DAMAGE DETECTION OF LONG-SPAN BRIDGE STRUCTURES BASED ON RESPONSE SURFACE MODEL

by

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In order to solve the problem of high risk and low precision of existing damage detection methods for long-span Bridges, a new method based on fourth-order polynomial response surface model is proposed. Response surface model is constructed by using fourth order polynomial function. The parameters of the finite element model of the bridge are modified according to the response surface model. Based on the finite element model, the modal strain energy before and after the damage of the element was calculated, and the damage index of the element was obtained, so as to realize the damage detection of the long-span bridge structure. Experimental results show that the proposed method can accurately detect the damage location of long-span Bridges under different damage conditions, and the detection error of damage degree is less than 1%, which has a broad application prospect.

Key words: *response surface model, long-span bridge, fourth-order polynomial, structural damage, model fitting, detection*

Introduction

As an important vehicle for urban development, Bridges connect various regions for economic transactions, information exchange and other activities [1]. With the continuous increase in the number of long-span Bridges and the continuous extension of service life, the bridge structure is gradually damaged, seriously affecting the normal service function of the bridge [2, 3]. Timely monitoring of the health condition of long-span Bridges is the key to ensure the normal use of Bridges. Response surface model (RSM) has been widely used in bridge damage detection. The method of approximating a set of input variables and output variables with mathematical model is called RSM method [4]. It is widely used in structural optimization, where input variables and output variables are also called independent variables and response variables [5, 6]. The RSM has the function of predicting the finite element model parameters, and can modify the finite element model parameters to detect the structural damage of Bridges. There are many different RSM. Radial basis function model, polynomial model and Chebyshev model are common RSM. Many researches show that the fourth-order polynomial model is the best model for damage detection of long-span Bridges. Therefore, this paper chooses the fourth-order polynomial model as the RSM to complete the damage detection of long-span Bridges.

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Detection of long-span bridge structures based on fourth-order polynomial response surface model

Construction of fourth-order polynomial response surface model

The expression between the real response value and the approximate value in the fourth-order polynomial response model [7] is shown:

$$y(x) = \bar{y}(x) + \phi \tag{1}$$

where $y(x)$ represents the real value of the response, which is not known, $\bar{y}(x)$ – the approximate value of the response, which is a known polynomial, ϕ – the random error between the approximate value and the real value, which follows the $(0, \sigma^2)$ standard normal distribution, and σ^2 – the standard deviation.

All types of functions are calculated by polynomial regression analysis, because Weierstress polynomial approximation principle shows that all functions can be approximated by polynomials [8]. The high-order polynomial model can be used to solve the high-order non-linear problem between the structural response of long-span bridges and the design parameters of bridges [9, 10]. The fourth-order polynomial RSM is constructed as shown in formula (2):

$$\begin{gathered}\bar{y} = \partial_0 + \partial_1 x_1 + \partial_2 x_2 + \cdots + \partial_H x_H + \\ +\partial_{H+1} x_1^2 + \partial_{H+2} x_2^2 + \cdots + \partial_{2H} x_H^2 + \\ +\partial_{2H+1} x_1^3 + \partial_{2H+2} x_2^3 + \cdots + \partial_{3H} x_H^2 + \\ +\partial_{3H+1} x_1^4 + \partial_{3H+2} x_2^4 + \cdots + \partial_{4H} x_H^4 + \\ +\sum_{i \neq j} \partial_{ij} x_i x_j\end{gathered} \tag{2}$$

where $\bar{y}$ denotes the approximate response of fourth-order polynomial, $x_i \in [x_i^l, x_i^u]$, $i \in (1, H)$, x_i^l and x_i^l – the maximum and minimum values of bridge design parameters, H – the number of input variables, ∂_i – the unknown coefficient, which needs to be determined by fitting the sample data.

The minimum number of samples needed to restore is $(H+1)(H+2)/2 + 2H$, but these data cannot meet the high-precision detection criteria for damage of long-span bridges. To this end, variance analysis method is used to test bridge design parameters [11], *i. e.* significance test, optimization of parameters, and the method of finding the sum of residual squares:

$$RSS = \sum_{i=1}^{n} (y_i - \tilde{y}_i)^2 \tag{3}$$

where n is the number of sample points of the RSM.

