\overrightarrow{\mu_j}\wedge\vec{q}\right) = \left[\overrightarrow{\mu_j}\,\vec{q}\right],\ \overrightarrow{\mu_j} = \mu_j\left(\frac{\vec{j}}{j}\right)$$

is the magnetic moment vector of the nucleus with spin spin, j, and $\vec{j}$ – the nuclear spin momentum vector, $\mu_j = \chi_m\mu_N$ – the magnetic moment of the nucleus, χ_m – the number of nuclear magneton, $\mu_N = e\hbar/(2M_pc)$ – the nuclear magneton [13], M_p – the proton mass, $\vec{\alpha}$ – the Dirac matrices, L^3 – the normalization volume, and $\vec{q} = \vec{P}_\gamma - \vec{P}_- - \vec{P}_+$ – the momentum of the recoil target nucleus, and $\vec{P}_\gamma$, P_-, $\vec{P}_+$ are the momentum vectors of photon, lepton (l^-) and antilepton (l^+), respectively.

When the longitudinal polarization of the particles of pairs is taken into account, the differential cross-section of the lepton (l^-l^+)-pairs in the field of a nucleus with magnetic moment, μ_j, averaged over the initial and summed over the final spin states of the nucleus ($j = \vec{j}$) has the form [14]:

$$d\sigma_{s_\gamma s_- s_\gamma}\left(\theta_-,\theta_+,Ze,\mu_j\right) = d\sigma^E_{s_\gamma s_- s_+}\left(\theta_-,\theta_+\right) + d\sigma^M_{s_\gamma s_- s_+}\left(\theta_-,\theta_+\right) \tag{2}$$

where

$$\begin{aligned} d\sigma^E_{s_\gamma s_- s_+}\left(\theta_-,\theta_+\right) &= \frac{1}{4}d\sigma_{OE}\left(\theta_-,\theta_+\right) + s_-s_+d\sigma_{1E}\left(\theta_-,\theta_+\right) + \\ &+ s_\gamma s_+ d\sigma_{2E}\left(\theta_-,\theta_+\right) + s_\gamma s_- d\sigma_{3E}\left(\theta_-,\theta_+\right) \end{aligned} \tag{3}$$

$$\begin{aligned} d\sigma^M_{s_\gamma s_- s_+}\left(\theta_-,\theta_+\right) &= \frac{1}{4}d\sigma_{OM}\left(\theta_-,\theta_+\right) + s_-s_+d\sigma_{1M}\left(\theta_-,\theta_+\right) + \\ &+ s_\gamma s_+ d\sigma_{2M}\left(\theta_-,\theta_+\right) + s_\gamma s_- d\sigma_{3M}\left(\theta_-,\theta_+\right) \end{aligned} \tag{4}$$

and

$$d\sigma_{OE}\left(\theta_-,\theta_+\right) = 4\eta\frac{1}{q^4}F_{OE}\left(\theta_-,\theta_+\right)d\Omega_-d\Omega_+ \tag{5}$$

$$d\sigma_{iE}\left(\theta_-,\theta_+\right) = \eta\frac{1}{q^4}F_{iE}\left(\theta_-,\theta_+\right)d\Omega_-d\Omega_+,\ \left(i = 1,2,3\right) \tag{6}$$

$$d\sigma_{OM}\left(\theta_-,\theta_+\right) = 4\eta\left(\frac{\mu_j}{Ze}\right)^2\alpha_j\frac{1}{q^2}F_{OM}\left(\theta_-,\theta_+\right)d\Omega_-d\Omega_+ \tag{7}$$

$$d\sigma_{iM}\left(\theta_-,\theta_+\right)=\eta\left(\frac{\mu_j}{Ze}\right)^2\alpha_j\frac{1}{q^2}F_{iM}\left(\theta_-,\theta_+\right)d\Omega_-d\Omega_+,\ \left(i=1,2,3\right) \tag{8}$$

$$\eta=\left(\frac{Z^2\alpha^3}{4\pi^2}\right)\frac{P_-P_+}{P_\gamma^3}dE_+,\quad \alpha_j=\frac{j+1}{3j},\quad \alpha=\frac{1}{137}$$

The functions F_{OE}, F_{iE}, F_{OM}, and F_{iM} (i = 1, 2, 3) are functions of the angles (θ_-, θ_+) of the emission of the lepton (l^-) and antilepton (l^+) and have complex form s.

