

SURFACE TEMPERATURE AND FREE-STREAM VELOCITY OSCILLATION EFFECTS ON MIXED CONVECTION SLIP FLOW FROM SURFACE OF A HORIZONTAL CIRCULAR CYLINDER

by

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The present phenomena address the slip velocity effects on mixed convection flow of electrically conducting fluid with surface temperature and free stream velocity oscillation over a non-conducting horizontal cylinder. To remove the difficulties in illustrating the coupled PDE, the primitive variable formulation for finite difference technique is proposed to transform dimensionless equations into primitive form. The numerical simulations of coupled non-dimensional equations are examined in terms of fluid slip velocity, temperature, and magnetic velocity which are used to calculate the oscillating components of skin friction, heat transfer, and current density for various emerging parameters magnetic force parameter; ζ , mixed convection parameter; λ , magnetic Prandtl number; γ , Prandtl number, and slip factor, S_L . It is observed that the effect of slip flow on the non-conducting cylinder is reduced the fluid motion. A minimum oscillating behavior is noted in skin friction at each position but maximum amplitude of oscillation in heat transfer is observed at each position $\alpha = \pi/4$ and $2\pi/3$. It is further noticed that a fluid velocity increases sharply with the impact of slip factor on the fluid-flow mechanism. Moreover, due to frictional forces with lower magnitude between viscous layers, the rise in Prandtl number leads to decrease in skin friction and heat transfer which is physically in good agreement.

Key words: *mixed-convection, oscillatory slip-flow, non-conducting cylinder, slip velocity, current density*

Introduction

The fluid slip is an interesting problem in fluid mechanics due to the absence of the surface effect, fundamental data are still lacking, and it is not sufficiently grasped in the experimental study. The fluid with slip-boundary has various applications in modern technology like artificial heart valves polishing and internal heart cavities. The effects of magnetic field on heat and fluid mechanisms have great interest in molten metal purification, macro and micro-electronic devices, metallurgy, and geophysical systems. Eshghy *et al.* [1] treated a free-convection problem on vertical surface with longitudinal oscillations effects analytically. They examined a

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slight decrease in the average coefficients of heat transfer in the laminar regime. The physical behavior of heat transfer in unsteady boundary-layer problem past a heated vertical plate under surface temperature oscillations has been investigated theoretically [2]. Chawla [3] analyzed an oscillatory flow problem of electrically conducting fluid along a flat-plate with harmonic oscillation effects by using Karman-Pohlhausen method analytically. Pedley [4] discussed free stream velocity effects on 2-D boundary-layers flow which oscillates without reversing numerically. Mahuri and Gupta [5] presented the free-convective flow past a plate with periodically varying surface-temperature of small fluctuations mathematically. They found that inner or shear layer with double deck structure and solution far down-stream exposes double boundary-layers when Prandtl number is unity. Verma [6] analyzed the asymptotic solution for unsteady flow on a semi-infinite flat surface with the effects of small fluctuations in surface temperature, where free stream is supposed to be at rest. An unsteady flow on a stationary sphere under the influence of free stream velocity with small fluctuations at finite Reynolds number has been studied analytically [7]. They observed that at low frequency, the classical Stokes solution does not depict the unsteady drag behavior correctly.

Eckerle and Awad [8] examined the detail of horseshoe vertex formation along a cylinder with flow parameters that affect the flow pattern in front of the cylinder. They measured pressures on cylindrical surface and size of separation region on the end wall of cylinder surface. Mei and Adrian [9] scrutinized the corresponding asymptotic solution for unsteady flow around a fixed sphere at small Reynolds number with low frequency oscillations in free-stream velocity theoretically. Klausner [10] obtained the flow phenomena over a spherical bubble by considering small oscillations in free-stream velocity for different Reynolds number ranges. He predicted that for large Reynolds numbers with low frequency, the history force which is combined form of viscous diffusion of the vorticity and acceleration of the flow field is finite. Hosain *et al.* [11] use a linearized theory to investigate free convective flow past a plate in the influence of concentration and fluctuating surface temperature which is assumed as n power of distance measure from the leading edge. The entrained flow due to a stretching surface with partial slip by applying similarity transformations has been solved in [12]. Fan and Barber [13] examined the discretized periodic heating problems by applying the transfer matrix method analytically. They predicted that it is suitable for transient response of various sets of initial or boundary temperature. Andersson [14] considered a technical note on Newtonian slip fluid along a linearly stretching surface analytically. Cossali [15] reported the periodic convection in a steady boundary-layer flow past a semi-infinite plate analytically. Martin and Boyd [16] performed a slip-flow problem on both momentum and heat transfer in a laminar boundary-layer numerically. They concluded that the slip velocity will lead to increased heat transfer without a thermal jump-condition.

