MODELING OF DEFORMATION PROCESSES IN LITHOSPHERIC STRUCTURES DURING THEIR STATIC INTERACTION

by

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Original scientific paper
https://doi.org/10.2298/TSCI19S2591T

We consider a model of lithospheric structures contacting along rectilinear geological faults as a system of composite plates on an elastic foundation. A simplification of the block element method for different-sized blocks is proposed. We also describe an approach that is a modification of the block element method using the method of eigenfunctions. The method is considered on the example of a static interaction problem of extended plates on the surface of an elastic layer for a given surface load. As a result we obtain the representations of solutions describing the surface displacements. The application of the proposed approach will allow us to draw conclusions about the effect of the physical and mechanical properties of lithospheric structures and the type of fault on the nature of displacements in the geological environment which are applicable for studying the structure of faults in the upper part of the earth's crust.

Key words: *elastic foundation, composite coating, fault, static interaction, localized load, factorization*

Introduction

The creation of the seismic process theory is connected with the establishment of regularities in the deformation processes of the crustal level. The upper part of the Earth's crust is an important object of study, since human activity is concentrated in it. Despite the rare manifestations of seismicity in the form of induced geomechanical processes, the environmental damage caused by technogenic earthquakes can be significant. The reaction caused by external influences depends both on their intensity and on the energy saturation of the crustal structures, the distribution and magnitude of the stresses in them. To date, seismic phenomena are being actively studied in the framework of models based on the theory on the block structure of the lithosphere [1, 2, *etc*.]. Natural heterogeneity of the geological environment, manifested in the form of structural defects (inclusions of different scale, multiple cracks, tectonic faults, *etc*.), determines its properties (strength and deformation) forming a response to external effects.

The purpose of this work is to study the stress-strain state of lithospheric structures that contact along a rectilinear fault and modeled by the coating-substrate system, in case of their static interaction.

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At the scale of the Earth's structure, lithospheric plates can be considered as coatings of relatively small thickness. Interest in the study of problems for bodies with coatings in a wide range of settings [3-5] is currently growing due to the active use of plates and shells as the main elements of structures in modern technology.

In this paper we consider a model of lithospheric structures contacting along rectilinear geological faults in the form of a system of composite plates on an elastic foundation. A deformable foundation can be modeled in different ways, it can be considered: an elastic half-space (including layered), a layer or a stack of layers, a three-dimensional block structure, *etc*. The Winkler foundation can be taken as the simplest model. In this paper, the case of an elastic layer is considered and a simplification of the block element method for different-sized blocks is proposed. Representations of solutions describing surface displacements and allowing to study the effect of the structure, the properties of its elements and the characteristics of the faults on the stress-strain state are obtained.

Formulation of the problem

We have studied the static problem of the extended plates, interaction on the surface of an elastic layer $\{-\infty < x_1, x_2 < +\infty, -H \le x_3 \le 0\}$ for a given localized surface load. The horizontal co-ordinate plane is aligned with the middle surface of semibounded deformable plates with parameters averaged over the thickness, bordering along one of the horizontal axes. The displacements of the plate surface are described by the linearized equations presented in [6]:

$$\mathbf{R}_j(\partial x_1, \partial x_2)\mathbf{u}_j(x_1, x_2) - \mathbf{E}_j\mathbf{g}_j(x_1, x_2) = \mathbf{b}_j(x_1, x_2), \quad x_1 \in \Omega_j, \quad x_2 \in \mathrm{R} \tag{1}$$

where $\mathbf{u}_j = \{u_{j1}, u_{j2}, u_{j3}\}$ is the vector of the displacements of the middle surface of the plate with the number *j*, and elements of matrix differential operators $\mathbf{R}_j(\partial x_1, \partial x_2)$ have representations:

