NATURAL-CONVECTION OF NEWTONIAN FLUIDS BETWEEN TWO CONCENTRIC CYLINDERS OF A SPECIAL CROSS-SECTIONAL FORM

by

Houssem LAIDOUDI[a*], Mustapha HELMAOUI[a], Mohamed BOUZIT[a], and Abdellah GHENAIM[b]

[a] Laboratoire des Sciences et Ingénierie Maritime (LSIM), Faculty of Mechanical Engineering, El -Menaouer, Oran, Algeria
[b] Laboratoire de Mécanique et Environnement ICUBE/INSA, Institut National des Sciences Appliquées, Strasbourg, France

Original scientific paper
https://doi.org/10.2298/TSCI200201190L

In this paper, we performed a numerical simulation of natural-convection of Newtonian fluids between two cylinders of different cross-sectional form. The inner cylinder is supposed to be hot and the outer cylinder is assumed to be cold. The diameter of inner cylinder to the diameter of outer cylinder defines the radii ratio (RR = 2.5). The governing equations describing the physical behavior of fluid-flow and heat transfer are solved using finite volume method. The effects of Prandtl number (Pr = 0.71-100), Rayleigh number (Ra = 10^3-10^5), and inclination angle of inner cylinder (ω = 0-80°) on streamlines, isotherms, and dimensionless velocity are presented and discussed. Also, the mean average Nusselt number of inner cylinder is plotted vs. the governing parameters. All present simulations are considered in 2-D for steady laminar flow regime. The obtained results showed that the flow between cylinders is more stable for the inclination angle ω = 0°. Increase in Rayleigh number increases the heat transfer rate for all values of inclination angle. Furthermore, the effect of Prandtl number on the mean average Nusselt number becomes negligible when Prandtl number is over the value 7.01. For example at Pr = 0.71 and Ra =10^5, increase in inclination from 0° to 40° decreases the average Nusselt number by 5.4%. A new correlation is also provided to describe the average Nusselt number as function of Prandtl number and Rayleigh number at ω = 0°.

Key words: *natural-convection, annulus, heat transfer, Nusselt number, inclination angle, steady-state, Rayleigh number*

Introduction

The natural-convection heat transfer between two concentric cylinders of different cross-sectional shape (for example: circular, square, and elliptic) is encountered widely in many different field of engineering applications such as solar collectors, fire preventions, cooling towers, refrigeration systems, heat exchangers, cooling of electronic component and so on. Over the last years, the study of this mechanism of heat transfer has been of interest to many researchers due to its engineering importance. The required information in industrial applications of this problem is about the behavior of fluid-flow between cylinders and its impact on heat transfer rate. Subsequently, it is mandatory to analyze the thermal pattern by natural-convection before designing such systems. Physically, the free convection is defined as the motion of fluid

* Corresponding author, e-mail: houssem.laidoudi@univ-usto.dz

particles due to thermal buoyancy that means the coldest parts of the fluid becoming heavier than other parts. Accordingly, the thermal energy is the source of fluid motion.

Indeed, many experimental and numerical studies have been accomplished to analyze and determine quantitatively the rate of natural-convection heat transfer between two concentric cylinders [1-20]. Mack and Bishop [1] they performed the earlier study of natural-convection between two horizontal concentric cylinders of circular cross-sectional shape. This work is limited only for low values of Rayleigh number. The computational results of this work have been compared with experimental findings and they showed a good agreement. Kuehn and Goldstein [2] represented an important work on natural-convection heat transfer between two horizontal cylinders of circular cross-section. Indeed, this work combines the numerical and experimental aspects of scientific research. The finite-element method is employed in this work to solve the governing equations. At the end of this work both experimental and numerical results have been represented and they have been compared for the same conditions, a good agreement is showed between them. Kumar [3] numerically studied the natural-convection in horizontal annuli whose cylinders are circular. The work focused on the effect of gap distance between cylinders on fluid and thermal patterns. The results showed the distributions of velocity and temperature profiles at different angular position. It has been concluded that the natural-convection heat transfer is a function of gap distance and Rayleigh number. Mastiani *et al.* [5] numerically investigated the melting of material inside a horizontal annulus. The cross-sectional form of the cylinders is circular. The numerical simulations are done at fixed value of Rayleigh number Ra = 10^4. Iqbal *et al.* [6] numerically carried out a study on optimum configurations of annulus with adding longitudinal fins of triangular cross-section the inner circular cylinder which became a finned annulus. The obtained results showed that the number of fins on cylinder surface has a significant effect on heat transfer. El-Maghlany *et al.* [7] studied the simultaneous forced the natural-convection heat transfer between an eccentric horizontal annulus. The inner cylinder is maintained at fixed temperature which is the source of thermal buoyancy and the rotation of the inner cylinder generates the forced convection. The obtained results showed that the eccentricity factor and the rotating velocity of cylinder have a positive effect of heat transfer. Tayebi *et al.* [8] studied a numerical investigation on nanofluid in horizontal annulus and the inner and outer cylinders are elliptical. The computations are done for the range of Rayleigh number of Ra = $10^3 \cdot 10^5$. The main results of this study are shown in terms of streamline and isotherm contours. Nada and Said [9] they studied the natural-convection heat transfer of air fluid in finned and un-finned annulus. The presented results showed the effects of Rayleigh number and the geometrical configurations of inner cylinders on the global characteristics of heat transfer rates and flow patterns. It was found that by adding the fins to the inner cylinder there is an enhancement of heat transfer. Matin and Khan [10] numerically studied the non-Newtonian power-law fluids in a horizontal concentric annulus of circular cross-section. The governing equations of momentum and energy have been solved using finite volume technique under the range of following governing parameters: Rayleigh number Ra = 10^3-10^5, the Prandtl number Pr = 10-1000, and the power-index n = 0.6-1.4. The radii ratio of studied geometry is RR = 2.5. The obtained results are shown as streamline and isotherm contours, also as profiles of dimensionless velocity and temperature. It was concluded that the usage of power-law fluids (n < 1) increases the heat transfer rate. Also, the effect of Prandtl number on thermal patterns becomes negligible when Pr > 10. Raghi *et al.* [11] numerically investigated the natural-convection inside an enclosure of square cross-section where there are small cross-sectional squares. The work studied the effects of small square number and their arrangement, Rayleigh number on hydrodynamic and thermal behaviors. Dash and Dash [12] studied in 3-D the natural-convection from a thick hollow horizontal cylinder placed at the bottom of block which has a rectangular cross-section. The study is affected by varying the thermophysic parameters, Rayleigh number, and the geometrical dimensions (length of cylinder diameter, the thickness). At the end of this paper, a correlation has been provided for Nusselt number as function of Rayleigh number and geometrical parameters which could be useful for engineering application. Masoumi *et al.* [13] carried out a laminar natural-convection heat transfer of Bingham fluids in annular spaces of horizontal direction. The cylinders are circular cross-sectional form. As all studies, the work tested the effects of Rayleigh number, yield number and the gap space distance on global hydrodynamic and thermal patterns. It was found that the yield number has a significant effect on heat transfer rate and above a certain value of yield number the natural-convection mechanism disappears at all. Pandey *et al.* [14] investigated by using numerical and experimental methodologies the natural-convection in different enclosures with and without confined bodies. Square, circular and elliptical cylinder. The research produced the effects Rayleigh number and geometrical parameters (cross-sectional form, aspect ratio, inclination angle) on fluid patterns and temperature distribution. Roy [15] studied the natural-convection reduced by exothermic reaction in a concentric annulus. The cylinders of annulus are circular with some undulations which give a new form of cross-section as oval, cross, star cross-sectional shape. The result showed that the form of cylinders have significant effect on fluid patterns. Ragui *et al.* [16] carried out the effect of thermo-fluidic parameters on fluid patterns and thermal distribution behavior within a porous annulus.