Ariel *et al.* [17] investigated the slip velocity effect on the flow characteristics in an elasto-viscous fluid over a stretching sheet analytically. Ackerberg and Phillips [18] obtained numerical and asymptotic solutions of time-mean boundary-layer on semi-infinite surface with free stream velocity and small fluctuations. They depicted that most of flow quantities approaches their asymptotic conditions far downstream through damped oscillations. Later, Ariel [19] considered 2-D, steady elasto-viscous fluid-flow problem with partial slip effects at the wall. Berg *et al.* [20] solved a slip-flow problem in capillary tubes for two-phase flow in porous media theoretically. This phenomena depicted that the flux increase due to slip depends on the equivalent capillary radius of the flow channels. Fang *et al.* [21] examined accurate solutions of electrically conducting fluid over a stretching sheet under slip conditions analytically. They noticed that an enhancement in slip parameter the wall drag-force decreases and velocity slip

increases. Abbas *et al.* [22] has been studied an oscillatory heat transfer phenomena in a viscous fluid along a stretching surface numerically. They observed that the heat transfer past a sheet to fluid becomes slow due to higher Prandtl number. Later, Martin and Boyd [23] modified a numerical problem on Falkner-Skan flow over a wedge under slip condition. The results of this phenomena show decreased velocity and momentum thickness under slip boundary-condition. Mukhopadhyay [24] considered a slip-flow mechanism for axis-symmetric flow of MHD fluid around a stretching cylinder numerically. Roy and Hossain [25] illustrated the unsteady boundary-layer features of mixed-convection flow by a vertical-wedge subjected to free-stream and surface-temperature with the magnitude of oscillations numerically. Hirschhorn *et al.* [26] analyzed the phenomena of an electrically conducting slip-flow for power law fluid over a flat-plate by using shooting technique. They found that the velocity of fluid is unchanged by variations in temperature slip-parameter. The problem on mixed convection flow under the influence of surface temperature and free stream with small amplitude of oscillations around a cylinder has been considered in [27] numerically. The impact of algebraic decay and viscous dissipation on different magnetized shapes has been proposed in [28-30]. Tiainen *et al.* [31] discussed the complexity in calculation of the boundary-layer thickness and losses in compressor blades. They used hybrid method to overcome the boundary-layer losses in centrifugal compressor. Garg *et al.* [32] examined the convective boundary-condition outcomes on unsteady MHD flow over a porous plate in slip-flow regime numerically. Recently, variable density effects on oscillating mixed convection flow around a non-conducting cylinder has been investigated by Ashraf and Fatima [33] and then Ashraf and Ullah [34]. The non-linear ODE arising from non-isothermal heat transport was solved by Kilicman *et al.* using HPM in [35] and Kadomtsev-Petviashvili and fractional KdV-Burgers and Potential KdV Equations, with existence and uniqueness results was investigated by Inc *et al.* [36] and Hashemi *et al.* [37].

Taking idea from the aforementioned literature review, it is addressed that the surface temperature oscillation and free stream velocity effects on oscillatory free-forced convective slip-flow along a horizontal non-conducting circular cylinder has not been yet examined by any researcher. Taking idea from the following [3, 16], we obtained a numerical method for previous slip-flow phenomena around a horizontal non-conducting circular cylinder under slip-velocity. We explore fluid slip-velocity, temperature and magnetic velocity which are used to conclude oscillating quantities of skin friction, fluctuating heat transfer and oscillatory current density.

The governing model and flow geometry

The 2-D boundary-layer fluid-flow phenomena around a horizontal non-conducting circular cylinder is considered. The fig. 1 represents distance along the surface is x , the y -direction is normal to the surface and the velocities u and v along the x - and y -direction. The magnetic field, H_x along the surface, H_y to the normal of surface, the temperature field is T and the external flu-

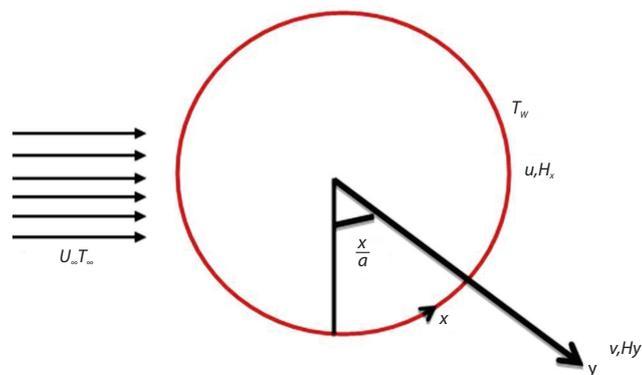


Figure 1. Non-conducting cylinder and flow geometry

id velocity is $U(x, t)$. Moreover, magnetic field intensity proceeds along normal direction of the surface of horizontal non-conducting cylinder. The dimensionless form of boundary-layer equations are given:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (1)$$

$$\frac{\partial \bar{u}}{\partial \tau} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{d\bar{U}_1}{d\tau} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \xi \left(\bar{h}_x \frac{\partial \bar{h}_x}{\partial \bar{y}} + \bar{h}_y \frac{\partial \bar{h}_x}{\partial \bar{y}} \right) + \lambda \bar{\theta} \sin \alpha \quad (2)$$