The R^2 test and RMS test are commonly used to test the accuracy of RSM in the following forms:

$$R^2 = 1 - \frac{\sum_{j=1}^{n} [y_{RS}(j) - y(j)]^2}{\sum_{j=1}^{n} [y(j) - \bar{y}(j)]^2} \tag{4}$$

$$RMS = \frac{1}{2\overline{y}}\sqrt{\sum[y(j) - \overline{y}(j) - y_{RS}(j)]^2} \tag{5}$$

where y_{RS} represents the calculated value of the RSM of the j^{th} sample, $y(j)$ – the calculated value corresponding to the finite element calculation value of the j^{th} sample, and $\overline{y}$ – the mean value of the finite element calculation result.

In addition, when the accuracy of bridge damage structure detection is high, the value of R^2 is close to 1, the value of RMS is close to 0.

Figure 1 is the process of constructing the fourth-order polynomial response model. From fig. 1, it can be seen that the polynomial RSM can be constructed through the steps of function construction, function fitting and accuracy test to ensure that the prediction accuracy of the model conforms to the damage detection standard. In the follow-up, the function model fitting is introduced in detail [12].

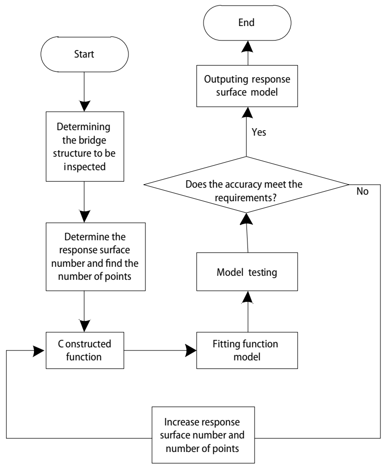

Figure 1. Construction process of polynomial RSM

Fitting of response surface function model

The fourth-order polynomial response surface function (RSF) is used to define n independent variables $x_1, x_2, \cdots, x_n$ and sample m groups of sample points. The form of the RSF after m-order experimental analysis is as follows:

$$y^g = e_0 + \sum_{i=1}^{m} e_i x_i{}^g + \sum_{1\le i\le j\le n}^{m} e_j t x_i{}^g + \eta \tag{6}$$

where $g = 1, 2, \cdots, m$, y^g means response, $x_i{}^g$ and $x_j{}^g$ are design variables, e_0, e_i, and e_j are regression coefficients.

Fourth-order polynomials have $s = (n+1)(n+2)/2$ coefficients. The model is expressed in matrix form as shown:

$$y^g = x^g e \tag{7}$$

where $e = [e_0, e_1, \cdots, e_{s-1}]^G$.

The matrix in eq. (8) is used to complete the estimation:

$$Y \approx XE \tag{8}$$

In the formula:

$$Y = [y^1, y^2, \cdots, y^m]^G \tag{9}$$

$$X = \begin{bmatrix} 1 & x_1^1 & x_1^1 & \cdots & (x_n^1)^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_1^m & x_2^m & \cdots & (x_n^m)^2 \end{bmatrix} \tag{10}$$

The least square method is used to solve the problem.

$$\hat{e} = \left(X^G X\right)^{-1} X^G Y \tag{11}$$

The RSF model is obtained by substituting $\hat{e}$ into eq. (7).

Strain energy damage index of element modal

Fourth-order polynomial model is used to modify the finite element model of bridge. According to the finite element model, the element modal strain energy before and after structural damage is calculated to obtain the element damage index, including the information of damage location and degree.

The change of dynamic characteristics of long-span bridges is often caused by the reduction of stiffness, while the stiffness decreases due to structural damage. Therefore, the damage of long-span bridges must be detected and repaired as soon as possible [13-15]. The dynamic characteristics of long-span bridges are reflected in the changes of natural frequencies and modes [16]. For long-span bridges, the method for calculating the modal strain capacity of the i^{th} order mode of the damaged element j is as follows:

$$MSE_{ij} = \frac{1}{2}\int_i \left(EI\right)_j \times \left\{\left[\psi_i''(x)\right]^2\right\} \mathrm{d}x \tag{12}$$