There are also other studies examined the number and cross-sectional shape of the bodies inside a square enclosures [17-22]. The laminar natural-convection is also studied inside a square enclosure without bodies [23-25]. The previous studies showed also that the thermal buoyancy has a significant impact on fluid patterns and heat transfer rate in other geometrical configuration rather than annulus [26-31]

From the aforementioned survey, it can be concluded that the laminar natural-convection heat transfer in annular space depends mainly on Rayleigh number, Prandtl number, and geometrical configurations of inner and outer cylinders especially. Also, the famous shapes of cross-section of cylinder are (square, circular, and elliptical). Furthermore, many researchers have examined the natural-convection within annular space but few investigations have treated the natural-convection from inner cylinder of a special cross-sectional form. In present investigation, the laminar natural-convection heat transfer in annular space is considered. The inner cylinder is an orthogonality of two equal-sized elliptical cylinders and the outer cylinder is kept circular. The effects of Rayleigh number (Ra = 10^3-10^5), Prandtl number (Pr = 0.71-100) and inclination angle (ω = 0-80°) on flow patterns and heat transfer rates is the main purpose of present study.

Physical problem, governing equation and boundary conditions

Figure 1 reflects a schematic illustration of present problem. It is an annular space which is surrounded by two closed cylinders; the inner cylinder is formed by orthogonality of two identical elliptical cylinders, the geometrical definition of the elliptical part is given by the aspect ratio

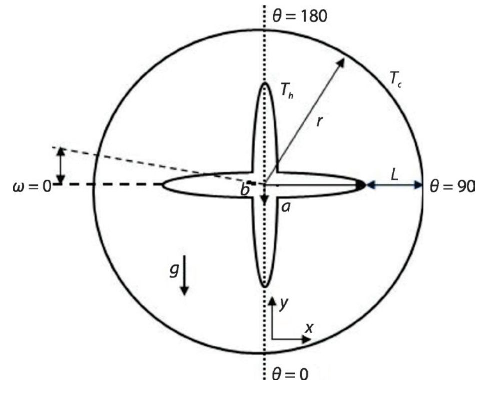

Figure 1. Schematic illustration of present problem

(E = b/a = 0.1), the outer cylinder is circular and it is defined by radius, r. The wing length, a, to outer cylinder diameter, r, defines the radii ratio $RR = a/r = 2.5$. The minimum distance between inner and outer cylinder is defined by $L = r - a$. Also, the inner cylinder is subjected to an inclination angle (ω = 0-80°). The purpose of present study is to examine the effects of governing and geometrical parameters on hydrodynamic flow and thermal patterns. To do so, the inner cylinder is heated by uniform constant temperature T_h ($T_h > T_c$) and the outer cylinder is maintained cold by uniform constant temperature, T_c. The annular space is filled by incompressible Newtonian fluid. All numerical simulations are performed in 2-D and for steady-state regime. The effect of thermal buoyancy force induced by the temperature difference toward the y-direction is computed by using the Boussinesq approximation.

The steady PDE for present 2-D problem are continuity, momentum and energy are writing in dimensionless from in a Cartesian co-ordinate system, respectively:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} = -\frac{\partial p}{\partial x} + \mathrm{Pr}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial vu}{\partial x} + \frac{\partial vv}{\partial y} = -\frac{\partial p}{\partial y} + \mathrm{Pr}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \mathrm{RaPr}\phi \tag{3}$$

$$\frac{\partial u\phi}{\partial x} + \frac{\partial v\phi}{\partial y} = \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) \tag{4}$$

In the governing equations u, v, p, ϕ, Ra, and Pr are the flow dimensionless velocity along x-, y-directions, dimensionless pressure, dimensionless temperature, Rayleigh number and Prandtl number, respectively. These dimensionless variables are defined from their dimensional variables:

$$x = \frac{\bar{x}}{L},\ y = \frac{\bar{y}}{L},\ u = \frac{\bar{u}L}{\alpha},\ v = \frac{\bar{v}L}{\alpha},\ p = \frac{\bar{p}L^2}{\rho\alpha^2},\ \phi = \frac{T - T_c}{T_h - T_c} \tag{5}$$

$$RR = \frac{a}{r} = 2.5,\ L = r - a \tag{6}$$

$$\mathrm{Ra} = \frac{g\beta(T_h - T_c)L^3}{\nu\alpha},\ \mathrm{Pr} = \frac{\nu}{\alpha} \tag{7}$$

The thermo-fluid properties are defined by the density, ρ, kinematic viscosity, ν, thermal diffusivity, α, and expansion coefficient, β. The acceleration gravitation is g.

The quantitative value of local Nusselt number along the inner cylinder is computed by flowing expression:

$$\mathrm{Nu}_l = \left.\frac{\partial \phi}{\partial n}\right|_{\mathrm{wall}} \tag{8}$$

The integration of local Nusselt number along the inner cylinder surface defines the average Nusselt number:

$$\mathrm{Nu} = \frac{1}{s}\int_0^s \mathrm{Nu_l}\,\mathrm{d}s \tag{9}$$

where n and s are the direction normal to the wall and the length of inner cylinder surface.

The boundary conditions used for present problem are:

On the surface of internal and external cylinders, respectively:

$$u = 0,\ \ v = 0,\ \ \phi = 1 \tag{10}$$

$$u = 0,\ v = 0,\ \ \phi = 0 \tag{11}$$

Simulation procedure

In present work, the finite-volume method with single-block unstructured triangular cells of non-uniform grid spacing, fig. 2, has been used to convert the partial equations of continuity, momentum and energy subjected to boundary conditions into a system of discrete algebraic equations. High resolution discretization scheme is used for the spatial discretization of the convective terms and semi-implicit method for pressure-linked equations-consistent (SIMPLEC) algorithm is used as the pressure-velocity coupling scheme. The convergence criteria based on relative error for the inner iterations are set as 10^{-8} for the discretized continuity and momentum equations and 10^{-6} for the discretized energy equation.