$$\frac{\partial \bar{h}_x}{\partial \bar{x}} + \frac{\partial \bar{h}_y}{\partial \bar{y}} = 0 \quad (3)$$

$$\frac{\partial \bar{h}}{\partial \tau} + \bar{u} \frac{\partial \bar{h}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{h}}{\partial \bar{y}} - \bar{h}_x \frac{\partial \bar{u}}{\partial \bar{x}} - \bar{h}_y \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{1}{\gamma} \frac{\partial^2 \bar{h}}{\partial \bar{y}^2} \quad (4)$$

$$\frac{\partial \bar{\theta}}{\partial \tau} + \bar{u} \frac{\partial \bar{\theta}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{\theta}}{\partial \bar{y}} = \frac{1}{\text{Pr}} \frac{\partial^2 \bar{\theta}}{\partial \bar{y}^2} \quad (5)$$

The dimensionalized boundary conditions:

$$\begin{aligned} \bar{u} = S_L \frac{\partial \bar{u}}{\partial \bar{y}}, \quad \bar{v} = 0, \quad \bar{h}_y = \bar{h}_x = 0, \quad \bar{\theta} = 1 + \epsilon \cos \omega \tau \quad \text{at } \bar{y} = 0 \\ \bar{u} \rightarrow 1 + \epsilon \cos \omega \tau, \quad \bar{\theta} \rightarrow 0, \quad \bar{h}_y \rightarrow 1 \quad \text{as } \bar{y} \rightarrow \infty \end{aligned} \quad (6)$$

where ξ is the magnetic force number, λ – the mixed convection parameter, γ – the magnetic Prandtl number, Pr – the Prandtl parameter, S_L – slip parameter, θ – the dimensionless fluid temperature. We take stream velocity $U(\tau) = 1 + \epsilon e^{i\omega\tau}$, with $|\epsilon| \ll 1$, where ϵ is oscillating component with small magnitude and ω – the frequency factor. The fluid velocity, magnetic velocity and temperature components u , v , h_x , h_y , and θ are in the sum form of steady and unsteady equations:

$$\begin{aligned} \bar{u} = u_s + \epsilon u_t e^{i\omega\tau}, \quad \bar{v} = v_s + \epsilon v_t e^{i\omega\tau}, \quad \bar{h}_x = h_{xs} + \epsilon h_{xt} e^{i\omega\tau} \\ \bar{h}_y = h_{ys} + \epsilon h_{yt} e^{i\omega\tau}, \quad \bar{\theta} = \theta_s + \epsilon \theta_t e^{i\omega\tau} \end{aligned} \quad (7)$$

We can substitute (7) into eqs. (1)-(5) along with boundary conditions (6), we separate steady and unsteady part by collecting orders $O(\epsilon^0)$ and $O(\epsilon e^{i\omega\tau})$. Then by following [33], we use primitive variable formulation obtain primitive form of steady part and unsteady part in terms of real and imaginary parts.

Computational analysis

The system of primitive dimensionless governing steady and unsteady coupled equations is discretized numerically by applying finite-difference scheme. The numerical results of obtained algebraic equations with unknown variable U , V , θ , and φ then be solved by tri-diagonal matrix form the Gaussian-elimination technique for unknown variables. The eq. (8) is used to obtain results of oscillating components of skin friction τ_w , heat transfer q_w and current density j_w at various positions of a horizontal non-conducting circular cylinder:

$$\begin{aligned}
 \tau_w &= \left(\frac{\partial U}{\partial Y} \right)_{y=0} + \epsilon |A_s| \cos(\omega t + \alpha_s), \quad q_w = \left(\frac{\partial \theta}{\partial Y} \right)_{y=0} + \epsilon |A_t| \cos(\omega t + \alpha_t) \\
 j_w &= \left(\frac{\partial \phi}{\partial Y} \right)_{y=0} + \epsilon |A_m| \cos(\omega t + \alpha_m), \quad A_s = (u_1^2 + u_2^2)^{1/2}, \quad A_t = (\theta_1^2 + \theta_2^2)^{1/2} \\
 A_m &= (\phi_{x1}^2 + \phi_{x2}^2)^{1/2}, \quad \alpha_s = \tan^{-1} \left(\frac{u_2}{u_1} \right), \quad \alpha_t = \tan^{-1} \left(\frac{\theta_2}{\theta_1} \right), \quad \alpha_m = \tan^{-1} \left(\frac{\phi_{x2}}{\phi_{x1}} \right)
 \end{aligned} \tag{8}$$

where A_s , A_t , and A_m are amplitudes while, α_s , α_t , and α_m are phase angles to calculate transient rate of skin-friction, oscillating heat transfer and present density.

