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A REVISED MODEL OF "STEADY LAMINAR NATURAL CONVECTION OF NANOFLUID UNDER THE IMPACT OF MAGNETIC FIELD ON 2-D CAVITY WITH RADIATION" [AIP Advances 9, 065008 (2019); https://doi.org/10.1063/1.5109192]

by

Hossam A. NABWEY *

Department of Mathematics, College of Science and Humanities,
Prince Sattam bin Abdulaziz University, Al-Kharj, Saudi Arabia
Department of Basic Engineering Science, Faculty of Engineering,
Menoufia University, Shebin El-Kom, Egypt

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This discussion exhibits the major scientific errors on the recent published paper, entitled "Steady Laminar Natural Convection of Nanofluid Under the Impact of Magnetic Field on 2-D Cavity with Radiation" and their corrections indifferently. In Saleem et al. [1], the authors stated in both of abstract and problem assumptions that the non-Darcy model is used for the porous medium, while the porous terms are incompatible with this assumption. In addition, the authors used a non-inclined geometry in their investigation, but the governing equations are conflicting with this hypothesis. Further, the used range of the Darcy number is between 10^{-2}-10^{2} and this range is very large and did not represent the porous media flow. All of these observations make the mathematical formulations and the obtained results of Saleem et al. [1] are wrong. In the following sections, these scientific errors and their corrections will be presented minutely.

Wrong mathematical formulations

Saleem *et al*. [1] declared in the abstract that *Non-darcy model was utilized to employ porous terms in momentum equations*. Also, in the problem assumptions, they wrote *For porous media, non-Darcy model is involved*. Further, they presented the governing equations in the following forms:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\mu_{\mathrm{nf}}}{\rho_{\mathrm{nf}}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\beta_0^2 v(\sin\lambda)\sigma_{\mathrm{nf}}(\cos\lambda)-\frac{1}{\rho_{\mathrm{nf}}}\frac{\partial p}{\partial x}-\frac{\mu_{\mathrm{nf}}}{\rho_{\mathrm{nf}}K}u-$$
$$-\left(T_c-T\right)\beta_{\mathrm{nf}}\mathrm{g}\sin\gamma+\sigma_{\mathrm{nf}}\beta_0^2\left[-u\,(\sin\lambda)^2\right]=u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} \tag{2}$$

*Authors' e-mail: eng_hossam21@yahoo.com

$$\frac{\mu_{\rm nf}}{\rho_{\rm nf}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\left(T_c-T\right)\beta_{\rm nf}\,{\rm g}\cos\gamma-\frac{\partial p}{\partial y}\frac{1}{\rho_{\rm nf}}-\frac{\mu_{\rm nf}}{\rho_{\rm nf}K}v+$$
$$+\sigma_{\rm nf}\beta_0^2\left[u(\sin\lambda)(\cos\lambda)-v\,cos^2\lambda\right]=v\frac{\partial v}{\partial y}+u\frac{\partial v}{\partial x} \tag{3}$$

$$\frac{1}{\left(\rho C_p\right)_{\rm nf}}\frac{\partial q_r}{\partial y}+\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_{\rm nf}\left[\left(\rho C_p\right)_{\rm nf}\right]^{-1}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$
$$\left[T^4\cong 4T_c^3T-3T_c^4,\quad q_r=-\frac{4\sigma_e}{3\beta_R}\frac{\partial T^4}{\partial y}\right]$$

Also, the dimensionless forms of these governing equations were presented as:

$$\frac{\partial^2\Psi}{\partial X^2}+\frac{\partial^2\Psi}{\partial Y^2}=-\Omega \tag{5}$$

$$U\frac{\partial\Omega}{\partial X}+V\frac{\partial\Omega}{\partial Y}={\rm Pr}\frac{A_5}{A_1}\frac{A_2}{A_4}\left[\frac{\partial^2\Omega}{\partial X^2}+\frac{\partial^2\Omega}{\partial Y^2}\right]+$$
$$+\frac{A_6}{A_1}\frac{A_2}{A_4}{\rm Ha}^2{\rm Pr}\left(\frac{\partial U}{\partial X}\sin\lambda\cos\lambda-\frac{\partial V}{\partial X}cos^2\lambda-\frac{\partial V}{\partial Y}\sin\lambda\cos\lambda+\frac{\partial U}{\partial Y}\sin^2\lambda\right)+$$
$$+\frac{A_3}{A_1}\frac{A_2^2}{A_4^2}{\rm Ra\,Pr}\left[\frac{\partial\theta}{\partial X}\cos\gamma-\frac{\partial\theta}{\partial Y}\sin\gamma\right]-\frac{A_5}{A_1}\frac{A_2}{A_4}\frac{\rm Pr}{\rm Da}\Omega \tag{6}$$