$$R_{11}^j = \frac{\partial^2}{\partial x_1^2} + \varepsilon_{j1}\frac{\partial^2}{\partial x_2^2}, \quad R_{22}^j = \frac{\partial^2}{\partial x_2^2} + \varepsilon_{j1}\frac{\partial^2}{\partial x_1^2}, \quad R_{12}^j = R_{21}^j = \varepsilon_{j2}\frac{\partial^2}{\partial x_1 \partial x_2}$$

$$R_{33}^j = \varepsilon_{j3}\left(\frac{\partial^4}{\partial x_1^4} + 2\frac{\partial^4}{\partial x_1^2 \partial x_2^2} + \frac{\partial^4}{\partial x_2^4}\right), \quad R_{13}^j = R_{23}^j = R_{31}^j = R_{32}^j = 0$$

Here

$$\varepsilon_{j1} = \frac{1-\nu_j}{2}, \quad \varepsilon_{j2} = \frac{1+\nu_j}{2}, \quad \varepsilon_{j3} = \frac{h_j^2}{12}, \quad \varepsilon_{j5} = \frac{1-\nu_j^2}{E_j h_j}$$

where ν_j is the Poisson's ratio, E_j – the Young's modulus, ρ_j – the density, h_j – the thickness of the j^{th} plate. In eq. (1) matrix $\mathbf{E}_j = \mathrm{diag}\{-\varepsilon_{j5}, -\varepsilon_{j5}, \varepsilon_{j5}\}$, vector $\mathbf{g}_j = \{g_{jk}\}$ describes the acting from the side of the substrate on the lower boundary of the stress plate; $\mathbf{b}_j = -\varepsilon_{j5}\mathbf{t}_j$, $\mathbf{t}_j = \{t_{jk}\}$, $k = \overline{1.3}$ – vector of surface load, and $\Omega_1 = \{x_1 : 0 < x_1 < +\infty\}$, $\Omega_2 = \{x_1 : -\infty < x_1 < 0\}$.

The boundary conditions determined by the type of contact interaction between plates are given in the area of their joining:

$$\mathbf{L}_1(\partial x_1, \partial x_2)\mathbf{u}_1(0, x_2) + \mathbf{L}_2(\partial x_1, \partial x_2)\mathbf{u}_2(0, x_2) = \mathbf{f}(x_2) \tag{2}$$

where $\mathbf{L}_j(\partial x_1, \partial x_2)$, $(j = 1,2)$ – given differential operators, which form, as well as the vector function, **f**, determines the nature of the contact interaction between plates. The use of the straight normal hypothesis requires the specification of four boundary conditions for each point of the boundary. The formulations of the boundary conditions are given, for example, in [6].

Displacements $\mathbf{u}(\mathbf{x},t)=\{u_k\}$ $(k=\overline{1,3})$, $\mathbf{x}=(x_1,x_2,x_3)$, of the elastic isotropic layer are described by the Lame equations of the form:

$$(\lambda+\mu)\,\mathrm{grad\,div}\mathbf{u}+\mu\Delta\mathbf{u}=0$$

Here, the elastic properties of the medium are characterized by the modules of λ, μ, and ρ is the foundation density.

For an elastic foundation with the use of the Green's matrix-function, it is possible to write out integral relations on its surface that connect the stresses and displacements:

$$\mathbf{u}(x_1,x_2,0)=\frac{1}{4\pi^2}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\mathbf{K}(\alpha_1,\alpha_2)\mathbf{G}(\alpha_1,\alpha_2,0)\exp\left[-\mathrm{i}(\alpha_1x_1+\alpha_2x_2)\right]\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \tag{4}$$

where $\mathbf{K}=\|K_{nm}\|_{n,m=1}^{3}$ is the Green matrix of an elastic medium which elements depend on the physical and mechanical characteristics of the substrate (λ, μ, ρ), $\mathbf{G}$ = $\mathrm{V}_2\mathbf{g}$, V_2 – the 2-D operator of the integral Fourier transform with parametres α_1, α_2. Methods for constructing Green matrices for various types of elastic media are given in [7]. For media models with complex properties, these relationships, also called influence functions, can be obtained experimentally.