Results and discussion

Slip velocity, temperature and magnetic field profiles to check the accuracy of the obtained results by boundary conditions

The impact of slip parameter on non-dimensional slip velocity, fluid temperature and magnetic field around two appropriate positions $\alpha = \pi/4$ and $2\pi/3$ of non-conducting cylinder are presented in figs. 2(a)-2(c). From these figures, it is depicted that the wall slip-velocity amplitude increases with increase of slip-factor $S_L = 35.0$ at angle $\alpha = \pi/4$ and asymptotically approaches to given boundary condition. The fluid temperature distribution shows similar behavior at each position for three values of slip parameter, S_L , but magnetic field profile found to be in good agreement for large magnetic-Prandtl number, $\gamma = 3.0$. It is also observed that the effect of slip on the non-conducting cylinder is to reduce the boundary-layer displacement thickness. It is also depicted that the convective heat transfer accomplished by temperature dependent velocity. Whereas maximum slip parameter, permitting the more fluid to slip around the cylinder at certain heights. Figures 3(a)-3(c) are drafted for slip velocity, temperature field, and magnetic profile against three choice of mixed convection parameter $\lambda = 5, 0, 20.0$, and 40 . for constant slip factor, $S_L = 6.1$. The amplitude of oscillating slip velocity keeps on damping out due to large value of slip parameter, $S_L = 6.1$. The velocity overshoot up to certain heights and then asymptotically approach the given boundary condition. These figures present the oscillatory behavior of slip flow around two positions of non-conducting cylinder. Figure 3(a) shows maximum oscillating amplitude of slip velocity at $\alpha = 2\pi/3$ with the increase of mixed-convection parameter, λ . Effects of mixed-convection parameter on fluid temperature are presented in fig. 3(b). The temperature is found to decrease with the increasing λ at $\alpha = 2\pi/3$ position and thermal boundary-layer decreases with the increase of λ . The fluid temperature is found to be increase with the decreasing λ at $\alpha = \pi/4$ position and thermal boundary-layer increases and showed good variations at each station. The magnetic velocity found to be similar at each position and approaches to given boundary condition with good agreement. The obtained results confirm the loss of self-similarity of flow. The boundary-layer thickness decreases due to positive value of slip flow velocity normal to the surface of cylinder. Figures 4(a)-4(c) address the effects of magnetic Prandtl number, γ , on velocity for non-conducting cylinder at positions $\alpha = \pi/4$ and $2\pi/3$ while other parameters are kept constant. The magnetic Prandtl number has an extensive effect on the results. It is also concluded that higher restrictions to the fluid in the presence of magnetic field, which reduces the fluid velocity. This feature prevails up to certain heights due to slip condition and maximum amplitude is obtained in velocity and temperature profiles showed small variations at each position in figs. 4(a) and 4(b). It is obvious that the presence of magnetic field causes maximum behavior in magnetic profile at $\alpha = 2\pi/3$ and ob-

tained good variations in fig. 4(c). Figures 5(a)-5(c) depict the impact of Prandtl number on slip velocity, temperature field and magnetic profile under slip condition around non-conducting cylinder at two positions $\alpha = \pi/4$ and $2\pi/3$ while other pertinent parameters are kept constant. The slip velocity overshoot is noted for $Pr = 0.1$ at $\alpha = 2\pi/3$ position in fig. 5(a). The obtained result prevails up to certain heights and then asymptotically approaches to given boundary-condition. The maximum variations are obtained in temperature and magnetic velocity increase at $\alpha = 2\pi/3$ position for $Pr = 0.1$ in figs. 5(b) and 5(c). The analysis of given solutions found to be in good agreement for three values of Prandtl number. The substantial effect of magnetic force parameter, ζ , on transient slip flow around a non-conducting circular horizontal cylinder has been displayed in figs. 6(a)-6(c). The fluid slip-velocity is found to be maximum at position $\alpha = 2\pi/3$ due to slip parameter $S_L = 0.01$ in fig. 6(a). Temperature profile illustrates excellent variations at both positions $\alpha = \pi/4$ and $2\pi/3$ in fig. 6(b) for small force parameter $\zeta = 6.0$ but slightly change occurred at position $\alpha = \pi/4$. The impact of physical pertinent parameters on oscillatory skin friction, rate of heat transfer and oscillatory current density for slip flow around two positions $\alpha = \pi/4$ and $2\pi/3$ of horizontal non-conducting circular cylinder are deliberated in this section.