In eqs. (2)-(4), $d\sigma^E_{s_\gamma s_- s_+}(\theta_-, \theta_+)$ and $d\sigma^M_{s_\gamma s_- s_+}(\theta_-, \theta_+)$ are the differential cross-sections of the electric and magnetic photo production of lepton pairs in the field of a nucleus with charge Ze and magnetic moment μ_j. The terms proportional to s_+, s_-, s_γ, s_+, and $s_\gamma s_-$ in eqs. (3) and (4) represents the spin correlations between the spins of photon (s_γ), lepton (s_-), and antilepton (s_+) for the electric and magnetic photo production of the pairs.

The $d\Omega_- = \sin\theta_-\, d\theta_- d\theta_-$ and $d\Omega_+ = \sin\theta_+\, d\theta_+ d\theta_+$ are the solid angles of the emission of the lepton (l^-), and antilepton (l^+), $\cos\theta_\mp = (\vec{P}_\gamma\cdot\vec{P}_\mp)/(P_\gamma P_\mp)$.

The quantity $s_\gamma = \mp1$ defines the direction of the circular polarization of the incident photon, $s_\gamma = \mp1$ for a right-hand circularly polarized, and $s_\gamma = -1$ for a left-hand polarized photon. The quantities $s_- = \mp1$ and $s_+ = \mp1$ are the eigenvalues of the projection operator $(\vec{\sigma}_\mp\cdot\vec{P}_\mp)/(\vec{P}_\mp)$ representing the helicity or the longitudinal spin polarization of the lepton (l^-) and antilepton (l^+) of the produced pair. For $s_- = 1(s_+ = 1)$, the lepton (antilepton) is polarized in the direction of the momentum P_- (P_+), *i. e.* it is right-hand polarized, while for $s_- = -1(s_+ = -1)$, they are polarized opposite to the momentum P_- (P_+), *i. e.* it is left-hand polarized [15, 16].

The angular distribution cross-sections

By integrating eq. (2) over the solid angle $d\Omega_-$ of the lepton (l^-) emission, and after a series of long calculations, we get the following expressions for the angular distribution cross-section of the antilepton $l^+(e^+, \mu^+, \tau^+)$:

$$d\sigma_{s_\gamma s_- s_\gamma}\left(\theta_+,Ze,\mu_j\right)=d\sigma^E_{s_\gamma s_- s_+}\left(\theta_+\right)+d\sigma^M_{s_\gamma s_- s_+}\left(\theta_+\right) \tag{9}$$

where

$$d\sigma^E_{s_\gamma s_- s_+}\left(\theta_+\right)=\frac{1}{4}d\sigma_{OE}\left(\theta_+\right)+s_-s_+d\sigma_{1E}\left(\theta_+\right)+s_\gamma s_+d\sigma_{2E}\left(\theta_+\right)+s_\gamma s_-d\sigma_{3E}\left(\theta_+\right) \tag{10}$$

$$d\sigma^M_{s_\gamma s_- s_+}\left(\theta_+\right)=\frac{1}{4}d\sigma_{OM}\left(\theta_+\right)+s_-s_+d\sigma_{1M}\left(\theta_+\right)+s_\gamma s_+d\sigma_{2M}\left(\theta_+\right)+s_\gamma s_-d\sigma_{3M}\left(\theta_+\right) \tag{11}$$

and

$$d\sigma_{OE}\left(\theta_+\right)=8\pi\eta\Phi_{OE}\left(\theta_+\right)d\Omega_+ \tag{12}$$

$$d\sigma_{iE}\left(\theta_+\right)=2\pi\eta\,\Phi_{iE}\left(\theta_+\right)d\Omega_+,\ \left(i=1,2,3\right) \tag{13}$$

$$d\sigma_{OM}\left(\theta_+\right)=8\pi\eta\left(\frac{\mu_j}{Ze}\right)^2\alpha_j\,\Phi_{OM}\left(\theta_+\right)d\Omega_+ \tag{14}$$

$$d\sigma_{iM}\left(\theta_+\right)=2\pi\eta\left(\frac{\mu_j}{Ze}\right)^2\alpha_j\,\Phi_{iM}\left(\theta_+\right)d\Omega_+,\ \left(i=1,2,3\right) \tag{15}$$