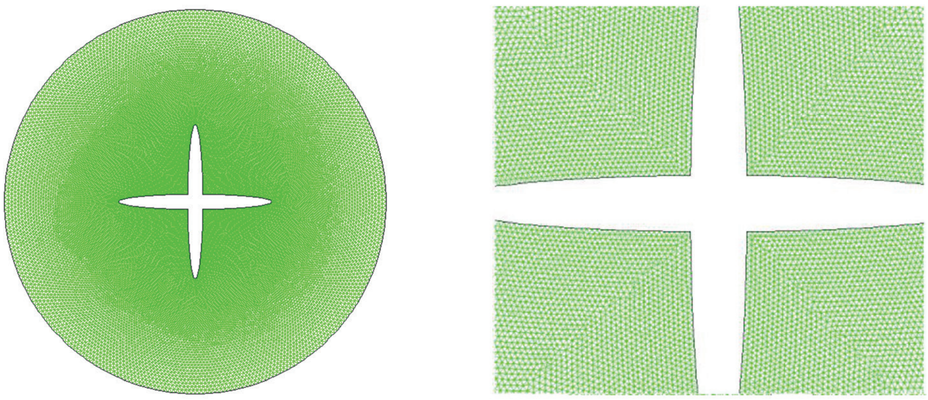

Figure 2. Presentation of computational grid structure with expanded view close to the inner cylinder for inclination angle (ω = 0)

The computational grid cell elements of the global domain were chosen based on the effect of grid density on dimensionless temperature distribution along the gap space distance between cylinders at the position $\theta = 90°$ and $\theta = 180°$ for $\omega = 0°$ and for the governing parameters Pr = 7.01 and Ra = 10^4. Fives meshes of different grid density were created for domain independency test as it is presented in tab. 1. Figure 3 shows the obtained results of grid independency test. It can be observed that there is no big difference in distribution of dimensionless temperature and the Mesh3 could be satisfactory for the rest of present work.

Table 1. Elemental values of generated meshes for grid independency test

Mesh	M_1	M_2	M_3	M_4	M_5
Elements	10.800	20.500	51.500	148.240	161.100

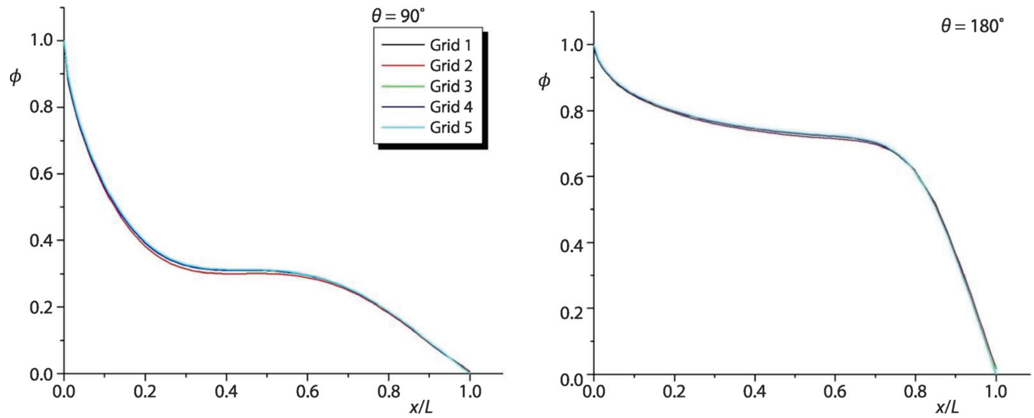

Figure 3. Distribution of dimensionless temperature along the gap space distance at θ = 90°, 180° for different values of grid elements

Validation of numerical method

In order to check the accuracy of present numerical methodology, the values of average Nussel number of annuli of inner circular cylinder are obtained and compared with previous experimental work of Kuehn and Goldstein [2] and the numerical results of Matin and Khan [10] for radii ratio RR = 2.6 and Pr = 0.7 as it is represented in fig. 4. A good agreement is observed between present results and those of previous works. Also, another set of average Nusset number of inner circular cylinder for radii ratio RR = 2.5 and Pr = 0.7 are obtained and compared with prior numerical results of Abu-Nada *et al.* [25] and Matin and Khan [10], fig. 4.

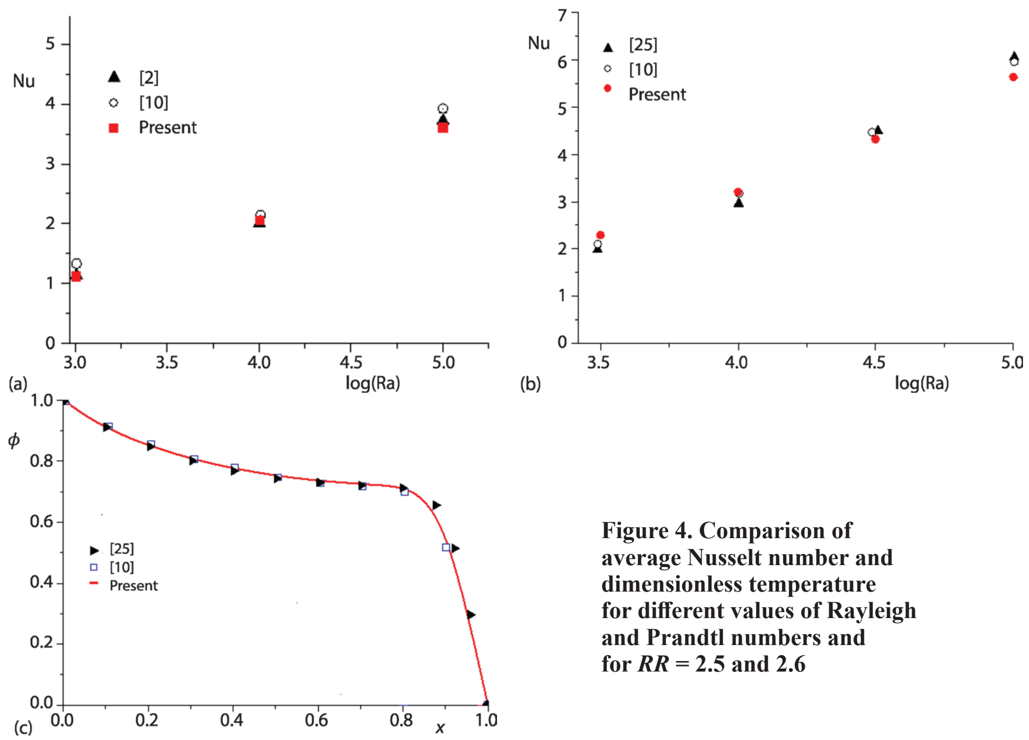

Figure 4. Comparison of average Nusselt number and dimensionless temperature for different values of Rayleigh and Prandtl numbers and for RR = 2.5 and 2.6

A good agreement is also detected between the three results. Moreover, fig. 4(c) shows the comparative evolution of dimensionless temperature along gap spacing between the two circular cylinders at the angle $\theta = 180°$ for radii ratio $RR = 2.5$ and Pr = 0.7. An excellent agreement is observed between the results.