Transient shape of fluctuating skin friction, heat transfer and current density

Figures 7(a)-7(c) shows the behavior of magnetic force parameter, ζ , by keeping slip velocity on magnitude of fluctuations in skin friction, rate heat transfer and current density around two positions $\alpha = \pi/4$ and $2\pi/3$ of non-conducting cylinder. The oscillating

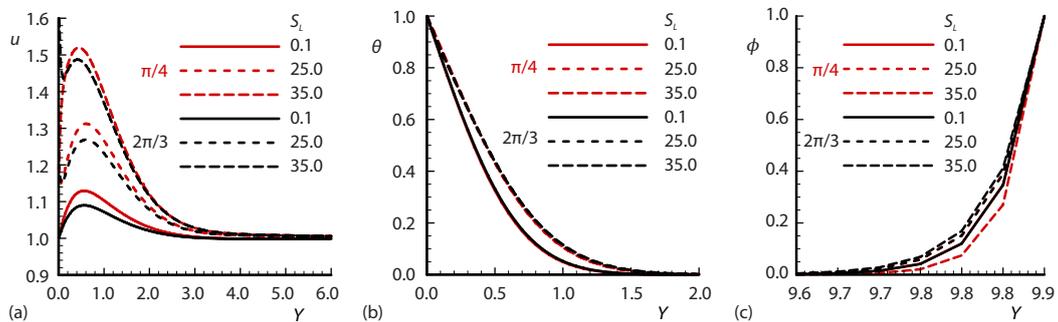


Figure 2. The geometrical profiles for (a) slip-velocity, (b) temperature, and (c) magnetic field at positions $\alpha = \pi/4$ and $2\pi/3$ for three choice of slip parameter $S_L = 0.1, 25.0,$ and 35.0 while $\gamma = 3.0, \zeta = 0.1, Pr = 7.0,$ and $\lambda = 0.005$

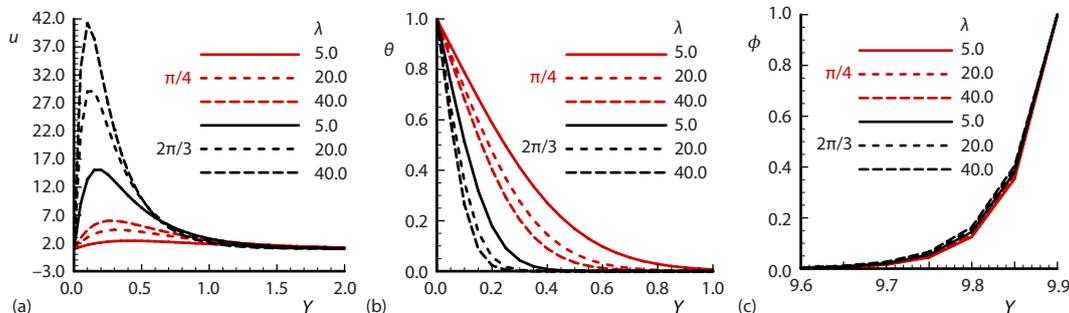


Figure 3. The geometrical profiles for (a) slip-velocity, (b) temperature, and (c) magnetic field at positions $\alpha = \pi/4$ and $2\pi/3$ with three choice of mixed-convection parameter $\lambda = 5.0, 20.0,$ and 40.0 while $\gamma = 3.0, \zeta = 0.1, Pr = 7.0,$ and $S_L = 6.1$

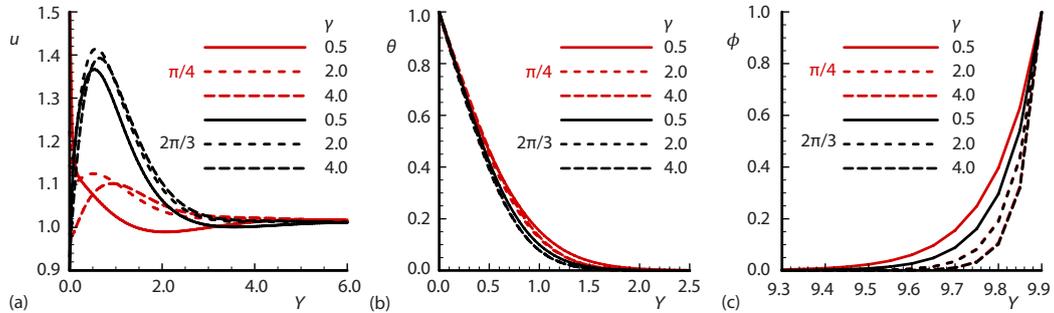


Figure 4. The geometrical profiles for (a) slip-velocity, (b) temperature, and (c) magnetic field at positions $\alpha = \pi/4$ and $2\pi/3$ with three choice of magnetic Prandtl-number $\gamma = 0.5, 2.0,$ and 4.0 while $S_L = 7.1, \zeta = 8.0, Pr = 0.1,$ and $\lambda = 0.01$