In the ultrarelativistic case (ω, E_-, $E_+ \gg m_{op}c^2$) where ω, E_-, and E_+ are the energies of the photon, lepton (e^-, μ^-, τ^-,) and antilepton (e^+, μ^+, τ^+,), m_{Ol} is the rest mass of the lepton (antilepton), and after a series of long calculations, we get, the following forms for the functions Φ_{OE}, $\Phi_{iE}\,(i = 1, 2, 3)$, Φ_{OM}, and $\Phi_{iM}\,(i = 1, 2, 3)$:

$$\Phi_{OE}(\theta_+)=\frac{1}{8m_{op}^2\omega^2\gamma^2\Delta_0}\left(\frac{7\gamma-2}{\gamma\Delta_0}+\frac{2(1-\gamma)}{\gamma}-\frac{1-\gamma^2}{A^2\Delta_0}+\frac{1}{1-\gamma}\cdot\right.$$
$$\left.\cdot\left\{\frac{B}{\gamma\Delta_0}\left(4\gamma^2-4\gamma+2+(2\gamma-1)\Delta_0\right)-2\gamma e_0+\frac{\gamma e_T}{A}\left(3-\frac{1-\gamma^2}{A^2}\right)\right\}\right) \tag{16}$$

$$\Phi_{1E}(\theta_+)=\frac{2-\Delta_0}{4m_{ol}^2\omega^2\gamma(1-\gamma)\Delta_0^2}-\Phi_{OE}(\theta_+) \tag{17}$$

$$\Phi_{2E}(\theta_+)=\frac{1}{8m_{ol}^2\omega^2\gamma^2\Delta_0}\cdot$$
$$\cdot\left\{\frac{6-\gamma}{\gamma\Delta_0}-\frac{1-\gamma^2}{A^2\Delta_0}-\frac{4}{\gamma}+B\frac{(2\gamma-1)(2-\Delta_0)}{\gamma(1-\gamma)\Delta_0}+\frac{\gamma e_T}{A(1-\gamma)}\left(3-\frac{1-\gamma^2}{A^2}\right)\right\} \tag{18}$$

$$\Phi_{3E}(\theta_+)=\frac{2-\Delta_0}{4m_{ol}^2\omega^2\gamma(1-\gamma)\Delta_0^2}-\Phi_{2E}(\theta_+) \tag{19}$$

$$\Phi_{oE}(\theta_+)=\frac{1}{4\gamma}\left(6\gamma+\frac{1}{\Delta_0}+\frac{1-\gamma^2}{\beta^2\Delta_0}+\frac{1}{1-\gamma}\cdot\right.$$
$$\left.\cdot\left\{B+2e_0\left[\frac{2\gamma^2-2\gamma+1}{\Delta_0}-\gamma(1-\gamma)\right]-\frac{\gamma e_T}{A}\left(1-\frac{1-\gamma^2}{A^2}\right)\right\}\right) \tag{20}$$

$$\Phi_{1M}(\theta_+)=\frac{2+\Delta_0}{2\gamma(1-\gamma)\Delta_0}-\Phi_{oM}(\theta_+) \tag{21}$$

$$\Phi_{2M}(\theta_+)=\frac{1}{4\gamma}\left\{2+\frac{1}{\Delta_0}+\frac{1-\gamma^2}{A^2\Delta_0}-\frac{1}{1-\gamma}\left[B-2e_0\left(\gamma+\frac{2\gamma-1}{\Delta_0}\right)+\frac{\gamma e_T}{A}\left(1-\frac{1-\gamma^2}{A^2}\right)\right]\right\} \tag{22}$$

$$\Phi_{3M}(\theta_+)=\frac{2+\Delta_0}{2\gamma(1-\gamma)\Delta_0}-\Phi_{2M}(\theta_+) \tag{23}$$

where

$$\gamma=\frac{E_+}{\omega},\ \ \Delta_0=1-\cos\theta_+,\ \ A=\sqrt{(1-\gamma)^2+2\gamma\Delta_0},\ \ B=2\ln\left[2\omega\gamma(1-\gamma)\right]$$
$$e_0=2\ln\left[2\omega(1-\gamma)\right],\ \ e_T=\ln\frac{A-\gamma+1}{A+\gamma-1}$$

