VERIFICATION OF APPLICABILITY OF THE TWO-EQUATION TURBULENCE MODELS FOR TEMPERATURE DISTRIBUTION IN TRANSITIONAL FLOW IN AN ELLIPTICAL TUBE

by

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Original scientific paper
https://doi.org/10.2298/TSCI19S4113L

To increase the efficiency, elliptical tubes are often used in cross-flow heat exchangers. For these kinds of heat exchangers the flow field in the tubes exhibits irregularities. Therefore, various flow regimes can be observed: the turbulent, the transitional, and even the laminar one. Therefore, applying typical turbulence models for numerical calculations may cause significant errors, when flow in the heat exchanger tubes is in the transitional or laminar regime. Hence, the average values of flow velocities and temperature in heat exchanger tubes can be calculated incorrectly. The paper presents empirical verification of applying the basic two-equation turbulence models for a transitional flow of water in an elliptical pipe of a heat exchanger.

Key words: *transitional flow, heat transfer, experimental investigation, turbulence model*

Introduction

Cross-flow heat exchangers with elliptical tubes are designed for turbulent flow conditions, but the nature of their damages, as described in papers [1-4], show that the mass-flow rate of the fluid-flow in particular tubes, as well as the flow velocity may vary significantly. It implies that Reynolds numbers get values from the transitional or even the laminar range.

In the typical CFD analysis of heat and fluid-flow in heat exchangers the turbulence models are applied, in order to analyze the turbulence phenomenon. However, the application of these models may cause significant errors, when the fluid-flow regime in particular tubes is not turbulent but transitional or laminar. Because the previously described phenomenon may occur in exploitation practice, so it becomes important to check, which turbulent models can be applied to different analysis of the mentioned equipment, especially at low Reynolds numbers. It is assumed, that such values are often typical for tube damages in heat exchangers. These were the areas, where temperatures reached very high values, and fluid with lower velocity (water in most cases) was heated the most [2, 3, 5, 6]. Consequently, the large temperature of tube wall caused the extensive thermal loading and stresses, what finally led to tube fracture.

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Many studies on fin-and-tube heat exchangers, and heat transfer correlation for internal flow in heat exchanger tubes have been performed so far [1-4, 7-11]. The two-equation turbulence models *e. g*. *k*-epsilon, *k*-omega, and SST [12-14], usually applied in engineering practice, are best fit for modeling a developed turbulent flow. However, their usefulness for transitional and/or laminar flow cases is questionable. One of the options for them is the transitional turbulence SST model, modified by Menter *et al*. [15] and Langtry and Menter [16], referred in the paper as SST-TR. It is a so called Gamma-Theta transitional turbulence model, that allows the CFD analysis, when the Reynolds numbers have low values (this model can be applied not only for the developed turbulent flow) and flow regime changes from turbulent via transitional to the laminar.

Temperature distributions in a cross-section of fluid-flow in an elliptical tube, obtained from the numeric calculations (using the mentioned turbulent flow models), have been analyzed in the paper, and compared with the measurement results (achieved on an experimental stand). Verification is needed for instance for appropriate modeling of the coupled field (fluid-flow-thermal-structural) analysis, that allow to determine stress in a heat exchanger construction upon the known pressure and temperature fields (referring to fluid and a vessel wall respectively).

Experimental set-up

Empirical verifications of the turbulence models, available in the literature, cover usually flow issues or flow-heat ones for the simple geometries, *e. g*. fluid-flow on a flat wall [14]. Such verification is missing for more complex geometries, undoubtedly for those such as an elliptic-profile pipe. For that reason, an appropriate testing stand was constructed in Institute of Power Engineering at Cracow University of Technology, Cracow, Poland. The elliptical tube, which is a part of the stand, is the same as used in typical high performance heat exchangers with cross-stream flow [17].