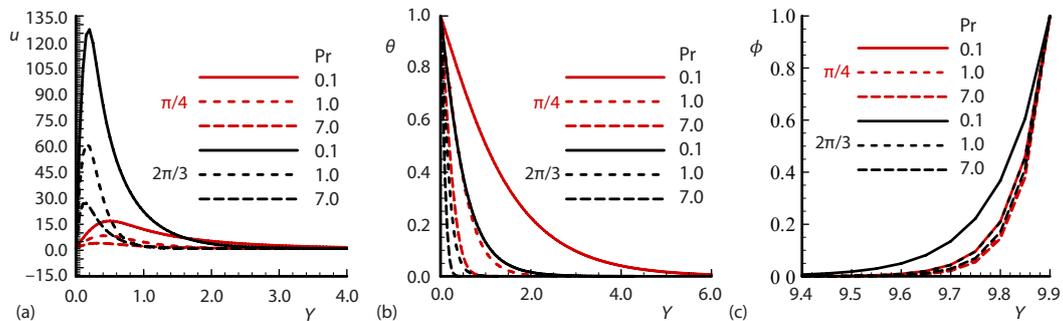


Figure 5. The geometrical profiles for (a) slip-velocity, (b) temperature, and (c) magnetic field at positions $\alpha = \pi/4$ and $2\pi/3$ with three choice of Prandtl number $Pr = 0.1, 1.0,$ and 7.0 while $\gamma = 2.5, \zeta = 0.2, S_L = 0.1,$ and $\lambda = 17.0$

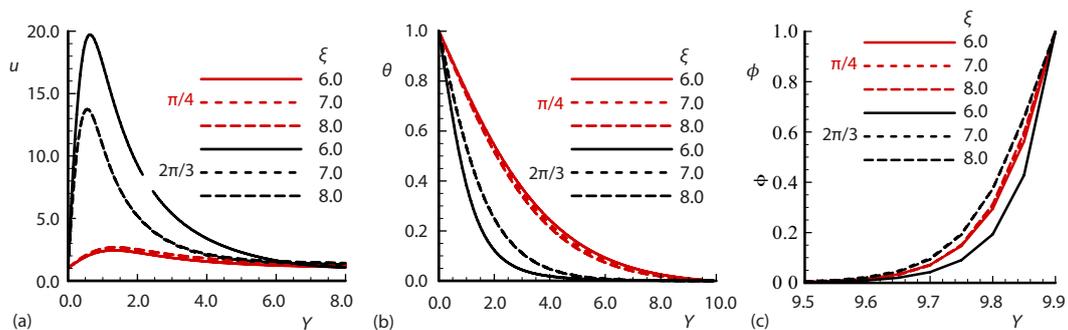


Figure 6. The geometrical profiles for (a) slip-velocity, (b) temperature, and (c) magnetic field at positions $\alpha = \pi/4$ and $2\pi/3$ with three choice of magnetic-force parameter $\zeta = 6.0, 7.0,$ and 8.0 while $\gamma = 1.0, S_L = 0.01, Pr = 0.1,$ and $\lambda = 0.2$

magnitude skin-friction increases as magnetic force parameter increases at $\alpha = 2\pi/3$ position and a good response of amplitude of oscillation is noted in fig. 7(a). A maximum oscillating heat transfer behavior has been observed for maximum value of $\zeta = 2.0$ at $\alpha = 2\pi/3$ position in fig. 7(b). Figure 7(c) depicts the same oscillating response in current density at each position of cylinder. The impact of mixed convection parameter, λ , for magnitude of oscillating skin friction, rate of heat transfer and oscillatory current density has been plotted

in figs. 8(a)-8(c). A minimum oscillating behavior is noted in skin friction at each position in fig. 8(a) but maximum behavior of oscillation in heat transfer is observed at each position $\alpha = \pi/4$ and $2\pi/3$ in figs. 8(b). Figure 8(c) shows similar and good oscillating behavior is also noted in current density. Figures 9(a)-9(c) present the geometrical interpretation of the amplitude of skin friction, oscillating heat transfer and current density for three choice of Prandtl number with higher value of $\lambda = 20.1$ along non-conducting cylinder. Very low amplitude of skin friction is examined in fig. 9(a) but fig. 9(b) plotted better oscillations in heat transfer against Prandtl number. The maximum and highest amplitude of oscillation is deliberated in current density at each position of non-conducting cylinder $\alpha = \pi/4$ and $2\pi/3$ in fig. 9(c). It is examined that the heat transfer increases as Prandtl number increases but skin friction decreases at each position. The physical behavior of magnetic Prandtl number, γ , for magnitude of oscillatory slip flow along two positions of cylinder has been drafted in figs. 10(a)-10(c). The magnitude of oscillation in skin friction is enhancing and good variation at each position for γ is noted in fig. 10(a). The highest fluctuating response is noted as γ decreases at $\alpha = 2\pi/3$ position in fig. 10(b). A similar trend is calculated for every value of γ but maximum fluctuation is pointed as magnetic Prandtl number increases in fig. 10(c). Due to slip flow, the frictional resistance between the viscous fluid and the surface is eliminated and the fluid velocity boosting the fluctuation in heat transfer and skin friction at each position. It can be predicted that an increase in amplitude of skin friction corresponds to a thinning of the velocity boundary-layer. The similar behavior of fluctuating magnitude of skin friction, oscillating rate of heat transfer and magnetic current density