If we define the degree of longitudinal polarization of lepton pairs ($l^-\,l^+$) in which l^- and l^+ have opposite polarizations, *i. e.* for which $s_+ = -s_- = \pm 1$:

$$P_L(\theta_+)=\frac{\left(d\sigma_{s_\gamma s_+ s_-}\right)_{s_+=-s_-=1}-\left(d\sigma_{s_\gamma s_+ s_-}\right)_{s_+=-s_-=-1}}{\left(d\sigma_{s_\gamma s_+ s_-}\right)_{s_+=-s_-=1}+\left(d\sigma_{s_\gamma s_+ s_-}\right)_{s_+=-s_-=-1}} \tag{24}$$

Using eqs. (9)-(15) , the formula (24) for $P_L(\theta_+)$ takes the form:

$$P_L\left(\theta_+\right)=s_\gamma P_L^E\left(\theta_+\right)\frac{1+\left(\frac{\mu_j}{Ze}\right)^2 a_j\,X_{23}\left(\theta_+\right)}{1+\left(\frac{\mu_j}{Ze}\right)^2 a_j\,X_{01}\left(\theta_+\right)} \tag{25}$$

where

$$X_{23}\left(\theta_+\right)=\frac{\Phi_{2M}\left(\theta_+\right)-\Phi_{3M}\left(\theta_+\right)}{\Phi_{2E}\left(\theta_+\right)-\Phi_{3E}\left(\theta_+\right)} \tag{26}$$

$$\mathrm{X}_{01}\left(\theta_+\right)=\frac{\Phi_{OM}\left(\theta_+\right)-\Phi_{1M}\left(\theta_+\right)}{\Phi_{OE}\left(\theta_+\right)-\Phi_{1E}\left(\theta_+\right)} \tag{27}$$

$$P_L^E\left(\theta_+\right)=\frac{\Phi_{2E}\left(\theta_+\right)-\Phi_{3E}\left(\theta_+\right)}{\Phi_{OE}\left(\theta_+\right)-\Phi_{1E}\left(\theta_+\right)} \tag{28}$$

where $P_L^E(\theta_+)$ is the degree of longitudinal polarization of pairs in which $s_+ = -s_-$ in the case of a target nucleus with zero magnetic dipole moment ($\mu_j = 0$) [17- 20]. Equation (25) for $\mu_j = 0$ reduces:

$$:P_L\left(\theta_+\right)=s_\gamma P_L^E\left(\theta_+\right) \tag{29}$$

The energy distribution cross-sections

By integrating eq. (27) over the solid angle of the (l^+) emission Ω_+, we get the following relation for the energy distribution spectrum of the longitudinally polarized ($l^- l^+$)-pairs:

$$d\sigma_{s_\gamma s_+ s_-}\left(E_+,E_-,Ze,\mu_j\right)=d\sigma^E_{s_\gamma s_+ s_-}\left(E_+,E_-\right)+d\sigma^M_{s_\gamma s_+ s_-}\left(E_+,E_-\right) \tag{30}$$

In the ultralativistic case ($\omega, E_+, E_-, \gg m_{ol}c^2$), we get the following relation for $d\sigma^E(E_+, E_-)$ and $d\sigma^M(E_+, E_-)$:

$$d\sigma^E_{s_\gamma s_+ s_-}\left(E_+,E_-\right)=\eta_E\left\{\begin{matrix}\frac{1}{3}\left[\ln 2\omega\gamma\left(1-\gamma\right)-\frac{1}{2}\right]\\ \left[\left(4\gamma^2-4\gamma-3\right)-s_+s_-\left(2\gamma-1\right)^2+s_\gamma s_+\left(4\gamma-1\right)+s_\gamma s_-\left(3-4\gamma\right)\right]\end{matrix}\right\} \tag{31}$$

$$d\sigma^M_{s_\gamma s_+ s_-}\left(E_+,E_-\right)=2m_{ol}^2\left(\frac{\mu_j}{Ze}\right)^2\alpha_j\eta_E\,\cdot$$