The testing stand diagram and its assemblies are presented in figs. 1 and 2. Water from the supplementary tank – 1 is pumped to the main (supplying) tank – 2, where the water level is kept constant. Then it flows through the elliptical tube – 4 (A = 14 mm, B = 36 mm,

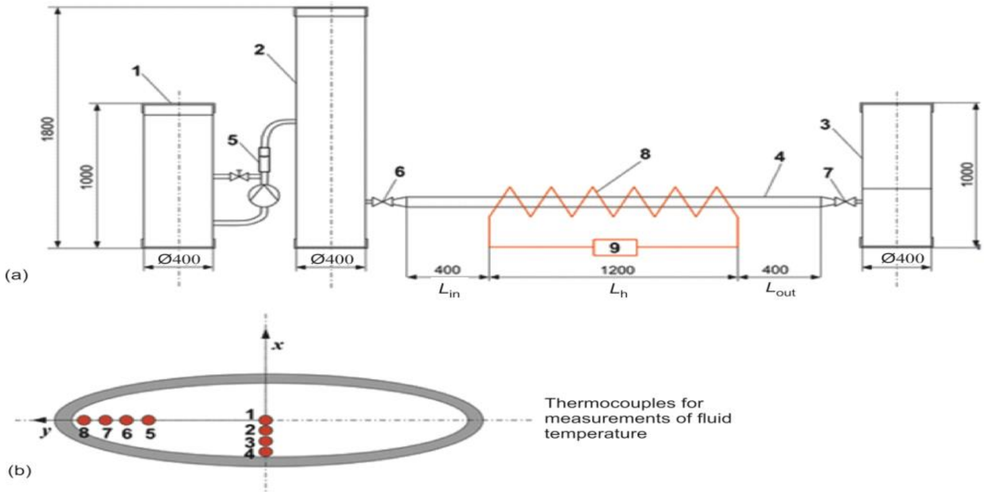

Figure 1. Scheme of experimental set-up (a) and the location of thermocouples (b) for water temperature measurement, dimensions in the drawing in [mm]

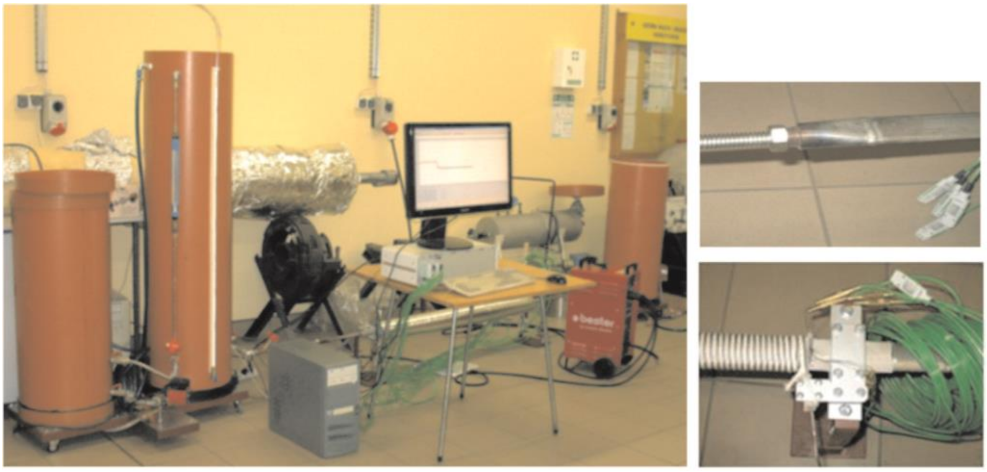

Figure 2. View of the testing stand and its selected assemblies

δ_t = 1.8 mm) and to the outflow tank – 3, equipped with overflow, that enables keeping the constant water level. The by-pass is installed in order to minimize the amount of water directed to the tank – 2 in case of lower flow through the tested tube. The flow is realized through the water level difference in both tanks – 2 and 3. The volumetric flow rate through the tube, is controlled with the control valve – 6 located in the inlet section. The valve – 7 in the outlet section enables the flow cut. The actual water volume flow rate in the elliptical tube is measured with the rotameter – 5.

The water flowing through the tube is heated with the heating wire – 8 with wound diameter ∅ = 2.8 mm, which is assembled on the outer wall of the tube and connected to the supplying device – 9. The welding machine Bester STB 250 is used for it because it allows an easy control of electric current. The length of the heating segment is 1200 mm. Water temperature is measured with jacket thermocouple elements NiCr-Ni (K-type, the jacket outer diameter ∅ = 0.5 mm). The elements are affixed in two surfaces, the first in front of and the second behind the heating segment. In both cases 3 mm apart from the ends of the heating segment. Location of the hot junctions of the thermocouples is presented in fig. 1(b). As one can see, they are placed along the major and minor axis of the ellipse. They enable control of the temperature distribution of water across the cross-section of tube.