Results and discussion

Effects of Rayleigh and Prandtl numbers for fixed inclination angle ($\omega = 0$)

In this section, the effects of Rayleigh and Prandt numbers on fluid-flow and thermal patterns between cylinders are presented and discussed at fixed value of inclination angle ($\omega = 0$). Also, the effects of these governing parameters on dimensionless velocity, temperature and average Nusselt number are also incorporated.

Figures 5 and 6 depict the plotted streamline and isotherm contours between the two cylinders for different values of Rayleigh number (Ra = 10^3-10^5) and Prandtl number (Pr = 0.71-100) at fixed inclination angle ($\omega = 0$), respectively. For all values of Rayleigh and Prandtl numbers the streamlines and isotherms are perfectly symmetric about y-axis ($y = 0$). Through the simultaneous notes taken from figs. 5 and 6 and based on the concept of thermal buoyancy, it can be analyzed that the warm fluid parts in the vicinity of hot inner cylinder move up (along the y-direction) towards the cold surface of outer cylinder. Simultaneously, the cold fluid parts near the outer cylinder become heavier and they move down towards the lower part of annular enclosure following the curvature surface of outer cylinder. Consequently, two symmetrical counter-rotating regions appear in annular space. One is in right side with clockwise rotation and the second is in left side with anti-clockwise rotation. The effect of thermal buoyancy strength (increase in the Rayleigh number) on streamlines and isotherms is more remarkable than the effect of enhancement in thermo-physical characteristics (increase in the Prandtl number). The vortices inside the space are observed to be stretched laterally with increasing

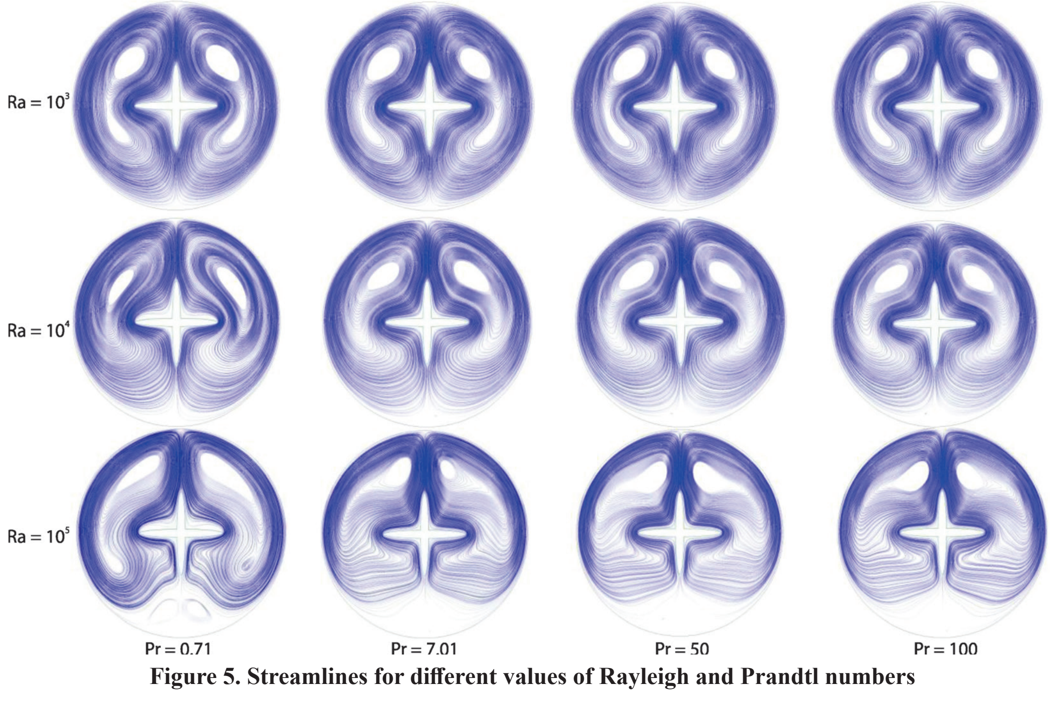

Figure 5. Streamlines for different values of Rayleigh and Prandtl numbers

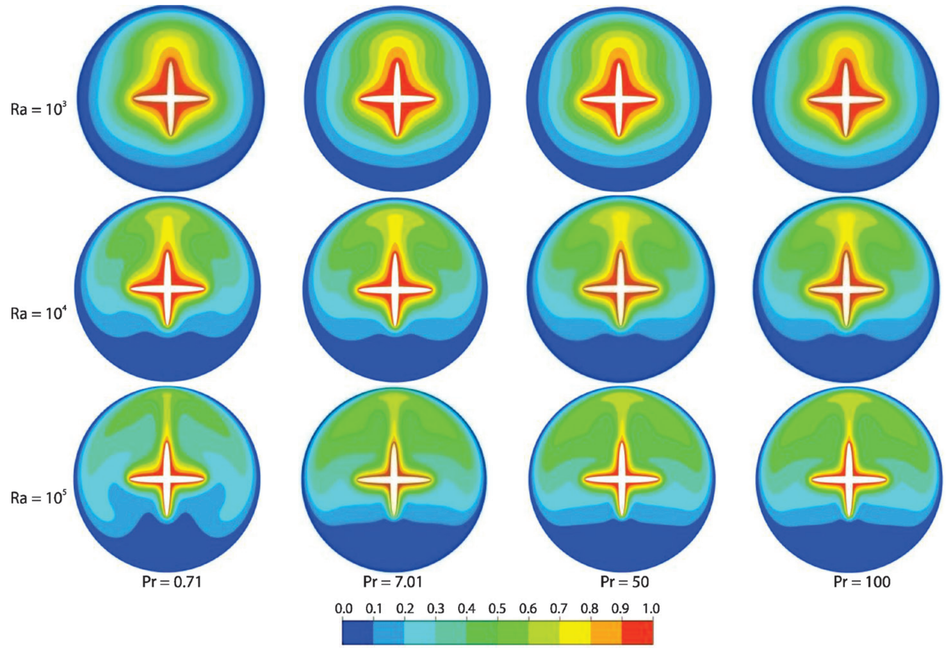

Figure 6. Isotherms for different values of Rayleigh and Prandtl numbers

the value of Rayleigh number. It is also observed that for all governing parameters, the mean centre of re-circulating zone is in the upper part of annular space and it moves up tightly with increasing in Rayleigh number. This note is different from annulus of inner circular cylinder [10]. The current position of rotating-zone centre is due to geometrical shape of inner cylinder, *i.e.* the right and left sharp wings of inner cylinder remain the re-circulation zone center of both zones in upper part of annular space. Also, in some developed cases (for example for Pr = 0.71 and Ra = 10^5) the wing of inner cylinder divides the re-circulation zone into two unequal zones. But lower zone is negligible in comparison main zone. Furthermore, for Pr = 0.71, the current cross-sectional shape of inner cylinder creates two extra small rotating zones at the lower part of annular space.