The contact between the substrate and the coating is considered ideal, which indicates the continuity of displacements and stresses at the interface of the coating/substrate:

$$\mathbf{u}_j(x_1,x_2)=\mathbf{u}(x_1,x_2),\ \ \mathbf{g}_j(x_1,x_2)=\mathbf{g}(x_1,x_2) \tag{3}$$

for $j=1,\ x_1>0$, for $j=2,\ x_1<0$, $-\infty<x_2<+\infty$.

Solution method

The application of the Fourier transform with respect to the variable x_2 to the equations for plate displacements (1) leads to a system of ODE solution of which takes the form:

$$\bar{\mathbf{u}}_j(x_1,\alpha_2)=\mathrm{V}^{-1}(x_1)\left\{\mathbf{R}_j^{-1}(-\mathrm{i}\alpha_1,-\mathrm{i}\alpha_2)\left[\mathbf{E}_j\mathbf{G}_j(\alpha_1,\alpha_2)+\mathbf{B}_j(\alpha_1,\alpha_2)\right]\right\}+$$
$$+\sum_{k=1}^{2}C_{jk}(\alpha_2)\bar{\mathbf{v}}_k^{(j)}(x_1,\alpha_2),\ \ \text{for}\ j=1,\ x_1>0,\ \text{for}\ j=2,\ x_1<0$$

where $\bar{\mathbf{u}}_j(x_1,\alpha_2)=\mathrm{V}(\alpha_2)\mathbf{u}_j(x_1,x_2)$, $\mathrm{V}(\alpha_2)$, $\mathrm{V}^{-1}(x_1)$ is the operator of the integral Fourier transform with respect to x_2 and inverse operator with respect to α_1, respectively, $\mathbf{R}_j(-\mathrm{i}\alpha_1,-\mathrm{i}\alpha_2)=\mathrm{V}_2(\alpha_1,\alpha_2)\mathbf{R}_j(\partial x_1,\partial x_2)$, $\mathbf{G}_j=\mathrm{V}_2\mathbf{g}_j$, $\mathbf{B}_j=\mathrm{V}_2\mathbf{b}_j$, $\bar{\mathbf{v}}_k^{(j)}=\{\bar{v}_{km}^{(j)}\}$ – general solutions of homogeneous systems (1), bounded at infinity in the corresponding half-planes.

After the application of 2-D Fourier transform the conditions for the ideal conjugation of plates with a substrate (3) take the form:

$$\mathbf{U}(\alpha_1,\alpha_2)=\mathbf{U}_1(\alpha_1,\alpha_2)+\mathbf{U}_2(\alpha_1,\alpha_2),\ \ \mathbf{G}(\alpha_1,\alpha_2)=\mathbf{G}_1(\alpha_1,\alpha_2)+\mathbf{G}_2(\alpha_1,\alpha_2) \tag{5}$$

where $\mathbf{G}(\alpha_1,\alpha_2,0)\equiv\mathbf{G}(\alpha_1,\alpha_2)=\mathrm{V}_2\mathbf{g}(x_1,x_2,0)$, $\mathbf{U}(\alpha_1,\alpha_2,0)\equiv\mathbf{U}(\alpha_1,\alpha_2)=\mathrm{V}_2\mathbf{u}(x_1,x_2,0)$, $\mathbf{U}_j=\mathrm{V}_2\mathbf{u}_j$ $(j=1,2)$.