In order to eliminate impact of the inlet section on the temperature distribution in the cross-section of water flowing through the elliptical pipe, the heating segment of the testing stand is located in the distance L_{in} = 400 mm from the inlet, fig. 1(a). The distance is significantly greater than the twenty-diameter hydraulic distances recommended in such cases [18] (for the tested elliptical pipe of d_h = 16.2 mm it equals L_{in}/d_h = 24.7 > 20). It is assumed therefore, that fluid-flow in the heating segment is stable and fully developed. The inlet and outlet sections of the tube were separated from the tube heating section to avoid longitudal heat transfer. Completing the data on the testing stand, it can be stated, that the distance between the individual coils of the wire is maintained by placing the silicon fabric cord ϕ 3 mm (in order to prevent the short-circuits). Moreover in order to minimize the heat losses, the heating wire, a part of which wound on the elliptical pipe can be seen in fig. 2, is covered with the insulation silicon fabric 1.8 mm thick. The whole set is insulated additionally with three layers of mineral wool, covered with aluminum foil from outside. So, the total insulation thickness is ca. 150 mm. Therefore, the heat losses to the surrounding are considered as negligible.

Experimental results and their analysis

The conducted tests cover the flows of the transitional regime, *i. e*. for Reynolds numbers from 2200 to 3700. It is the regime, where flow turbulence is low, but it cannot be excluded from the numerical calculations. Given Reynolds numbers are close to the critical value for the laminar flow, *i. e*. $Re_c = 2100$. It has to be emphasized, that it is the conventional critical value, suitable only for flow in smooth round tubes. It can be different for the elliptical tubes, even if the hydraulic diameter is used to calculate Reynolds number. Flows, with Reynolds number close to 2100, can occur in the tubes of a heat exchanger, where fluid velocity is very low (it can occur also in case of reverse flows, *i. e*. opposite to the intended ones [4]).

The tests allowed verifying, which of the known turbulence models: k-ε, k-ω, SST, or SST-TR (transitional turbulence gamma-theta model) is suitable for modeling the transitional regime flows. The propriety was assessed comparing temperature distributions achieved with experimental tests on one hand with the ones determined with numerical model for an elliptical pipe, loaded with heat flux of constant density (the values as maintained during testing), on the other.

It has to be added, that choosing the turbulence model properly enables to determine fluid temperature properly in the wall-adjacent areas (where the largest temperature gradients exists), that allows consequently to determine the proper values of wall temperatures, using the wall functions available in CFD codes. In the typical coupled field (fluid-flow – thermalstructural) analysis the obtained wall temperatures are used as thermal loads [3, 4]. Therefore validating the turbulence model is essential, because applying the model that does not work properly may yield final results that deviate significantly from the reality, *e. g*. the stress in tubes.

Water temperature distributions were determined in both, the inlet and the outlet cross-sections of flow in the elliptical tube, at heat flux, q_{appl}, between 8000 W/m^2 and 13000 W/m^2. For each load, the water flow rate was changed between 0.027 kg/s and 0.045 kg/s to ensure the transitional flow regime for these thermal conditions.

The experimental tests and numerical calculations were carried out for the steady-state conditions. The data achieved from tests enabled to verify the numerical calculations for the modeled system, carried out with the ANSYS CFX code. Due to the symmetry, the model considers only a quarter of an elliptical pipe, fig. 3.

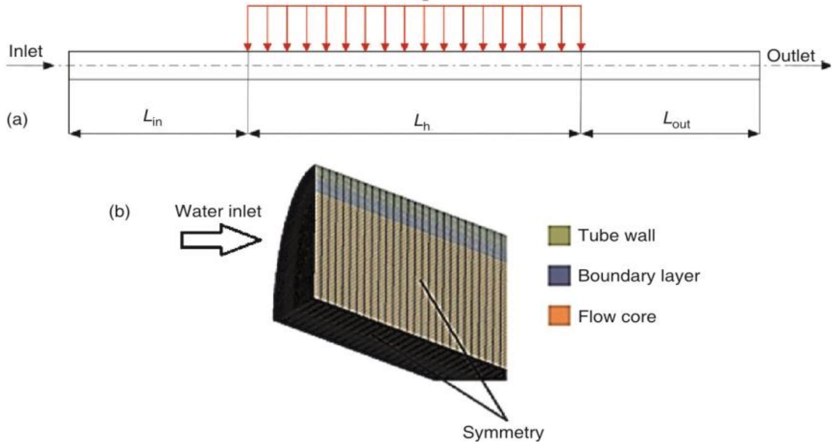

Figure 3. Thermal load of the modeled elliptical pipe (a) and a part of the discrete model (b) *(for colour image see journal web site)*