The effects of studied parameters on isotherms showed that there is an agreement between the isotherms and streamlines. The lateral expansion of isotherms is seen to be reduced with a gradual increase in Rayleigh number due to gradual increase in thermal buoyancy strength. As a result, the thermal layer thickness around the inner cylinder surface becomes thin and the thermal gradient becomes very important in these areas to hint an increase in heat transfer rate. A closed view on isotherm contours indicates that the isotherms are more crowded on bottom surface of inner cylinder showing higher values of heat transfer rate.

Figure 7 shows the distribution of dimensionless temperature along the radial space between cylinders at $\theta = 90°$ for different values of Rayleigh number and Prandtl number. It is clear that for all values of Prandtl number, the increase in the value of Rayleigh number increases the dimensionless velocity between cylinders. As we move from the value 0.71-7.01 of Prandtl number a small reduction of dimensionless velocity is appeared and over the value 7.01, the dimensionless velocity profiles are almost constant. It is also observed that the highest values of dimensionless velocities are near the inner and outer cylinder with distance of 0.1 L.

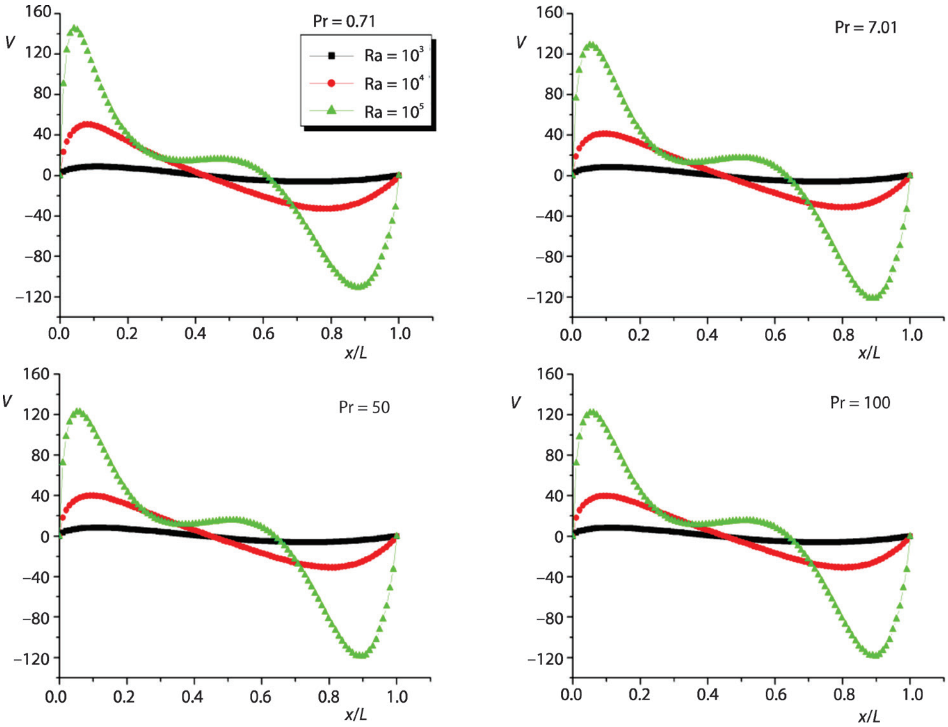

Figure 7. Dimensionless velocity profile along the line between cylinders at angular position $\theta = 90°$ for different values of Rayleigh and Prandtl numbers

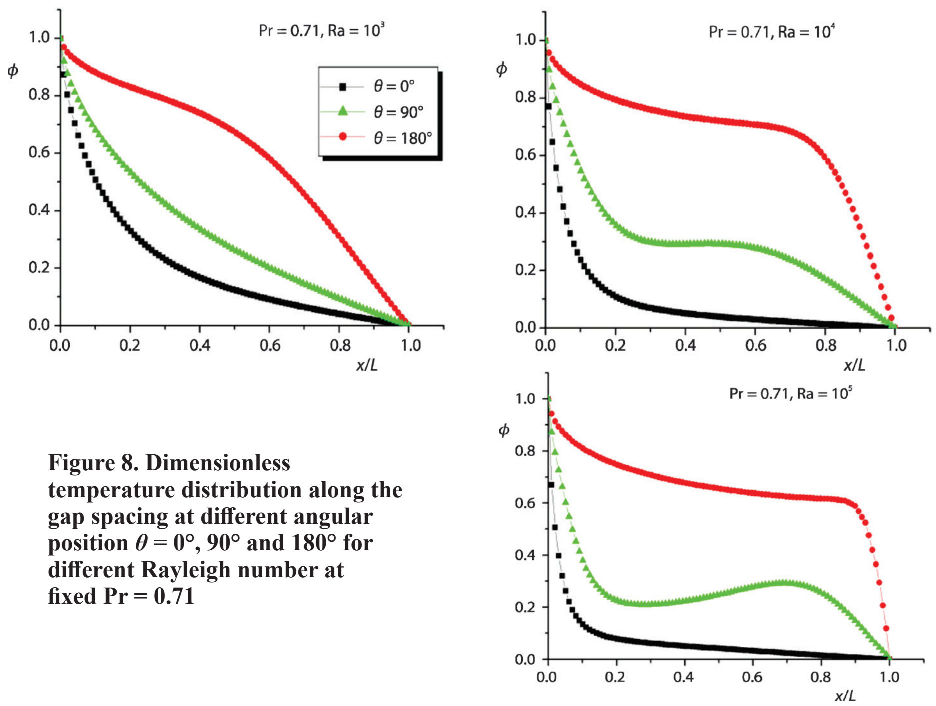

Figure 8. Dimensionless temperature distribution along the gap spacing at different angular position $\theta = 0°$, 90° and 180° for different Rayleigh number at fixed Pr = 0.71

Figure 8 depicts the dimensionless temperature profiles at different gap spacing position of different angular $\theta = 0°$, 90°, and 180° for different values of Rayleigh number at fixed values of Pr = 0.71 and inclination angle $\omega = 0°$. It is evident that at Ra = 103, the effect of thermal buoyancy is still small, therefore, the three plotted graphs have a semi-flattened profiles and the heat transfer mechanism of conduction is still clear. The three graphs show also a smooth decrease with increase in Rayleigh number indicating the increase in thermal gradient. Furthermore, the dimensionless temperature at angular position ($\theta = 0°$) has the lowest distribution which means the highest value of heat transfer rate.