$$\left\{\begin{matrix}\cdot\left\{\ln 2\omega\gamma\left(1-\gamma\right)+\left(2\gamma^2-2\gamma+1\right)\left(\ln 2\omega\gamma\right)\left[\ln 2\omega\left(1-\gamma\right)\right]\right\}-\\ -s_+s_-\left\{1-\ln 2\omega+\left(2\gamma^2-2\gamma+1\right)\left(\ln 2\omega\gamma\right)\left[\ln 2\omega\left(1-\gamma\right)\right]\right\}+\\ s_\gamma s_+\left\{\ln 2\omega\left(1-\gamma\right)+\left(2\gamma-1\right)\left(\ln 2\omega\gamma\right)\left[\ln 2\omega\left(1-\gamma\right)\right]-\left(1-\gamma\right)\right\}+\\ +s_\gamma s_-\left\{\ln 2\omega-\left(2\gamma-1\right)\left(\ln 2\omega\gamma\right)\left[\ln 2\omega\left(1-\gamma\right)\right]-\gamma\right\}\end{matrix}\right\} \tag{32}$$

where

$$\eta_E = \alpha\left(\frac{Ze^2}{m_{ol}}\right)^2 \frac{dE_+}{\omega}, \quad \alpha = \frac{1}{137}$$

Equations (31) and (32) represents the electric and magnetic energy distribution cross-section of the photoproduction of longitudinal polarized, taking the correlations between the spins s_γ, s_+, s_- of the photon, lepton, and antilepton *i. e.* terms proportional to $s_\gamma s_-$, $s_\gamma s_+$, and s_+s_- into accounts.

If we neglect the spin states of photons and leptons, we get the following formulas for the photoproduction in the unpolarized case [21, 22]:

$$d\sigma^E\left(E_+,E_-\right) = \eta_E\left\{\frac{1}{3}\left[\ln 2\omega\gamma\left(1-\gamma\right)-\frac{1}{2}\left(4\gamma^2-4\gamma-3\right)\right]\right\} \tag{33}$$

$$d\sigma^E\left(E_+,E_-\right) = 2m_{ol}^2\left(\frac{\mu_j}{Ze}\right)\alpha_j\eta_E\left\{\ln 2\omega\gamma\left(1-\gamma\right)+\left(2\gamma^2-2\gamma+1\right)\left(\ln 2\omega\gamma\right)\left[\ln 2\omega\left(1-\gamma\right)\right]\right\} \tag{34}$$

Results and discussion

Applying the eq. (30) for the energy distribution cross-section of the leptonic (e^-e^+, $\mu^-\mu^+$, $\tau^-\tau^+$)-pair production the two stable nuclei Na_{11}^{23} and Al_{13}^{27} at incident photon energies ω = 500, 2000, 3500, 5000, 6500, 8000, 100000 Mev, we get the following results which tabulated in tabs. 1 and 2:

Table 1. For Na_{11}^{23}-nucleus

	Na_{11}^{23}					
Energy [Mev]	*dBHp*			*d2Mp*		
	e^-e^+-pair	$\mu^-\mu^+$-pair	$\tau^-\tau^+$-pair	e^-e^+-pair	$\mu^-\mu^+$-pair	$\tau^-\tau^+$-pair
500	$1.09573\cdot10^{-33}$	$2.2858\cdot10^{-31}$	$3.81038\cdot10^{-30}$	$1.04058\cdot10^{-48}$	$2.48241\cdot10^{-46}$	$3.21199\cdot10^{-45}$
2000	$2.73931\cdot10^{-34}$	$5.7145\cdot10^{-32}$	$9.52595\cdot10^{-31}$	$2.73842\cdot10^{-49}$	$4.32369\cdot10^{-47}$	$5.98767\cdot10^{-46}$
3500	$1.56532\cdot10^{-34}$	$3.26543\cdot10^{-32}$	$5.4434\cdot10^{-31}$	$1.61419\cdot10^{-49}$	$2.57369\cdot10^{-47}$	$3.59325\cdot10^{-46}$
5000	$1.09573\cdot10^{-34}$	$2.2858\cdot10^{-32}$	$3.81038\cdot10^{-31}$	$1.15015\cdot10^{-49}$	$1.84376\cdot10^{-47}$	$2.58559\cdot10^{-46}$
6500	$8.42866\cdot10^{-35}$	$1.75831\cdot10^{-32}$	$2.93106\cdot10^{-31}$	$8.95769\cdot10^{-50}$	$1.4413\cdot10^{-47}$	$2.02729\cdot10^{-46}$
8000	$6.84829\cdot10^{-35}$	$1.42862\cdot10^{-32}$	$2.38149\cdot10^{-31}$	$7.34768\cdot10^{-50}$	$1.18557\cdot10^{-47}$	$1.67136\cdot10^{-46}$
10000	$5.47863\cdot10^{-35}$	$1.1429\cdot10^{-32}$	$1.90519\cdot10^{-31}$	$5.93708\cdot10^{-50}$	$9.60749\cdot10^{-48}$	$1.35758\cdot10^{-46}$