By applying the integral Fourier transform (4) we obtain functional relations for the elastic layer:

$$\mathbf{U}(\alpha_1,\alpha_2)=\mathbf{K}(\alpha_1,\alpha_2)\mathbf{G}(\alpha_1,\alpha_2)$$

Using the conditions for the plates with substrate (5), we obtain a system of functional equations for the Fourier images of the contact stresses between the coating and the substrate $\mathbf{G}_1(\alpha_1,\alpha_2)\equiv\mathbf{G}_1^+$ and $\mathbf{G}_2(\alpha_1,\alpha_2)\equiv\mathbf{G}_2^-$. In the static case, the system of loaded equations has the form:

$$\mathbf{K}_1\mathbf{G}_1^+(\alpha_1,\alpha_2)+\mathbf{K}_2\mathbf{G}_2^-(\alpha_1,\alpha_2)=\sum_{j=1}^{2}\mathbf{R}_j^{-1}(-\mathrm{i}\alpha_1,-\mathrm{i}\alpha_2)\left[\mathbf{E}_j\mathbf{G}_j(\alpha_1,\alpha_2)+\mathbf{B}_j(\alpha_1,\alpha_2)\right]-$$
$$-\mathbf{R}_{11}^{-1}(\alpha_1,\alpha_2)\left[\mathbf{E}_1\mathbf{G}_1(\mathrm{i}\lambda,\alpha_2)+\mathbf{B}_1(\mathrm{i}\lambda,\alpha_2)\right]-\mathbf{R}_{21}^{-1}(\alpha_1,\alpha_2)\left[\mathbf{E}_2\mathbf{G}_2(-\mathrm{i}\lambda,\alpha_2)+\mathbf{B}_2(-\mathrm{i}\lambda,\alpha_2)\right]-$$
$$-\mathbf{R}_{12}^{-1}(\alpha_1,\alpha_2)\left[\mathbf{E}_1\mathbf{G}'_{1,\alpha_1}(\mathrm{i}\lambda,\alpha_2)+\mathbf{B}'_{1,\alpha_1}(\mathrm{i}\lambda,\alpha_2)\right]-\mathbf{R}_{22}^{-1}(\alpha_1,\alpha_2)\left[\mathbf{E}_2\mathbf{G}'_{2,\alpha_1}(-\mathrm{i}\lambda,\alpha_2)+\mathbf{B}'_{2,\alpha_1}(-\mathrm{i}\lambda,\alpha_2)\right]+$$
$$+\sum_{j=1}^{2}\sum_{k=1}^{2}C_{jk}(\alpha_2)\mathbf{V}_k^{(j)}(\alpha_1,\alpha_2)$$

where $\mathbf{V}_k^j(\alpha_1,\alpha_2)=\mathrm{V}(\alpha_1)\bar{\mathbf{v}}_k^{(j)}$ – Fourier transform with respect to x_1, j,k = 1,2; $\lambda=|\alpha_2|$:

$$\mathbf{R}_{j1}^{-1}(\alpha_1,\alpha_2)=(\alpha_1\mp\mathrm{i}\lambda)^{-1}\operatorname*{Res}_{\eta_1=\pm\mathrm{i}\lambda}\mathbf{R}_j^{-1}(-\mathrm{i}\eta_1,-\mathrm{i}\alpha_2)+(\alpha_1\mp\mathrm{i}\lambda)^{-2}\left[\mathbf{R}_j^{-1}(-\mathrm{i}\eta_1,-\mathrm{i}\alpha_2)(\eta_1\mp\mathrm{i}\lambda)^2\right]\Big|_{\eta_1=\pm\mathrm{i}\lambda}$$
$$\mathbf{R}_{j2}^{-1}(\alpha_1,\alpha_2)=(\alpha_1\mp\mathrm{i}\lambda)^{-1}\left[\mathbf{R}_j^{-1}(-\mathrm{i}\eta_1,-\mathrm{i}\alpha_2)(\eta_1\mp\mathrm{i}\lambda)^2\right]\Big|_{\eta_1=\pm\mathrm{i}\lambda}$$
$$\mathbf{K}_j(\alpha_1,\alpha_2)=\pm\left[\mathbf{K}(\alpha_1,\alpha_2)-\mathbf{R}_j^{-1}(-\mathrm{i}\alpha_1,-\mathrm{i}\alpha_2)\mathbf{E}_j\right]$$

In previous and following equations, the superscript of the symbols «±», «∓» corresponds to the value $j=1$, the subscript corresponds to the value j = 2.