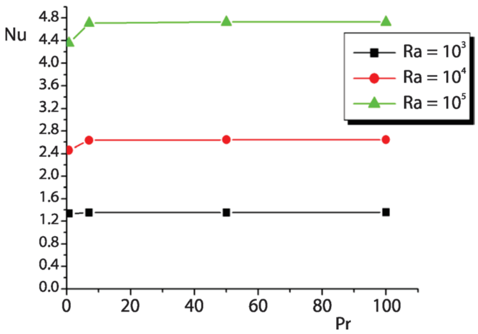

Figure 9. Average Nusselt number of inner cylinder *vs.* Prandtl number for different values of Rayleigh number

Figure 9 shows the average Nusselt number *vs*. Prandtl number for different values of Rayleigh number. As it is expected from the previous analyzes, increase in Rayleigh number increases the value of Nusselt number which means the heat transfer rate increases with increasing Rayleigh number. The effect of Prandt number on heat transfer rate is almost negligible. Finally, a new correlation is determined to show the evolution of average Nusselt number as function of Rayleigh and Prandtl numbers. This correlation is limited only for our geometrical configuration and for $Ra = 10^3\text{-}10^5$, Pr = 0.71-100 at fixed inclination angle $\omega = 0°$:

$$\mathrm{Nu} = 4.164 - 3.028\mathrm{e}^{\left[\left(\frac{\mathrm{Ra}}{15427.2}\right)-\left(\frac{\mathrm{Pr}}{4747.48}\right)\right]} \tag{12}$$

Effect of inclination angle

In this section, the effect of inclination angle, ω, on fluid-flow and thermal patterns is presented in terms of representative streamline and isotherm contours, respectively. The average Nusselt number under the effect of inclination angle is also incorporated for different values of Rayleigh and Prandtl numbers.

Figures 10 and 11 represent the effects of inclination angle, ω, on streamlines and isotherms for different values of Rayleigh number at fixed Prandtl number Pr = 7.01, respectively. As it can be seen in these figures, $\omega = 10°$ is the mirror of $\omega = 80°$ and $\omega = 30°$ is the mirror of $\omega = 60°$. It can be observed that the inclination angle generates an asymmetrical streamlines and isotherms. However, the effect of inclination angle, ω, on streamlines can be divided into two parts. When $0° < \omega < 45°$, the left wing of inner cylinder devises the left vortex into tow unequal vortices and by increasing the Rayleigh number both vertices move upwards to form one vortex again. Meanwhile, the right vortex keeps its unity. Also, in this range of inclination angle, there is a formation of new cell of counter-rotating on the left wing. The size of the cell is seen to be grown longitudinally and laterally with gradual increase in inclination angle and/or Rayleigh number. In other hand, the same behavior is seen to be happened on right part of annulus when $45° < \omega < 90°$. The representative isotherms of fig. 11 show the same physical phenomena detected from the interpretations of streamline contours. Moreover, the thermal boundary-layer is seen to be increased tightly for $0° < \omega < 45°$ which hints to reduce the heat transfer rate.

Figure 12 shows the variation of average Nusselt number *vs*. inclination angle for different values of Rayleigh and Prandtl numbers. As it is expected, the average Nusselt number decrease with increasing the inclination angle from 0-45°, the effect of inclination angle on average Nusselt number increases with increase in the value of Rayleigh number. Furthermore, for all values of ω and for fixed Ra, increase in Pr from 0.71-7.01 increases the average Nusselt number.

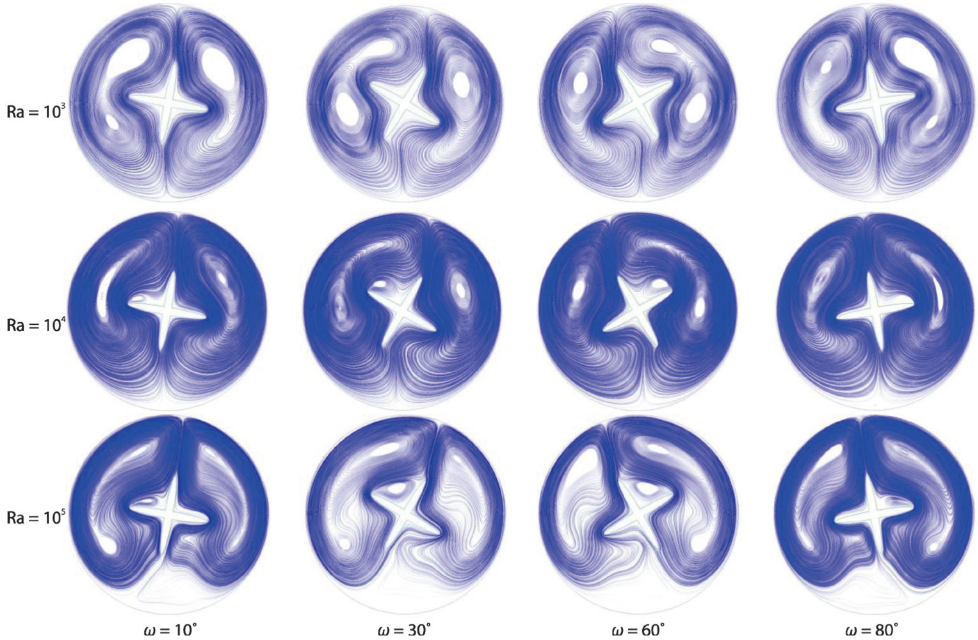

Figure 10. Streamlines for different values inclination angle and Rayleigh number at fixed Pr = 0.71