Table 2. For Al_{13}^{27}-nucleus

	Al_{13}^{27}					
Energy [Mev]	*dBHp*			*d2Mp*		
	e^-e^+-pair	$\mu^-\mu^+$-pair	$\tau^-\tau^+$-pair	e^-e^+-pair	$\mu^-\mu^+$-pair	$\tau^-\tau^+$-pair
500	$1.53039\cdot10^{-33}$	$3.19256\cdot10^{-31}$	$5.32194\cdot10^{-30}$	$2.22066\cdot10^{-48}$	$6.6886\cdot10^{-46}$	$8.65451\cdot10^{-45}$
2000	$3.82598\cdot10^{-34}$	$7.981141\cdot10^{-32}$	$1.33048\cdot10^{-30}$	$7.37849\cdot10^{-49}$	$1.16499\cdot10^{-46}$	$1.61334\cdot10^{-45}$
3500	$2.18628\cdot10^{-34}$	$4.5608\cdot10^{-32}$	$7.60277\cdot10^{-31}$	$4.34933\cdot10^{-49}$	$6.93464\cdot10^{-47}$	$9.68177\cdot10^{-46}$
5000	$1.53039\cdot10^{-34}$	$3.19256\cdot10^{-32}$	$5.32194\cdot10^{-31}$	$3.09901\cdot10^{-49}$	$4.9679\cdot10^{-47}$	$6.9667\cdot10^{-46}$
6500	$1.17723\cdot10^{-34}$	$2.45582\cdot10^{-32}$	$4.0938\cdot10^{-31}$	$2.41359\cdot10^{-49}$	$3.88349\cdot10^{-47}$	$5.4624\cdot10^{-46}$
8000	$9.56496\cdot10^{-35}$	$1.99535\cdot10^{-32}$	$3.32621\cdot10^{-31}$	$1.97978\cdot10^{-49}$	$3.19443\cdot10^{-47}$	$4.50337\cdot10^{-46}$
10000	$7.65197\cdot10^{-35}$	$1.59628\cdot10^{-32}$	$2.66097\cdot10^{-31}$	$1.59971\cdot10^{-49}$	$2.58868\cdot10^{-47}$	$3.65792\cdot10^{-46}$

From these tables, we can get the results:

- The values of energy cross-section decreases regularly with increasing the values of the incident photon energy in the two cases of Na_{11}^{23} and Al_{13}^{27} nuclei.
- The values of energy cross-sections for Al_{13}^{27} nucleus are larger than that for Na_{11}^{23} nucleus for all values of energy and for the three types of pairs, for example:

– at ω = 500 Mev:

For Al_{13}^{27}: $d\sigma_{e^-e^+}= 1.53039 \cdot 10^{-33}$, $d\sigma_{\mu^-\mu^+}= 3.19256 \cdot 10^{-31}$, $d\sigma_{\tau^-\tau^+}= 5.32194 \cdot 10^{-30}$

For Na_{11}^{23}: $d\sigma_{e^-e^+}= 1.09573 \cdot 10^{-33}$, $d\sigma_{\mu^-\mu^+}= 2.2858 \cdot 10^{-31}$, $d\sigma_{\tau^-\tau^+}= 3.81038 \cdot 10^{-30}$

– at ω = 6500 Mev:

For Al_{13}^{27}: $d\sigma_{e^-e^+}= 1.17723 \cdot 10^{-34}$, $d\sigma_{\mu^-\mu^+}= 2.45582 \cdot 10^{-32}$, $d\sigma_{\tau^-\tau^+}= 4.0938 \cdot 10^{-31}$

For Na_{11}^{23}: $d\sigma_{e^-e^+}= 8.42866 \cdot 10^{-35}$, $d\sigma_{\mu^-\mu^+}= 1.75831 \cdot 10^{-32}$, $d\sigma_{\tau^-\tau^+}= 2.93106 \cdot 10^{-31}$

- The values of cross-sections for the ($\tau^- \tau^+$) pair production are larger than that for the ($\mu^- \mu^+$) pair, which are also larger than that for the (e^-e^+) pair. This result is true for both Na_{11}^{23} and Al_{13}^{27} nuclei, for example:

– at ω = 6500 Mev:

For Na_{11}^{23}: $d\sigma_{e^-e^+}= 1.04058 \cdot 10^{-48}$, $d\sigma_{\mu^-\mu^+}= 2.48241 \cdot 10^{-46}$, $d\sigma_{\tau^-\tau^+}= 3.21199 \cdot 10^{-45}$

For Al_{13}^{27}: $d\sigma_{e^-e^+}= 2.11066 \cdot 10^{-48}$, $d\sigma_{\mu^-\mu^+}= 6.6887 \cdot 10^{-46}$, $d\sigma_{\tau^-\tau^+}= 8.65451 \cdot 10^{-45}$

- For the values of energy cross-section less than 500 Mev, our calculations gives the following results for Al_{13}^{27} nucleus for example:

– at ω = 6500 Mev:

$d\sigma_{e^-e^+}= 2.41359 \cdot 10^{-49}$, $d\sigma_{\mu^-\mu^+}= 3.88349 \cdot 10^{-47}$, $d\sigma_{\tau^-\tau^+}= 5.4624 \cdot 10^{-46}$

– at ω = 10000 Mev:

$1.59971 \cdot 10^{-49}$, $d\sigma_{\mu^-\mu^+}= 2.58868 \cdot 10^{-47}$, $d\sigma_{\tau^-\tau^+}= 3.65792 \cdot 10^{-46}$

From these figures we can show that:

– the values of cross-section is less than that at ω = 500 Mev.
– the values of cross-sections are decreases with increasing energy from 500-1000 *Mev*.
– the values of cross-sections for ($\tau^- \tau^+$) pair production are longer than that for the ($\mu^- \mu^+$) pair and consequently for ($e^- e^+$)- pair.

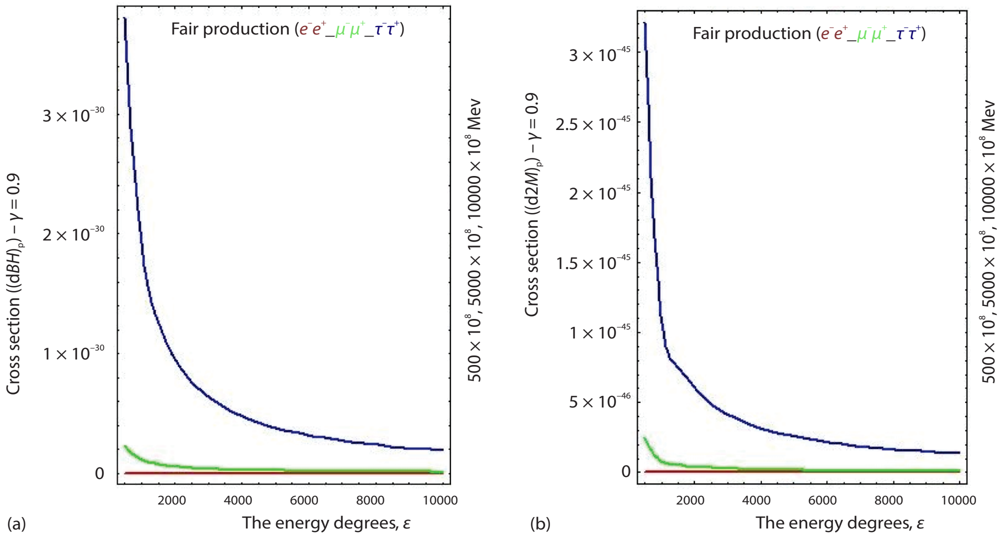

Figure 1.