$$\left[1+\frac{4}{3}\left(\frac{k_{\rm nf}}{k_f}\right)^{-1}{\rm Rd}\right]\frac{\partial^2\theta}{\partial Y^2}+\left(\frac{\partial^2\theta}{\partial X^2}\right)=-\frac{\partial\theta}{\partial Y}\frac{\partial\Psi}{\partial X}+\frac{\partial\theta}{\partial X}\frac{\partial\Psi}{\partial Y} \tag{7}$$

where all of the symbols in the previous equations were defined in Saleem *et al*. [1]. Also, the following dimensionless quantities were used:

$$U=\frac{uL}{\alpha_{\rm nf}},\ V=\frac{vL}{\alpha_{\rm nf}},\ \theta=\frac{T-T_c}{\Delta T},\ \Delta T=\frac{q''L}{k_f},\ \left(XL,YL\right)=\left(x,y\right),\ \Psi=\frac{\psi}{\alpha_{\rm nf}},\ \Omega=\frac{\omega L^2}{\alpha_{\rm nf}} \tag{8}$$

On the other hand, in all of the figures for Saleem *et al*. [1], the values of the Darcy number were varied from 0.01 to 100, which did not represent the Darcy regime rather than the non-Darcy regime.

Corrections of the previous formulations:

To present a beneficial discussion for the readers, corrections for all of these errors are included in this section. Let us consider the same flow assumptions and the same physical model of Saleem *et al*. [1]. By taking into account the local thermal equilibrium model between the porous and fluid phases as well as the Boussinesq approximation, the vector form of the governing equations are expressed as (Nield and Bejan [2]):

$$\nabla\vec{V}=0 \tag{9}$$

$$\frac{\rho_{\rm nf}}{\varepsilon^2}\left(\vec{V}\nabla\right)\vec{V}=-\nabla p+\frac{\mu_{\rm nf}}{\varepsilon}\nabla^2\vec{V}-\frac{\mu_{\rm nf}}{K}\vec{V}-\frac{C_F\rho_{\rm nf}}{\sqrt{K}}\left|\vec{V}\right|\vec{V}-\left(\rho\beta_{\rm nf}\right)\left(T-T_c\right)\vec{g}+\vec{I}\times\vec{B} \tag{10}$$

$$\left(\rho C_p\right)_{\text{nf}}\left(\vec{V}\vec{\nabla}T\right)=\vec{\nabla}\left(k_m\vec{\nabla}T\right) \tag{11}$$

$$k_m=\left(1-\varepsilon\right)k_s+\varepsilon k_{\text{nf}} \tag{12}$$

$$\nabla\vec{I}=0 \tag{13}$$

$$\vec{I}=\sigma_{\text{nf}}\left(-\nabla\varphi+\frac{\vec{V}}{\varepsilon}\times\vec{B}\right) \tag{14}$$

In the previous equations, $\vec{V}$ is the velocity vector, $\vec{B}$ – the magnetic field vector, $\vec{g}$ – the vector of the gravitational acceleration, φ – the electric potential, and ε – the porosity of the porous medium.

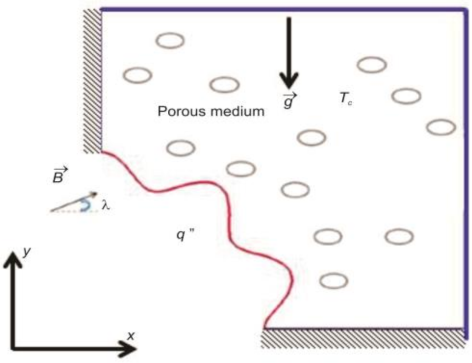

Figure 1. Physical model of Saleem *et al*. [1]

From fig. 1, the components of $\vec{B}$ and $\vec{g}$ are given by:

$$\vec{B}=\left(\beta_0\cos\alpha,\beta_0\sin\alpha,0\right),\quad \vec{g}=\left(0,-g,0\right) \tag{15}$$

With the help of eq. (15), the governing equations (taking into account the radiation term as Saleem *et al*. [1]) are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{16}$$

$$\frac{\rho_{\text{nf}}}{\varepsilon^2}\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+\frac{\mu_{\text{nf}}}{\varepsilon}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+$$
$$+\frac{\sigma_{\text{nf}}\beta_0^2}{\varepsilon}\left(v\sin\lambda\cos\lambda-u\sin^2\lambda\right)-\frac{\mu_{\text{nf}}}{K}u-\frac{C_F\rho_{\text{nf}}}{\sqrt{K}}\sqrt{u^2+v^2}\,u \tag{17}$$

$$\frac{\rho_{\text{nf}}}{\varepsilon^2}\left[u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial p}{\partial y}+\frac{\mu_{\text{nf}}}{\varepsilon}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\left(\rho\beta\right)_{\text{nf}}\,\text{g}\left(T-T_c\right)+$$
$$+\frac{\sigma_{\text{nf}}\beta_0^2}{\varepsilon}\left(u\sin\lambda\cos\lambda-v\cos^2\lambda\right)-\frac{\mu_{\text{nf}}}{K}v-\frac{C_F\rho_{\text{nf}}}{\sqrt{K}}\sqrt{u^2+v^2}\,v \tag{18}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_m}{\left(\rho C_p\right)_{\rm nf}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-\frac{1}{\left(\rho C_p\right)_{\rm nf}}\frac{\partial q_r}{\partial y} \tag{19}$$