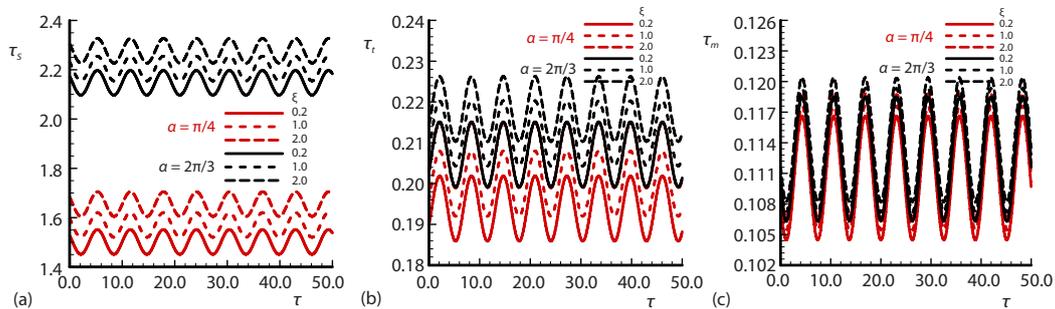


Figure 7. The geometrical profiles of (a) skin-friction, (b) heat transfer, and (c) current-density at $\alpha = \pi/4$ and $2\pi/3$ positions with three choice of magnetic-force parameter $\zeta = 0.2, 1.0,$ and 2.0 where other parameters $\gamma = 0.01, S_L = 10.0, Pr = 0.1,$ and $\lambda = 1.2$

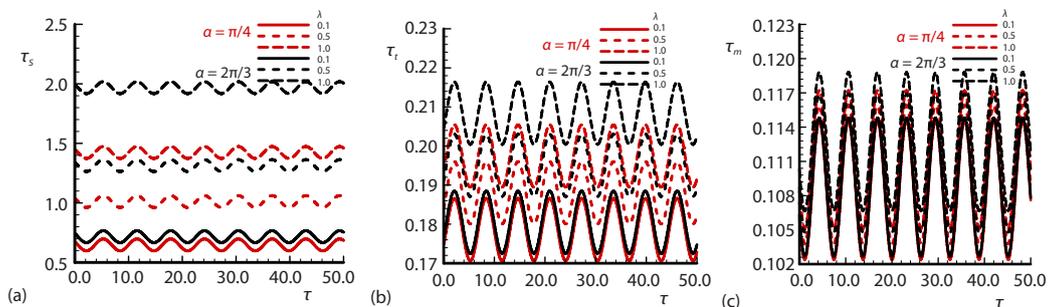


Figure 8. The geometrical profiles of (a) skin-friction, (b) heat transfer, and (c) current-density at $\alpha = \pi/4$ and $2\pi/3$ positions with three choice of mixed convection parameter $\lambda = 0.1, 0.5,$ and 1.0 where other parameters $\gamma = 0.01, \zeta = 1.1, Pr = 0.1,$ and $S_L = 3.0$

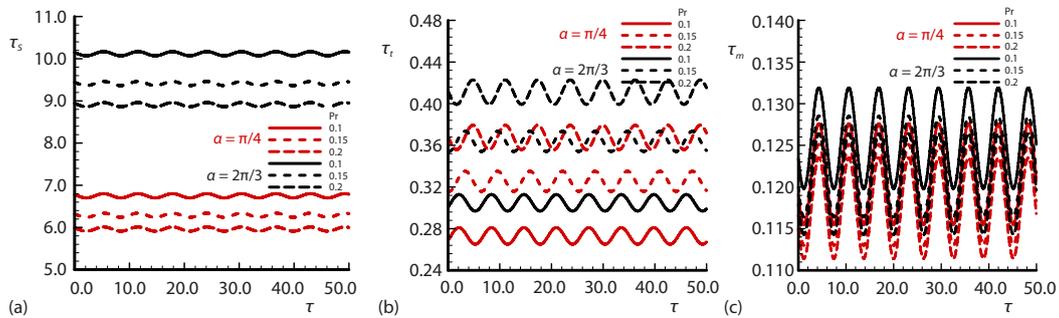


Figure 9. The geometrical profiles of (a) skin-friction, (b) heat transfer, and (c) current-density at $\alpha = \pi/4$ and $2\pi/3$ positions with three choice of Prandtl number $Pr = 0.1, 0.15,$ and 0.2 and where other parameters $\gamma = 0.01, \zeta = 0.8, S_L = 1.0,$ and $\lambda = 20.1$