The discrete model, presented in fig. 3, consists of the elliptical tube and fluid-flowing in it. On the outer surface of tube wall the heat flux, q_{appl}, is applied. In order to increase modeling accuracy of flow and-heat processes occurring in the elliptical pipe, relation between water physical properties and temperature is assumed. This was possible, using the steam-water charts IAPWS97 built into the ANSYS CFX program.

The discrete flow model was created according to the ANSYS CFX code recommendation [19], so that the y^+ values in the wall-adjacent layer did not exceed 5. In case of $y^+ > 5$, the SST-TR model that considers change from the turbulent into the transitional flow does not work properly, *i. e*. fluid temperatures in the wall-adjacent layers are too high.

As presented in fig. 3(a), fluid is heated at length $L_{\mathrm{h}} = 1.2$ m with the heating wire, producing constant thermal power. Heat loss analysis was carried out in order to determine heat flux, q_{appl}, that was subsequently used in numerical calculations (the loss will still exist, despite of applying a thick thermal insulating material, although it will be rather minimized). The lost heat flow, for the steady-state test conditions, was determined from the relation:

$$Q_{\mathrm{loss}} = PL_{\mathrm{h}}\alpha_{\mathrm{p}}\left(T_{\mathrm{ins}} - T_{\mathrm{a}}\right) \tag{1}$$

It was assumed in the considered case, that the heat transfer coefficient between insulation wall and the air equals $\alpha_{\mathrm{p}} = 10$ W/m^{2}K. Temperature of the outer surface of insulation was determined as the arithmetic mean of the measurements carried out in four zones of the heating segment, separated from the inlet cross-section by 100, 400, 800, and 1100 mm (the temperature of outer surface of insulation was measured for each of the zones, in three equally distributed points on the circumference).

Heat flow obtained from the heating system is calculated from the formula:

$$Q = UI \tag{2}$$

When both values, *i. e*. Q_{loss} and Q are known, it enables it to estimate heat losses that occurred at the tests. Their values (expressed as a percentage) were determined from the relation:

$$\xi_{\mathrm{p}} = \frac{Q_{\mathrm{loss}}}{Q}100\% \tag{3}$$

As expected, losses during the tests were very low and were in the range between 0.35% and 0.7%. It was assumed for the further calculations, that the value $\xi_{\mathrm{p}} \approx 1\%$, that corresponds to calculated heat flow equal:

$$Q_{\mathrm{calc}} = \left(1 - \xi_{\mathrm{p}}\right)Q \approx 0.99Q \tag{4}$$

Finally, heat flux, used in ANSYS CFX program as a boundary condition of the second kind on the pipe outer wall, was determined upon the equation:

$$q_{\mathrm{appl}} = \frac{Q_{\mathrm{calc}}}{A_o} = \frac{Q_{\mathrm{calc}}}{P_o L_{\mathrm{h}}} \tag{5}$$

where P_{o} is a circumference of an ellipse with semi-axis $a + \delta_{\mathrm{t}}$ and $b + \delta_{\mathrm{t}}$ (a and b are lengths of semi-axis of an ellipse creating flow cross-section, and δ_{t} is the wall thickness).

Results of numerical calculations for water temperature distributions in the outlet cross-section of the heating segment, which is $L = L_{\mathrm{in}} + L_{\mathrm{h}} = 1600$ mm, fig. 3(a) downstream the inlet cross-section were compared with the experimental results, as mentioned. The comparison was carried out for all the given values of water mass-flow rate, $\dot{m}$, and heat flux,

q_{appl}. The data for the both values are given in tab. 1, with the corresponding Reynolds numbers. In tab. 2 there are given locations for hot junctions of the thermocouples, *x*- and *y*-co-ordinates, fig. 1(b).