$$MSE_{ij}' = \frac{1}{2}\int_i \left(EI\right)_j' \times \left\{\left[\psi_i''(x)\right]'\right\}^2 \mathrm{d}x \tag{13}$$

where MSE_{ij} and MSE_{ij}' represent the i^{th} order modal strain energy of j element before and after damage, $(EI)_j$ and $(EI)_j'$ represent the stiffness of j element before and after damage, $[\psi_i''(x)]$ and $[\psi_i''(x)]'$ represent the i^{th} order modal mode shapes before and after damage. Then the damage index calculation method of the j^{th} element of bridge structure under the mode of the i^{th} order is as follows:

$$\vartheta_{ij} = \frac{\left|MSE_{ij} - MSE_{ij}'\right|}{MSE_{ij}} \tag{14}$$

where ϑ_{ij} represents the damage index of bridge damage structure in the i^{th} mode of ^{j} element.

Relevant research shows that the first four modes strain energy of element is the most sensitive to damage detection, and the detection effect of structural damage is the best. By defining the total damage index (index algebra) and enhancing the sensitivity of damage index to structural damage detection, the specific method is as follows: setting the damage index of structural j element is the algebraic sum of the first four order element modal strain energy damage index, as shown:

$$\vartheta_j = \vartheta_{1j} + \vartheta_{2j} + \vartheta_{3j} + \vartheta_{4j} \tag{15}$$

In the eq. (15), ϑ_j represents the damage index of j element of long-span bridge structure.

Experimental analysis

In order to verify the effectiveness of the RSM-based damage detection method for long-span Bridges, an experiment is needed.

Finite element model construction of long span bridges

Taking a long-span bridge as the research object, the performance of the proposed method to detect structural damage is verified. The experimental bridge is a four-span prestressed concrete bridge with a length of 182 m + 2 × 277 m + 182 m. It is a typical long-span bridge. The experimental bridge has been equipped with health monitoring system to obtain the bridge parameters and construct the finite element model. The bridge response values are: longitudinal second-order, transverse first-order, and vertical third-order natural frequencies. The allowable value of finite element model design and damage condition of bridge are shown in tabs. 1 and 2.

Table 1. Permissible parameters of finite element model design of bridges

Parameter number	Position	Permissible value of parameter (accurate to one decimal point)
AD1	Main girder at tower-girder joint	0.1-0.85
AD2	Left middle span of main girder	0.1-0.85
AD3	Left 1/4 span of main girder	0.1-0.85
AD4	Right midspan of main girder	0.1-0.85
AD5	Right 1/4 span of main girder	0.1-0.85

Based on the finite element model of the long-span constructed bridge, the experimental research is carried out.

Table 2. Bridge damage condition setting

	Parameter number	Working condition		
		1	2	3
Elastic modulus loss rate [%]	AD1	0.32	0.12	0.21
	AD2	0.24	0.35	0.15
	AD3	0.44	0.32	0.51
	AD4	0.51	0.15	0.71
	AD5	0.12	0.54	0.45

Accuracy test of response surface model R

The prediction accuracy of RSM affects the detection accuracy of bridge structure damage. The R^2 test and relative root mean square error (RMS) test are used to test the prediction accuracy of detection RSM. Four-order polynomial RSM is used to detect structural damage of long-span bridges. Radial basis function RSM and orthogonal polynomial RSM are selected to compare their performance. The results of model accuracy test under the two inspection modes are shown in figs. 2 and 3.

