MULTILAYER METHOD FOR SOLVING A PROBLEM OF METALS RUPTURE UNDER CREEP CONDITIONS

by

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The paper deals with a parameter identification problem for creep and fracture model. The system of ordinary differential equations of kinetic creep theory is applied for describing this model. As for solving the parameter identification problem, we proposed to use the technique of neural network modeling, as well as the multilayer approach. The procedures of neural network modeling and multilayer approximation constructing application is demonstrated by the example of finding parameters for uniaxial tension model for isotropic steel 45 specimens at creep conditions. The solution corresponding to the obtained parameters agrees well with theoretical strain-damage characteristics, experimental data, and results of other authors.

Keywords: *creep, fracture, parameter identification, the Cauchy problem, ODE, artificial neural networks, multilayer neural networks.*

Introduction

In recent years, there is an increasing need for the description of deformation and fracture processes in complex temperature-power regimes for materials with complicated rheological characteristics including viscosity. These problems find wide application in different fields of science and technology, *e. g*. mechanical engineering and aerospace industry. Special attention is paid to the possibility of creep accounting at high and moderate temperatures for metal and composite structures. However, up to now, there has been no common approach to the description of this phenomenon, and there are dozens of various creep theories and their modifications, *e. g*. the aging theory, the hardening theory, the heredity theory, and the Rabotnov theory of structural parameters. It is not usually possible to reliably determine which of the theories is better to use in a particular case. Applying equations of any theory may be a very complex process, as equations used usually contain several material constants (creep characteristics), which complicated to obtain. In other words, these parameters can be determined by using information about a deformation process. A primary source of information is an experiment. Creep characteristics may depend on the type of used material and its condition, regime of loading, temperature, type of anisotropy, and other factors. Problems of their identification are very complicated. All these reasons indicate the need for a common

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approach to determining the parameters of models for various equation types. This paper provides a unified method for identifying parameters of models describing creep and fracture processes of structures. The method considered involves the use of experimental data. The principles and the techniques of neural network modeling, as well as the multilayer approach, are adopted as a basis for the approach developed in the work.

Problem statement

To describe the behavior of metals under creep conditions up to the fracture moment we use the system of the Rabotnov theory of structural parameters ODE in the form [1, 2]:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = f_1(\sigma,T)\Psi(\omega,T), \quad \frac{\mathrm{d}\omega}{\mathrm{d}t} = f_2(\sigma,T)\Psi(\omega,T) \tag{1}$$

where ε is the creep strain, ω – the scalar damage parameter, σ – the stress, t – the time, and T – the temperature. Functional relationships in the eq. (1) are determined by experimental data.

The function $\Psi(\omega,T)$ may be chosen in the form [2]:

$$\Psi(\omega,T) = \omega^{-\alpha}\left(1-\omega^{\alpha+1}\right)^{-m} \tag{2}$$

where α and m are model parameters that can depend on the temperature, T, in a general case.

Considering the eq. (2) and constant temperature, we get:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{f_1(\sigma)}{\omega^{\alpha}\left(1-\omega^{\alpha+1}\right)^{m}}, \quad \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{f_2(\sigma)}{\omega^{\alpha}\left(1-\omega^{\alpha+1}\right)^{m}} \tag{3}$$

The functions $f_1(\sigma)$ and $f_2(\sigma)$ we choose in the power form [2]:

$$f_1(\sigma) = B_{\varepsilon}\sigma^{n}, \quad f_2(\sigma) = B_{\omega}\sigma^{k}$$

where B_{ε}, B_{ω}, n, k are creep characteristics.

As the initial conditions for the system of eq. (3) we use:

$$\omega(0) = 0, \ \varepsilon(0) = 0 \tag{4}$$

If $\sigma = \sigma_0 = \mathrm{const}$, we can also get the solution of the Cauchy problem (3)-(4) in the same way as it is done in the article [3]:

$$\omega(t) = \left\{1-\left[1-(\alpha+1)(m+1)B_{\omega}\sigma_0^{k}t\right]^{\frac{1}{m+1}}\right\}^{\frac{1}{\alpha+1}} \tag{5}$$

$$\varepsilon(t) = \frac{B_{\varepsilon}\sigma_0^{n}}{B_{\omega}\sigma_0^{k}}\omega(t) \tag{6}$$

Considering that the damage parameter is equal to the unit at the fracture moment, from the eq. (5), we obtain the value of long-term strength t^* of this construction:

$$t^* = \left[(\alpha+1)(m+1)B_{\omega}\sigma_0^{k}\right]^{-1} \tag{7}$$

In the paper, neural network and multilayer approaches are used for the solution to the identification problem of creep model parameter for the uniaxial tension of isotropic aviation steel 45 cylindrical specimens at the constant temperature T = 850 °C.