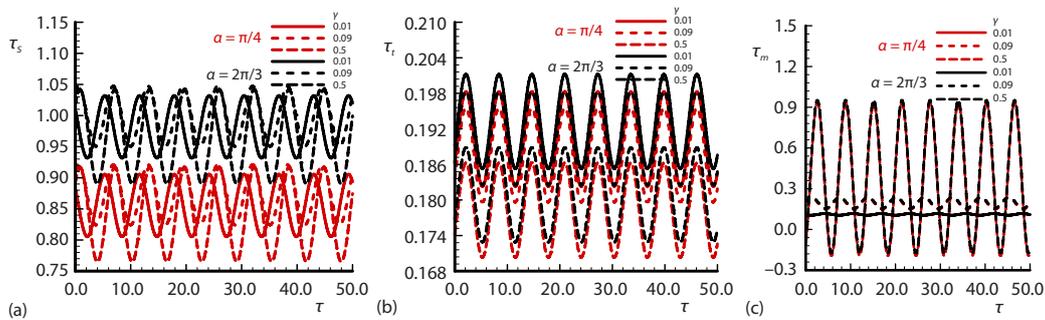


Figure 10. The geometrical profiles of (a) skin-friction, (b) heat transfer, and (c) current-density at $\alpha = \pi/4$ and $2\pi/3$ positions with three choice of magnetic-Prandtl number $\gamma = 0.01, 0.09,$ and 0.5 where other parameters $S_L = 5.0, \zeta = 2.1, Pr = 0.1,$ and $\lambda = 0.2$

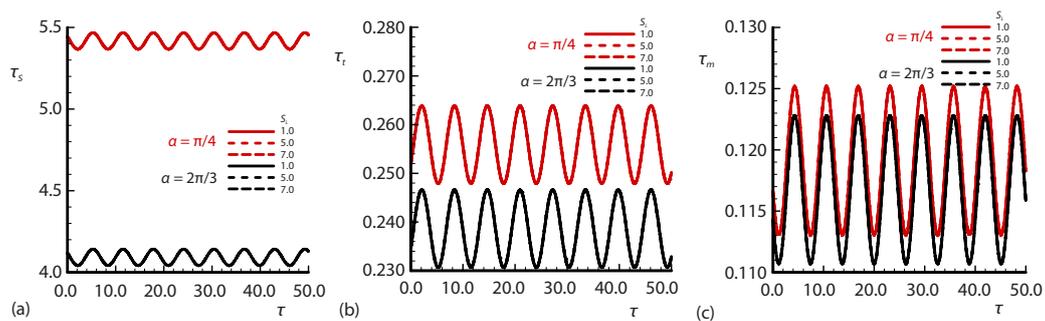


Figure 11. The geometrical profiles of (a) skin-friction, (b) heat transfer, and (c) current-density at $\alpha = \pi/4$ and $2\pi/3$ positions with three choice of slip parameter $S_L = 1.0, 5.0,$ and 7.0 where other parameters $\gamma = 0.01, \zeta = 0.2, Pr = 0.1,$ and $\lambda = 5.0$

for different values of slip parameter S_L at two positions $\alpha = \pi/4$ and $2\pi/3$ has been plotted in figs. 11(a)-11(c). The maximum fluctuating amplitude is examined for heat transportation and fluctuating current density at each position, but similar trend is observed for each value of slip parameter in figs. 11(b) and 11(c). It is concluded that due to slip condition, the fluid velocity and momentum thickness decreases as slip parameter increases.

Conclusion

The dimensionless equations are dimensionless MHD boundary-layer equations which illustrate the effect of surface temperature oscillation and free-stream velocity on unsteady mixed convective. The physical interpretation for slip factor and other governing parameters at angles $\alpha = \pi/4$ and $2\pi/3$ is graphically presented and discussed. It is examined that the wall slip-velocity amplitude increases with the increase of slip-factor $S_L = 35.0$ at angle $\alpha = \pi/4$ and asymptotically approaches to given boundary condition. Increase in slip parameter, permitting the more fluid to slip around the cylinder at certain heights. It is noted that the presence of magnetic field causes maximum behavior in magnetic profile at $\alpha = 2\pi/3$ and obtained good variations for each value of γ in fig. 4(c). The magnitude of oscillating skin-friction increases as magnetic force parameter increases at $\alpha = 2\pi/3$ position and a good response of amplitude of oscillation is noted in fig. 7(a). Due to slip flow, the frictional resistance between the viscous fluid and the surface is eliminated and the fluid velocity boosting the fluctuation in heat transfer and skin friction at each position. The maximum fluctuating amplitude is examined for heat transportation and fluctuating current density at each position, but similar trend is observed for each value of slip parameter.

Nomenclature

H_x, H_y – magnetic velocities along x - and y -direction, [Tesla]
 Pr – Prandtl number
 S_L – slip factor
 T – fluid temperature, [K]
 T_∞ – ambient-temperature, [K]
 u, v – velocity along x - and y -direction, [ms^{-1}]

Greek symbols

α – thermal diffusivity, [m^2s^{-1}]
 γ – magnetic-Prandtl number
 θ – dimensioned temperature
 λ – mixed convection parameter
 ζ – magnetic force parameter
 τ – Shearing stress, [Pa]

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