Table 1. Values of parameters (water mass-flow, applied heat flux, and Reynolds number), for which the experimental tests and the numerical calculations were carried out (for validating turbulence models for elliptical tube)

Mass-flow rate of water, $\dot{m}$ [kgs^{-1}]	Applied heat flux q_{appl} [Wm^{-2}]	Reynolds number
0.042	9000	3668
0.037		3100
0.032		2710
0.027		2230
0.045	11500	3718
0.037		3165
0.032		2795
0.027		2330

Table 2. Location of thermoelements in the inlet and outlet cross-sections of the heating segment of the tested elliptical tube, fig. 1(b)

Thermocouple	Co-ordinate [mm]		Thermocouple	Co-ordinate [mm]	
	x	*y*		*x*	*y*
1	0	0	5	0	11.00±0.25
2	–1.55±0.25	0	6	0	12.55±0.25
3	–3.10±0.25	0	7	0	14.10±0.25
4	–4.70±0.25	0	8	0	15.65±0.25

It can be stated, upon tab. 2, that location accuracy of the thermoelements equals half the outer diameter of their jackets, *i. e*. 0.25 mm.

Comparison of the test data collected at thermoelements locations 1 to 8, fig. 1(b) and tab. 2, with the computational results where the turbulence models were applied: k-ε, k-ω, SST, and SST-TR is presented in figs. 4 and 5. These are the typical examples, for $\dot{m}$ = 0.037 kg/s and q_{appl} = 9000 W/m^2, as well as for $\dot{m}$ = 0.042 kg/s and q_{appl} = 11500 W/m^2 respectively.

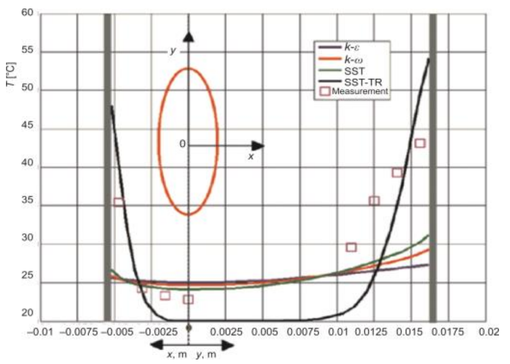

Figure 4. Comparing water temperature distributions in the outlet surface of the heating segment, achieved with different two-equation turbulence models and measured in points 1 to 8, fig. 1(b), for $\dot{m}$ = 0.037 kg/s and q_{appl} = 9000 W/m^2

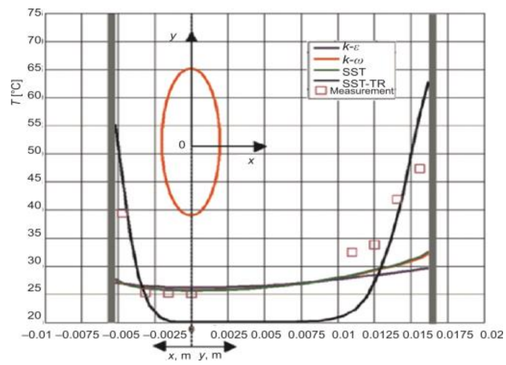

Figure 5. Comparing water temperature distributions in the outlet surface of the heating segment, achieved with different two-equation turbulence models and measured in points 1 to 8, fig. 1(b), for $\dot{m}$ =0.042 kg/s and q_{appl} =11500 W/m^2

Test results and numerical calculations, presented in figs. 4 and 5 in a form of temperature change of water flowing through the tube, lengthwise it is both elliptical axis, fig. 1(b), for the outlet cross-section of the heating segment, make it clear, that there is a satisfying accordance on one hand and visible differences on the other. It applies either to the core flow area or the wall-adjacent areas, depending on the applied turbulence model. Based on the accuracy obtained by the calculations in predicting the temperature field in comparison with the experimental values, it can be stated that:

– the two-equation models: k-ε, k-ω, and SST reflect properly the field of temperature in the flow core, but give high inconsistency close to the wall and
– the SST-TR model reflects temperatures in the flow core less accurately (in comparison to the above mentioned ones), but shows significantly higher accuracy in the wall-adjacent area.