System of functional equations is solved by the Wiener–Hopf method [8] by factorization with respect to the parameter α_1 with respect to the real axis. However, in contrast to the case of a steady-state oscillatory process [9], together with unknown values of vector functions $\mathbf{G}_j(\pm\mathrm{i}\lambda,\alpha_2)$, system also includes unknown values of the derivatives $\mathbf{G}'_{j,\alpha_1}(\pm\mathrm{i}\lambda,\alpha_2)=\mathrm{d}\mathbf{G}_j/\mathrm{d}\alpha_1\big|_{\alpha_1=\pm\mathrm{i}\lambda}$ at the points $\alpha_1=\pm\mathrm{i}\lambda$, where $\pm\mathrm{i}\lambda$ are zeros of order 2 of $\det\mathbf{R}_j(-\mathrm{i}\alpha_1,-\mathrm{i}\alpha_2)$. Therefore, in the static case, to determine $\mathbf{G}_j(\pm\mathrm{i}\lambda,\alpha_2)$ and $\mathbf{G}'_{j,\alpha_1}(\pm\mathrm{i}\lambda,\alpha_2)$, $j=1,2$, it is also necessary to construct functional relations by differentiating the expressions obtained for $\mathbf{G}_j(\alpha_1,\alpha_2)$. As a result, we obtain a system of linear algebraic equations for the determination of unknowns $\mathbf{G}_j(\pm\mathrm{i}\lambda,\alpha_2)$ and $\mathbf{G}'_{j,\alpha_1}(\pm\mathrm{i}\lambda,\alpha_2)$, which should then be substituted into the Fourier transforms of the contact stresses between the coating and the substrate.

Fourier transforms of displacements $\mathbf{U}_j$ have a representation:

$$\mathbf{U}_j(\alpha_1,\alpha_2)=\mathbf{R}_j^{-1}(-\mathrm{i}\alpha_1,-\mathrm{i}\alpha_2)\left[\mathbf{E}_j\mathbf{G}_j(\alpha_1,\alpha_2)+\mathbf{B}_j(\alpha_1,\alpha_2)\right]+$$
$$+\sum_{k=1}^{2}C_{jk}(\alpha_2)\mathbf{V}_k^{(j)}(\alpha_1,\alpha_2)-\mathbf{R}_{j1}^{-1}(\alpha_1,\alpha_2)\left[\mathbf{E}_j\mathbf{G}_j(\pm\mathrm{i}\lambda,\alpha_2)+\mathbf{B}_j(\pm\mathrm{i}\lambda,\alpha_2)\right]+$$
$$+\sum_{k=1}^{2}C_{jk}(\alpha_2)\mathbf{V}_k^{(j)}(\alpha_1,\alpha_2)-\mathbf{R}_{j1}^{-1}(\alpha_1,\alpha_2)\left[\mathbf{E}_j\mathbf{G}_j(\pm\mathrm{i}\lambda,\alpha_2)+\mathbf{B}_j(\pm\mathrm{i}\lambda,\alpha_2)\right]-$$
$$-\mathbf{R}_{j2}^{-1}(\alpha_1,\alpha_2)\left[\mathbf{E}_j\mathbf{G}'_{j,\alpha_1}(\pm\mathrm{i}\lambda,\alpha_2)+\mathbf{B}'_{j,\alpha_1}(\pm\mathrm{i}\lambda,\alpha_2)\right] \tag{6}$$

where

$$\mathbf{B}'_{j,\alpha_1}(\pm\mathrm{i}\lambda,\alpha_2)=\frac{\mathrm{d}\mathbf{B}_j}{\mathrm{d}\alpha_1}\bigg|_{\alpha_1=\pm\mathrm{i}\lambda}$$

The latter will contain as unknowns only $C_{jk}(\alpha_2)$, j,k = 1,2, determined from the boundary conditions in the area of plates joining (2). Further, the scheme of the solution coincides with the one described in [9] for steady oscillations of the plates. For this, it is necessary to apply the inverse Fourier transform $\mathrm{V}^{-1}(x_1)$ to the expressions (6) with respect to the parameter α_1.