Numerical results

In the case when stress is independent of creep strain and time, the solution of problem of eqs. (3) and (4) reduces to solving the Cauchy problem for the differential equation:

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{f_2(\sigma)}{\omega^{\alpha}\left(1-\omega^{\alpha+1}\right)^{m}} \tag{8}$$

with initial condition:

$$\omega(0) = 0 \tag{9}$$

The creep strain can be found by eq. (6) using the solution of problem (8)-(9).

Euler method

We apply our modification of the Euler method [4] to construct an approximate solution to the problem (8)-(9) [5-9]. For this purpose, we use the recurrence formula:

$$t_{i+1}(\omega) = t_i(\omega) + \frac{(\omega_{i+1}-\omega_i)}{f_2(\sigma)}\omega_i^{\alpha}\left(1-\omega_i^{\alpha+1}\right)^{m} \tag{10}$$

where $i = 0,...,N-1$. We determine the initial function by condition $t_0(\omega-0.5) = t_0$. Step sizes are chosen from the condition $\omega_i = 0.5 + i(\omega-0.5)/N$. As an approximate solution of the problem we shall use $t = t_N(\omega)$. From eq. (10) we obtain:

$$t_{i+1}(\omega) = t_i(\omega) + a(\omega-0.5)\mathrm{S}_i(\omega) \tag{11}$$

where

$$\mathrm{S}_i(\omega) = \left[0.5N + i(\omega-0.5)\right]^{\alpha}\left\{N^{\alpha+1} - \left[0.5N + i(\omega-0.5)\right]^{\alpha+1}\right\}^{m}, \quad a = \frac{N^{-\alpha-m-\alpha m-1}}{f_2(\sigma)}$$

Summing over i, we obtain:

$$t = t_0 + a(\omega-0.5)\sum_{i=0}^{N-1}\mathrm{S}_i(\omega) \tag{12}$$

Substituting the initial value, we have $t_0 = 0.5a\sum_{i=0}^{N-1}\mathrm{S}_i(0)$, whence:

$$t = a\left[0.5\sum_{i=0}^{N-1}\mathrm{S}_i(0) + (\omega-0.5)\sum_{i=0}^{N-1}\mathrm{S}_i(\omega)\right]$$

Material constants α, m, and a are obtained using ordinary least squares method and the experimental data, ω_q, t_q:

$$J = \sum_{q=1}^{l}\left\{t_q - a\left[0.5\sum_{i=0}^{N-1}\mathrm{S}_i(0) + (\omega_q-0.5)\sum_{i=0}^{N-1}\mathrm{S}_i(\omega_q)\right]\right\}^2 \to \min$$

On the figs. 1 and 2, we present the results of the calculations for $N = 6$.

The values of the error functional are for the exact solution $J = 0.332$ and the approximate solution $J = 0.284$, *i. e*. the approximate solution corresponds to the experimental data is somewhat better than the exact one. Analysis of the graphs shows that this correspondence is valid for all experimental points, except for three.

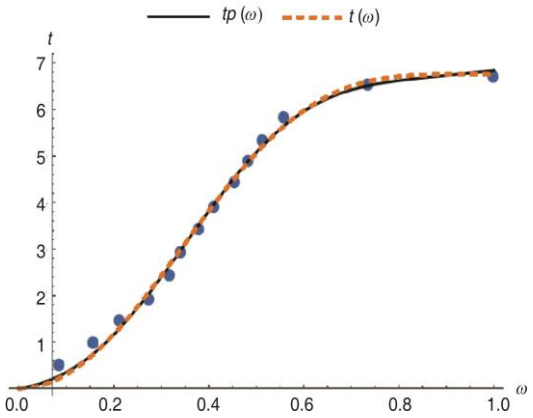

Figure 1. A graph of the exact solution of the Cauchy problem *t*(*ω*) and an approximate neural network solution *tp*(*ω*); material constants obtained by the ordinary least squares method; circles correspond to the experimental values