The following dimensionless quantities are introduced:

$$U=\frac{uL}{\alpha_f},\ V=\frac{vL}{\alpha_f},\ \theta=\frac{T-T_c}{\Delta T},\ \Delta T=\frac{q''L}{k_f},\ \left(XL,YL\right)=\left(x,y\right),\ \Psi=\frac{\psi}{\alpha_f},\ \Omega=\frac{\omega L^2}{\alpha_f} \tag{20}$$

Using eq. (20), the following dimensionless system (in the vorticity-stream function formulas) is obtained:

$$\frac{\partial^2\Psi}{\partial X^2}+\frac{\partial^2\Psi}{\partial Y^2}=-\Omega \tag{21}$$

$$\begin{aligned}\frac{1}{\varepsilon^2}\left[U\frac{\partial\Omega}{\partial X}+V\frac{\partial\Omega}{\partial Y}\right]&=\frac{\Pr}{\varepsilon}\frac{A_5}{A_1}\left[\frac{\partial^2\Omega}{\partial X^2}+\frac{\partial^2\Omega}{\partial Y^2}\right]+\frac{A_3}{A_1}\mathrm{Ra}\,\Pr\frac{\partial\theta}{\partial X}+\\&+\frac{A_6}{A_1\varepsilon}\mathrm{Ha}^2\Pr\left(\frac{\partial U}{\partial X}\sin\lambda\cos\lambda-\frac{\partial V}{\partial X}\cos^2\lambda-\frac{\partial V}{\partial Y}\sin\lambda\cos\lambda+\frac{\partial U}{\partial Y}\sin^2\lambda\right)-\\&-\frac{A_5}{A_1}\frac{\Pr}{\mathrm{Da}}\Omega-\frac{C_F}{\sqrt{\mathrm{Da}}}\left[\frac{\partial}{\partial X}\left(V\sqrt{U^2+V^2}\right)-\frac{\partial}{\partial Y}\left(U\sqrt{U^2+V^2}\right)\right]\end{aligned} \tag{22}$$

$$U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}=\frac{A_4}{A_2}\left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right)+\frac{4}{3A_2}\mathrm{Rd}\frac{\partial^2\theta}{\partial Y^2} \tag{23}$$

where

$$A_1=\frac{\rho_{\rm nf}}{\rho_f},\ A_2=\frac{\left(\rho C_p\right)_{\rm nf}}{\left(\rho C_p\right)_f},\ A_3=\frac{\left(\rho\beta\right)_{\rm nf}}{\left(\rho\beta\right)_f},\ A_4=\frac{k_m}{k_f},\ A_5=\frac{\mu_{\rm nf}}{\mu_f},\ A_6=\frac{\sigma_{\rm nf}}{\sigma_f},\ \Pr=\frac{\nu_f}{\alpha_f}$$

By comparing eqs. (1)-(7) and the corrected systems (16)-(19) and (21)-(23), it can be found that:

- The wrong eqs. (5)-(7) contains the term $[(\partial\theta/\partial X)\cos\gamma-(\partial\theta/\partial Y)\sin\gamma]$ which means that the geometry is inclined with angle γ however the corrected system (21)-(23) includes the term $\partial\theta/\partial X$ which agrees with physical model, fig. 1.
- The corrected system (21)-(23) includes the term

$$-\frac{C_F}{\sqrt{\mathrm{Da}}}\left[\frac{\partial}{\partial X}\left(V\sqrt{U^2+V^2}\right)-\frac{\partial}{\partial Y}\left(U\sqrt{U^2+V^2}\right)\right]$$

  which agrees with problem assumptions, but the wrong system (5)-(7) not includes this term.
- The corrected system (21)-(23) includes the seepage velocity which agrees with the non-Darcy assumptions while this hypothesis is not included in the wrong system of Saleem *et al*. [1].

Based on all these differences, it can be included that the mathematical formulations and all the obtained results presented in Saleem *et al*. [1] are wrong.

Notes on the range of Darcy number in Saleem *et al.* [1]

We will start with the following mathematical rule:

$$\frac{A_5}{A_1}\frac{\mathrm{Pr}}{\mathrm{Da}}\Omega \to 0, \quad \mathrm{Da} \to \infty \tag{24}$$

So, to simulate the fluid-flow in the porous medium, values of Darcy number must be small. The recent published studies revealed that the preferred range of Darcy number between 10^{-2}-10^{-6}, see for example Krishna *et al*. [3, 4] and Nguyen *et al*. [5].

Therefore, the range of Darcy number used in Saleem *et al*. [1] is not correct and did not represent the porous medium.

Important note

Here it should be mentioned that Saleem *et al*. [1] did their best in producing this study but this study will be used as a reference in the future woks, so it must be corrected.

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