Then, after applying the Fourier transform with respect to the variable x_2 to the boundary conditions (2), which as a result will take the form:

$$\mathbf{L}_1(\partial x_1,-\mathrm{i}\alpha_2)\bar{\mathbf{u}}_1(0,\alpha_2)+\mathbf{L}_2(\partial x_1,-\mathrm{i}\alpha_2)\bar{\mathbf{u}}_2(0,\alpha_2)=\mathbf{F}(\alpha_2) \tag{7}$$

where $\mathbf{F}(\alpha_2)=\mathrm{V}(\alpha_2)\mathbf{f}(x_2)$, we can substitute in eq. (7) $\bar{\mathbf{u}}(x_1,\alpha_2)=\mathrm{V}^{-1}(x_1)\mathbf{U}_j(\alpha_1,\alpha_2)$ and solve the resulting linear algebraic system with respect to the unknown $C_{jk}(\alpha_2)$, j,k = 1,2.

The values of the surface displacements of the coating / substrate system are obtained by applying the inverse Fourier transform $\mathrm{V}^{-1}(x_2)$ with respect to the parameter α_2 to $\bar{\mathbf{u}}_1(x_1,\alpha_2)$ for $x_1>0$ and to $\bar{\mathbf{u}}_2(x_1,\alpha_2)$ – for $x_1<0$:

$$\mathbf{u}_j(x_1,x_2)=\mathrm{V}^{-1}(x_2)\bar{\mathbf{u}}_j(x_1,\alpha_2) \text{ for } j=1,\ x_1>0,\ \text{ for } j=2,\ x_1<0,\ \ x_2\in\mathrm{R}$$

The Fourier inversion is carried out by means of the theory of residues.

The approach presented in this paper can be extended to the problems of thermoelasticity [10, 11]. In this case, equation (1) takes the form:

$$\mathbf{R}_j(\partial x_1,\partial x_2)\mathbf{u}_j(x_1,x_2)-\mathbf{E}_j\mathbf{g}_j(x_1,x_2)=\mathbf{b}_j(x_1,x_2)+\mathbf{s}_j(x_1,x_2),\ x_1\in\Omega_j,\ \ x_2\in\mathrm{R}$$

where

$$\mathbf{s}_j=\left\{2\varepsilon_{j2}\frac{\partial\theta_j}{\partial x_1},2\varepsilon_{j2}\frac{\partial\theta_j}{\partial x_2},-\frac{2\varepsilon_{j2}}{h_j}\alpha_{0j}\Delta\theta_j\right\}$$

is the vector of thermal stresses, α_{0j} – the thermal expansion coefficients, θ_j – the relative temperatures of plats.

The equations of a related problem for an isotropic elastic foundation will take the form [12]:

$$(\lambda+\mu)\mathrm{graddiv}\mathbf{u}+\mu\Delta\mathbf{u}-\gamma\mathrm{grad}\theta=0,\ \Delta\theta=0$$

where $\gamma=(3\lambda+2\mu)\alpha_0$ is the thermomechanical constant, α_0 – the thermal expansion coefficients of the foundation, θ – the relative temperature of the layer. In the linear theory of thermoelasticity it is implied that in the process of deformation small changes occur in the temperature $\theta=T-T_0$ in relation to the absolute temperature of the body, T_0. Connectivity of the equations is manifested through the temperature term $\gamma\mathrm{grad}\theta$.