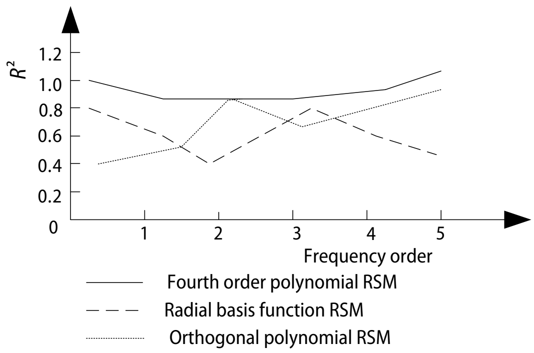

Figure 2. Comparison of model accuracy based on R^2 test

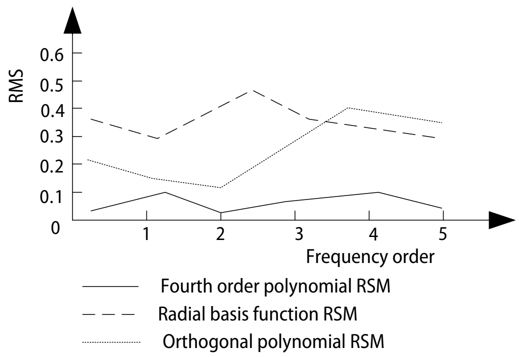

Figure 3. Comparison of model accuracy based on RMS test

The R^2 value is close to 1, and RMS value is close to 0, which proves that the RSM has high accuracy. Analysis of figs. 2 and 3 shows that in R^2 test, the fourth-order polynomial

RSM has the closest accuracy, which is above 0.9, while the radial basis function RSM and the orthogonal polynomial RSM have the highest accuracy of only 0.8 and 0.82. In RMS test, the fourth-order polynomial RSM has the closest accuracy of 0.1, and the radial basis function RSM has the highest accuracy of 0.25-0.48. Meanwhile, the accuracy of the orthogonal polynomial RSM is between 0.12 and 0.35. The data show that the fourth-order polynomial RSM is most in line with the high precision standard, and the accuracy of predicting the parameters of the finite element model is better. Therefore, the finite element model constructed has strong reliability, which provides effective data for the scientific detection of damage of long-span bridges and reduces the difficulty of detection.

Damage detection and analysis

The results of damage detection of different long-span bridges by the proposed method are shown in tabs. 3-5.

Table 3. Damage detection results of working condition 1

Design parameters of finite element model	Whether the damage position is judged accurately?	Damage degree		
		Actual value [%]	Detection value [%]	Damage degree detection error [%]
AD1	Yes	0.32	0.321	0.30
AD2	Yes	0.28	0.279	0.36
AD3	Yes	0.36	0.362	0.56
AD4	Yes	0.12	0.121	0.83
AD5	Yes	0.54	0.538	0.37

Table 4. Damage detection results of working condition 2

Design parameters of finite element model	Whether the damage position is judged accurately?	Damage degree		
		Actual value [%]	Detection value [%]	Degree of damage detection error [%]
AD1	Yes	0.12	0.1203	0.25
AD2	Yes	0.15	0.1506	0.40
AD3	Yes	0.23	0.229	0.43
AD4	Yes	0.34	0.338	0.59
AD5	Yes	0.25	0.251	0.40

Table 5. Damage detection results of working condition 3

Design parameters of finite element model	Whether the damage position is judged accurately?	Damage degree		
		Actual value [%]	Detection value [%]	Degree of damage detection error [%]
AD1	Yes	0.23	0.231	0.43
AD2	Yes	0.25	0.251	0.40
AD3	Yes	0.37	0.369	0.27
AD4	Yes	0.45	0.448	0.44
AD5	Yes	0.11	0.505	0.45

From tabs. 3-5, it can be seen that the proposed method can accurately detect the damage location of long-span bridges under different damage conditions. The detection error of damage degree is less than 1%, and most of the detection errors are less than 0.5%. Therefore, the damage degree of long-span bridges detected by the proposed method is basically consistent with the actual situation. It can be used as a high-precision damage detection method for bridges. The health status of building structures provides a new way.

Conclusions

In this paper, response surface model is used to detect the damage of long-span bridges. There are many kinds of RSM. According to the previous experimental results, the predictive performance of the fourth-order polynomial RSM is outstanding. Therefore, this paper studies a detection method of long-span bridges based on the fourth-order polynomial RSM. In practical application, the finite element model of bridge needs to be constructed. In this paper, the fourth-order polynomial RSM is used to predict the parameters of the finite element model. Experiments verify that the model has high prediction accuracy and provides accurate data basis for damage detection. This is also the reason why the method in this paper has high accuracy in detecting long-span bridge structures.

The simulation results show that the proposed method has excellent performance in detecting the damage of bridge structures. It can truly reflect the damage degree and location of long-span bridge structures. It is of great significance for monitoring the health status of building structures. However, the stability of the detection method has not been tested in this experiment, and there will be some disturbances in the actual detection process. How to ensure better detection stability with higher precision is the main direction of the following in-depth study.

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