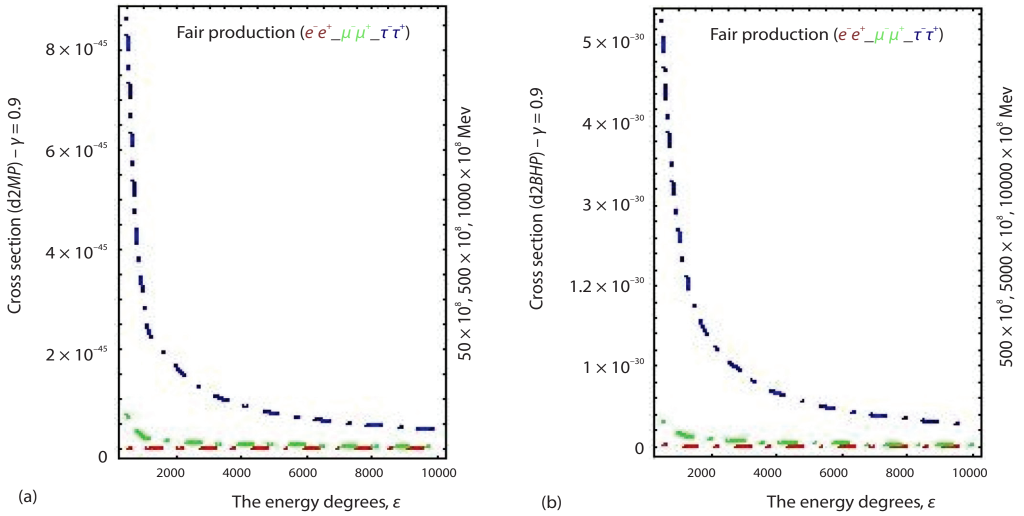

Figure 2.

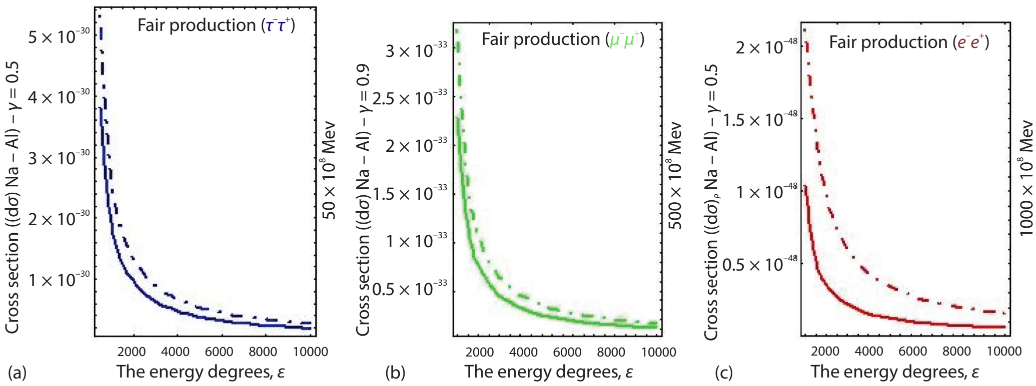

Figure 3.

From fig. 1(a) and 1(b) obtained for Na^{23}_{11}-Al^{27}_{13} nucleus, we see that the production of ($\tau^-\,\tau^+$)-pair is larger than that for the ($\mu^-\,\mu^+$) or ($e^-\,e^+$)-pairs in the two cases with studying polarization. The figs. 2(a) and 2(b) obtained for Al^{27}_{13}, we see the same result.

The comparisons between the production of the three pairs in the two cases of studying and ignoring polarizations are shown in figs. 3(a)-3(c). In fig. 3(a), the connected red line show the production of the ($e^-\,e^+$)-pair at different energies with the polarization in Al^{27}_{13}, while the disconnected red line show the ($e^-\,e^+$)-pair production with studying the polarization in Na^{23}_{11}. In figs. 3(b) and 3(c), the connected green and blue lines show the production of ($\mu^-\,\mu^+$) and ($\tau^-\,\tau^+$)-pairs, respectively in two nuclei, while the disconnected green and blue lines show the ($\mu^-\,\mu^+$) and ($\tau^-\,\tau^+$)-pairs production with studying the polarization.

Conclusion

In summary, we have examined the effect of the nuclear magnetic moment on the photo production of ($e^-\,e^+$), ($\mu^-\,\mu^+$), ($\tau^-\,\tau^+$) *i. e.* ($l^-\,l^+$), lepton pairs. Also, we investigated the role of the nuclear magnetic moment on the angular and energy distribution of the antilepton $l^+(e^+, \mu^+, \tau^+)$. The obtained results have confirmed that the energy cross-section decreases with increasing the energy of incident photon, and that the values of cross-sections in the case of studying the polarizations of photon and particles are larger than that in Al_{13}^{27}, and finally that the values of the cross-sections of $(\tau^- \tau^+)$ pair production are larger than that for $(\mu^- \mu^+)$ and $(e^- e^+)$-pairs for different energies and for the two nuclei Na_{11}^{23} and Al_{13}^{27}.

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