The form of the matrix, **K,** for relation (4) can easily be obtained similarly to the Green matrix of the quasistatic problem given in [12] for an isotropic thermoelastic layer with given heat flux densities on the upper and lower boundary surfaces $\tau_1(x_1,x_2)\equiv\tau_1(x_1,x_2,0)$ and $\tau_2(x_1,x_2)\equiv\tau_2(x_1,x_2,-H)$. In this case, extended vectors **u** and **g** will be taken as the $\mathbf{u}=\{u_1,u_2,u_3,\theta\}$ and $\mathbf{g}=\{g_1,g_2,g_3,\tau\}$, where $\tau=-k_0\,\partial\theta/\partial x_3$ is the heat flux density and k_0 – coefficient of internal thermal conductivity of the elastic substrate. The conditions for the continuity of temperature and heat flux are fulfilled in the area of contact between the plates and the substrate, the boundary conditions for the temperature function reflect the thermal regime in the area of plate joining.

Numerous publications devoted to the study of problems for (elastic, thermoelastic, *etc.*) plates, coatings and shells, contain the results of using numerical methods (finite element method, boundary element method, *etc.*), for example, [13, 14]. However, the traditionally applied direct numerical methods of analysis often do not provide an opportunity to track the influence of individual parameters of the system on the solution, as well as their mutual influence. Some methods become inapplicable in extended areas. The main difficulty of the proposed approach is the need for matrix factorization. Approximate factorization can be carried out using the methods described in [15].

Conclusions

Thus, the algorithm of the differential factorization method has been modified in the work with reference to the study of static spatial contact problems for plates on a deformable foundation, modeling lithospheric structures with faults or defective coatings. A method for solving boundary value problems of interaction between multi-type plates contacting along a rectilinear fault is proposed. Despite the universality of the topological method of the block element [16], in the case of rectilinear and flat boundaries, when the boundary problems for two semi-bound plates on a 3-D foundation are considered as models of a different-sized block structure under the effect of a localized stationary surface load, a simplified method can be used. This method opens the possibility of studying the effect of the plate and foundation properties, as well the effect of various boundary conditions in the contact area of the coating components on the main characteristics of the stress-strain state of the block structure under consideration under different contact conditions of the plates on the fault.

The study of the lithospheric plates interaction along rectilinear faults and the development of methods for determining the characteristics of the stress-strain state of such structures are very important, since at the regional scale the faults are mostly rectilinear. The models that consider lithospheric plates in the form of a system of contacting plates on the surface of an elastic 3-D substrate allow us to construct piecewise linear approximations for the curvilinear sections of the plate boundaries, using the developed approach for each straight section.

The approach to the construction of a model of multilayer extended plates leads to a single-layered plate model with reduced mechanical characteristics. The problems for such plates on an elastic foundation can be solved in the same way as the problems for homogeneous plates, in accordance with the mode under consideration, both stationary and steady.

The theoretical results obtained on the basis of the described approach can find applications not only in seismology, geophysics, and in engineering practice but can be also used to study the strength properties of materials with coatings, including defective ones, when evaluating the operational characteristics of the working parts of structural elements as well as machine parts, for example, in shipbuilding and aviation, in solving problems of the crack formation in widely applied multilayer coatings.

Acknowledgment

This publication was prepared as part of the SA SSC RAS 2018, project № 01201354241 and partially supported by the grant RFBR and administration of Krasnodar region 16-41-230184.

Nomenclature

x_j, $j=\overline{1,3}$ – Cartesian co-ordinates
i – imaginary unit
R – set of real numbers
π – Archimedes' constant

Δ – Laplace operator
V, V^{-1} – integral Fourier transform and it's inversion
V_2 – 2-D Fourier transform

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Paper submitted: October 22, 2018
Paper revised: November 9, 2018
Paper accepted: December 13, 2018