Quantitative comparison of the achieved results allows to state, that the k-ε, k-ω, and SST models have a tendency to *equalize* temperatures in the whole cross-section of the flowing fluid. The highest observed differences between the temperatures at the wall and in the flow core equal ca. 8 °C to 10 °C, for q_{appl} equal to 9000 W/m^2 and 11500 W/m^2, respectively. But in case of the SST-TR model, the differences can reach even ca. 39 °C to 50 °C for the same q_{appl} range, and it reflects the reality better. Unfortunately, the model shows the lowered temperatures in the flow core, and the highest observed differences reached up to ca. 5 °C for q_{appl} = 9000 W/m^2 and up to ca. 7 °C for q_{appl} = 11500 W/m^2

The SST-TR model gives an accurate temperature distribution within the boundary--layer and near-wall region, however in the flow core, the temperature is nearly equal to the inlet temperature. The model assumes that the major heat transfer process occurs within the near wall region, therefore the temperature profile is inflated.

Keeping in mind the objective of the potential use of the paper findings, for instance for the coupled field (flow-thermal and structural) analysis of working condition of heat exchangers with elliptical tubes, it is more important to reflect properly the fluid temperature changes in the wall-adjacent areas. In this case, the k-ε, k-ω and SST models produce significantly lower temperatures at the tube wall, and the differences between the measured values and the calculated ones with numerical modeling can reach even up to 20 °C for q_{appl} = 9000 W/m^2 and up to 25 °C for q_{appl} = 11500 W/m^2. For the SST-TR model they reach up to ca. 8 °C and up to ca. 10 °C, respectively, for q_{appl} as given previously. It has to be stated at the same time, that when applying this model for calculations, it gives the inflated temperature values in the wall-adjacent areas, i.e. they are higher than measured.

Conclusions

Summing up the results of the paper, it has to be stated, that the two-equation turbulence models k-ε, k-ω, and SST, used for calculating flow in elliptical tubes, have difficulties in determining the proper temperature values of fluid-flowing in the wall-adjacent area. Obviously, it could be expected, because they were deigned to model only the turbulent flows. The paper incorporates the appropriate calculations, using the mentioned models, to estimate the scope of *deviation* they can cause.

A better solution for the previous situation would be applying the SST-TR (SST with transitional turbulence Gamma-Theta model) model that allows analyzing the whole flow regime. It manifests significantly higher accuracy with the test results, especially in the key locations, *e. g*. the flow-heat ones in the wall-adjacent areas. As a result, the achieved values of wall temperature and indirectly the stresses in tubes of heat exchanger will be calculated more accurately.

The presented problems are especially important when carrying out coupled analyzes, of not only the separated subsystems of a device, but also of its whole construction, where other flow regimes may occur in certain zones apart from the assumed turbulent flows, *e. g.* the transitional flow regime. It is important in case of the SST-TR model, that the fluid temperatures determined in the wall-adjacent areas upon the model are a little higher than the measured ones. Therefore, the determined thermal stresses will be also higher, giving a certain security margin.

Nomenclature

A – minor axis of an ellipse, [mm]
A_o – outer surface area of tube, [m^2]
B – major axis of an ellipse, [mm]
a – length of minor semi-axis of an ellipse, [m]
b – length of major semi-axis of an ellipse, [m]
d_h – hydraulic diameter of a tube, [m]
I – electric current in a resistance wire, [A]
L_h – length of a heating segment, [m]
L_{in} – length of an inlet section, [mm]
L_{out} – length of an outlet section, [mm]
$\dot{m}$ – mass-flow rate, [kgs^{-1}]
P – circumference (calculated for an outer thermal insulation layer), [m]
P_o – outer elliptical tube circumference, [m]
Q – heat flow, [W]
Q_{calc} – calculated heat flow, [W]
Q_{loss} – heat flow losses, [W]
q_{appl} – applied heat flux used for CFD calculations, [Wm^{-2}]
Re – Reynolds number [$= \dot{m}\, d_h/(\pi ab\mu)$], [–]
T_a – ambient temperature, [°C]
T_{ins} – mean temperature of outer surface of insulation layer, [°C]
U – voltage over a resistance wire, [V]
y^+ – non-dimensional distance between the wall and the nearest node, [–]

Greek symbols

α_p – heat transfer coefficient from ambient to the outer surface of insulation, [Wm^{-2}K^{-1}]
δ_t – wall thickness of an elliptical tube, [mm]
μ – water dynamic viscosity, [kgm^{-1}s^{-1}]

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Paper submitted: December 3, 2018
Paper revised: December 28, 2018
Paper accepted: January 24, 2019