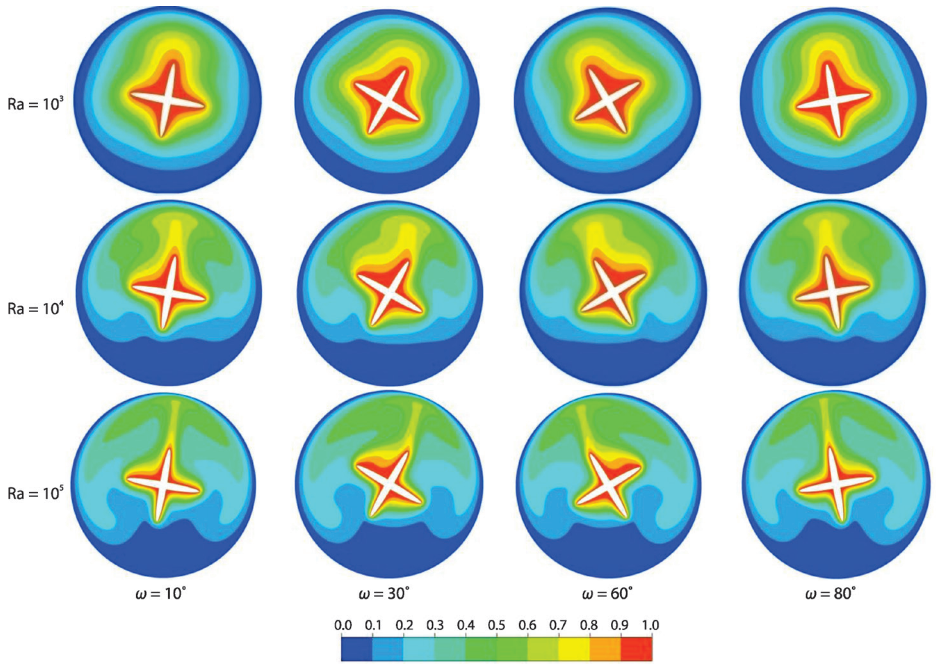

Figure 11. Isotherms for different values inclination angle and Rayleigh number at fixed Pr = 0.71

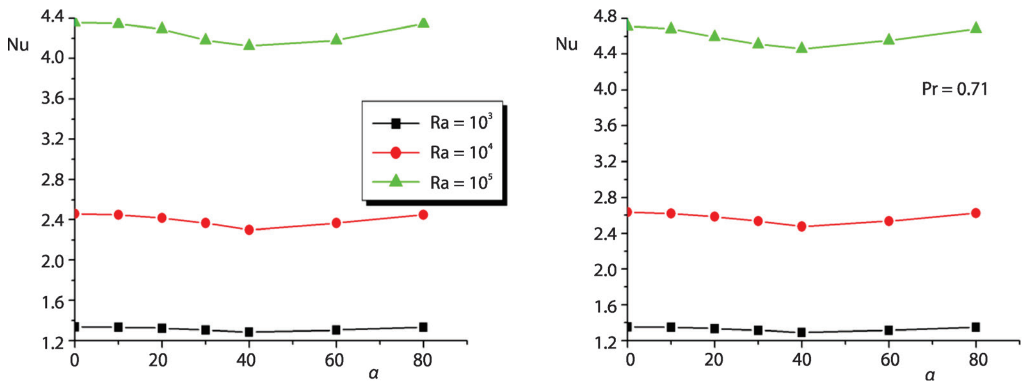

Figure 12. Average Nusselt number *vs.* inclination angle, *ω*, for different Rayleigh and Prandtl numbers

Conclusion

Numerical simulations are performed to study the effects of Rayleigh number (Ra = 10^3-10^5), Prandtl number (Pr = 0.71-100) and inclination angle (ω = 0-80°) on fluid-flow and heat transfer rate between two horizontal concentric cylinders. The inner cylinder is an orthogonality of two identical elliptical cylinders of aspect ratio of E = 0.1 and it is heated. In other hand the outer cylinder is maintained circular and it is supposed to be cold. The annular space between cylinders is defined by radii ratio RR = 2.5. At the end of this work, it was concluded that when ω = 0°, the flow patterns inside the annular space creates two symmetrical vortices and the main vortex center is almost in the upper annular part. Also, for Pr = 0.71 there is apparition of two extra small vortices at the lower part of annular. The effect of Prandt number is almost negligible above the value Pr = 7.01. The heat transfer rate which is quantified by average Nusselt number, it is observed to be increased with Rayleigh number. Moreover, the inclination angle generates an asymmetrical flow inside the annular space. And the degree of asymmetry depends positively on inclination angle for the range of 0° < ω < 45°. The increase in the inclination angle ω = 0-45° decreases the heat transfer rate. For example at Pr = 0.71 and Ra = 10^5, inclining the inner cylinder with 45° reduces the average Nusselt number by 5.4%. Finally, the values of average Nusselt number *vs*. Ra, Pr, and ω are provided in order to be useful for certain industrial applications. For this reason, a new expression of correlation is defined to give the variation of average Nusselt number as function of Ra and Pr for ω = 0°.

Nomenclature

a – big elliptical radius, [m]
b – small elliptical radius, [m]
E – aspect ratio, [–]
g – gravity acceleration, [ms^{-2}]
L – annular spacing length, [m]
n, s – normal direction, [–]
Nu – average Nusselt number, [–]
Nu$_l$ – local Nusselt number, [–]
p – dimensionless pressure, [–]
Pr – Prandtl number, [–]
r – radius of outer cylinder, [m]
RR – radii ratio, [–]
Ra – Rayleigh number, [–]
T – temperature, [K]
T_c – cold temperature, [K]
T_h – hot temperature, [K]
u – dimensionless axial velocity, [–]
v – dimensionless radial velocity, [–]
x – dimensionless axial co-ordinate, [–]
y – dimensionless radial co-ordinate, [–]

Greek symbols

α – thermal diffusivity, [m^{2}s^{-1}]
β – coefficient of volume expansion, [K^{-1}]
ν – kinematic viscosity, [m^{2}s^{-1}]
ρ – density of fluid, [kgm^{-3}]

θ – angular angle, [°]
ϕ – dimensionless temperature, [–]
ω – inclination angle, [°]

Superscripts

– – dimensional quantity

References

[1] Mack, L. R., Bishop, E. H., Natural-Convection between Horizontal Concentric Cylinders for Low Rayleigh Numbers, *Quart. J. Mech Appl Math*, *21* (1968), 2, pp. 223-241
[2] Kuehn, T. H., Goldstein R. J., An Experimental and Theoretical Study of Natural-Convection in the Annulus between Horizontal Concentric Cylinders, *Jozrnal Fluid Mechanics*, *74* (1976), 4, pp. 695-719
[3] Kumar, R., Study of Natural-Convection in Horizontal Annuli, *Int. J. Heat Mass Transfer*, *31* (1988), 6, pp. 1137-1148
[4] Char, M. I., Lee, G. C., Maximum Density Effects on Natural-Convection of Micropolar Fluids between *Horizontal Eccentric Cylinders*, *Int. J. Eng. Sci*., *36* (1998), 2, pp. 157-169
[5] Mastiani, M., *et al.*, Melting of a Phase Change Material in a Horizontal Annulus with Discrete Heat Sources, *Thermal Science*, *19* (2015), 5, pp. 1733-1745
[6] Iqbal, Z., *et al.*, Optimum Configurations of Annulus with Triangular Fins for Laminar Convection, *Thermal Science*, *21* (2017), 1A, pp. 161-173
[7] El-Maghlany, W., *et al*., Mixed Convection in an Eccentric Annulus Filled by Copper Nanofluid, *Thermal Science*, *20* (2016), 5, pp. 1597-1608
[8] Tayebi, T., *et al*., Natural Convective Nanofluid-Flow in an Annular Space Between Confocal Elliptic Cylinders, Num Heat Transfer, *Journal of Thermal Science and Engineering Appl*., *9* (2017), 011010
[9] Nada, S. A., Said, M. A., Effects of Fins Geometries, Arrangements, Dimensions and Numbers on Natural-Convection Heat Transfer Characteristics in Finned-Horizontal Annulus, *Int. J. of Thermal Sciences*, *137* (2019), Mar., pp. 121-137
[10] Matin, M. H., Khan, W. A., Laminar Natural-Convection of Non-Newtonian Power-Law Fluids between Concentric Circular Cylinders, *Int. Comm. in Heat and Mass Transfer*, *43* (2013), Apr., pp. 112-121
[11] Raghi, K., *et al*., Impact of Square Fuel Assemblies Arrangement on Fluid-Flow and Heat Transfer Enhancement Inside Water Filled Enclosures, *Int. Letters of Chemistry, Physics and Astronomy*, *64* (2016) Feb., pp. 144-158
[12] Dash, M. K., Dash, S. K., The 3-D Numerical Study of Natural-Convection Heat Transfer from a Hollow Horizontal Cylinder Placed on the Ground, *Int. J. of Thermal Sciences*, *140* (2019), June, pp. 429-441
[13] Masoumi, H., *et al.*, Laminar Natural-Convection of Yield Stress Fluids in Annular Spaces between Concentric Cylinders, *Int. J.of Heat and Mass Transfer*, *138* (2019), Aug., pp. 1188-1198
[14] Pandey, S., *et al*., An Exhaustive Review of Studies on Natural-Convection in Enclosures with and Without Internal Bodies of Various Shapes, *Int. J. of Heat and Mass Transfer*, *138* (2019), July, pp. 762-795
[15] Roy, N. C., Natural-Convection in the Annulus Bounded by Two Wavy Wall Cylinders Having a Chemically Reacting Fluid, *Int. J. of Heat and Mass Transfer*, *138* (2019), Aug., pp. 1082-1095
[16] Ragui, K., *et al*., Correlating Heat and Mass Transfer Coefficients for Thermosolutal Convection Within a Porous Annulus of a Circular Shape: Case of Internal Pollutants Spreading, *Heat and Mass Transfer*, *54* (2018), Feb., pp. 2061-2078
[17] Zhang, W., *et al*., Transient Behaviors of Mixed Convection in a Square Enclosure with an Inner Impulsively Rotating Circular Cylinder, *Int. Comm. in Heat and Mass Transfer*, *98* (2018), Nov., pp. 143-154
[18] Hussein, A. K., Computational Analysis of Natural-Convection in a Parallelogrammic Cavity with a Hot Concentric Circular Cylinder Moving at Different Vertical Locations, *Int. Comm. in Heat and Mass Transfer*, *46* (2013), Aug., pp. 126-133
[19] Mohamed-Ali, A., Natural-Convection over Circular Cylinders in a Porous Enclosure Filled with a Nanofluid under Thermo-Diffusion Effects, *Journal of the Taiwan Institute of Chemical Engineers*, *70* (2017) Jan., pp. 88-103
[20] Shyam, R., *et al*., Free Convection from a Heated Circular Cylinder in Confined Power-Law Fluids, *Int. J. of Thermal Sciences*, *74* (2013), Dec., pp. 156-173
[21] Chatterjee, D., Halder, P., The MHD Mixed Convective Transport in Square Enclosure with Two Rotating Circular Cylinders, *Numerical Heat Transfer*, *65* (2014), 8, pp. 802-824
[22] Park, Y. G., Natural-Convection in Square Enclosure with Hot and Cold Cylinders at Different Vertical Locations, *Int. J. of Heat and Mass Transfer*, *55* (2012), 25-26, pp. 7911-7925

[23] Boutra, A., *et al*., Natural-Convection Heat Transfer of a Nanofluid into a Cubical Enclosure: Lattice Boltzmann Investigation, *Arab. J. Sci. Eng*., *41* (2016), Feb., pp. 1969-1980
[24] Jani, S., *et al*., Numerical Investigation of Natural-Convection Heat Transfer in a Symmetrically Cooled Square Cavity with a Thin Fin on its Bottom Wall, *Thermal Science*, *18* (2014), 4, pp. 1119-1132
[25] Abu-Nada, E., *et al*., Natural-Convection Heat Transfer Enhancement in Horizontal Concentric Annuli Using Nanofluids, *Int.Comm.in Heat and Mass Transfer*, *35* (2008), 5, pp. 657-665
[26] Laidoudi, H., Makinde, O. D., Mixed Convection Heat Transfer around a Tandem Circular Cylinders in Incompressible Downward Flow, *Diffusion Foundations, 16* (2018), June, pp. 12-20
[27] Mokeddem, M., The 3-D Simulation of Incompressible Poiseuille Flow through 180° Curved Duct of Square Cross-Section under Effect of Thermal Buoyancy, *Periodica Polytechnica Mechanical Engineering*, *64* (2019), 4, pp. 257-269
[28] Mebarek-Oudina, F., Makinde, O. D., Numerical Simulation of Oscillatory MHD Natural-Convection in Cylindrical Annulus: Prandtl Number Effect, *Defect and Diffusion Forum*, *387* (2018), Sept., pp. 417-427
[29] Mebarek-Oudina, F., Bessaih, R., Numerical Simulation of Natural-Convection Heat Transfer of Copper-Water Nanofluid in a Vertical Cylindrical Annulus with Heat Sources, *Thermophysics and Aeromechanics*, *26* (2019), Aug., pp. 325-334
[30] Mebarek-Oudina, F., Convective Heat Transfer of Titania Nanofluides of Different Base Fluids in Cylindrical Annulus with Discrete Heat Source, *Heat Transfer – Asian Res*., *48* (2019), 1, pp. 135-147
[31] Laidoudi, H., Bouzit, M., Mixed Convection in Poiseuille Fluid from an Asymmetrically Confined Heated Circular Cylinder, T*hermal Science*, *22* (2018), 2, pp. 821-834

Paper submitted: February 1, 2020
Paper revised: March 9, 2020
Paper accepted: May 25, 2020