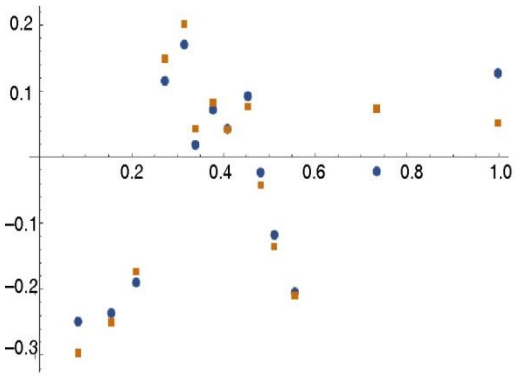

Figure 2. A matching error between the experimental data and the exact solution *t*(*ω*) of the Cauchy problem (squares), as well as between the experimental data an approximate solution *tp*(*ω*) (circles) in the case *N* = 6; neural network weights obtained by the ordinary least squares method

The trapezoidal rule

We apply our modification [5-9] of the trapezoidal rule [4]. For this purpose, we use the recurrence formula:

$$t_{i+1}(\omega) = t_i(\omega) + \frac{(\omega_{i+1} - \omega_i)}{2 f_2(\sigma)} \left[\omega_i^{\alpha} \left(1 - \omega_i^{\alpha+1}\right)^m + \omega_{i+1}^{\alpha} \left(1 - \omega_{i+1}^{\alpha+1}\right)^m \right] \tag{13}$$

where $i = 0,..., N-1$. The initial function and step sizes are chosen in the same way as before.

From eq. (13) we obtain:

$$t_{i+1}(\omega) = t_i(\omega) + a(\omega - 0.5)\left[\mathrm{S}_i(\omega) + \mathrm{S}_{i+1}(\omega)\right] \tag{14}$$

where $a = 0.5N^{-\alpha-m-\alpha m-1} / f_2(\sigma)$.

Summing over i, we obtain:

$$t = t_0 + a(\omega - 0.5)\left[2\sum_{i=1}^{N-1} \mathrm{S}_i(\omega) + \mathrm{S}_0(\omega) + \mathrm{S}_N(\omega) \right]$$

Substituting the initial value, we have $t_0 = 0.5a[2\sum_{i=1}^{N-1} \mathrm{S}_i(0) + \mathrm{S}_0(0) + \mathrm{S}_N(0)]$, whence:

$$t = a\left\{ 0.5\left[2\sum_{i=1}^{N-1} \mathrm{S}_i(0) + \mathrm{S}_0(0) + \mathrm{S}_N(0) \right] + (\omega - 0.5)\left[2\sum_{i=1}^{N-1} \mathrm{S}_i(\omega) + \mathrm{S}_0(\omega) + \mathrm{S}_N(\omega) \right] \right\}$$

Through substitution and simplifying, we get:

$$t = a\left\{ \begin{array}{l} \sum_{i=1}^{N-1} \mathrm{S}_i(0) + \omega N^{\alpha+m+\alpha m} 2^{-\alpha} \left(1 - 2^{-\alpha-1}\right)^m + \\ + (\omega - 0.5)\left[2\sum_{i=1}^{N-1} \mathrm{S}_i(\omega) + N^{\alpha+m+\alpha m} \omega^{\alpha} \left(1 - \omega^{\alpha+1}\right)^m \right] \end{array} \right\} \tag{15}$$

Material constants α, m, and a are obtained using ordinary least squares method and the experimental data:

$$J = \sum_{q=1}^{l} \left(t_q - a \left\{ \begin{array}{l} \sum_{i=1}^{N-1} \mathrm{S}_i(0) + \omega_q N^{\alpha+m+\alpha m} 2^{-\alpha} \left(1 - 2^{-\alpha-1}\right)^m + \\ +(\omega_q - 0.5)\left[2\sum_{i=1}^{N-1} \mathrm{S}_i(\omega_q) + N^{\alpha+m+\alpha m} \omega_q{}^{\alpha} \left(1 - \omega_q{}^{\alpha+1}\right)^m \right] \end{array} \right\} \right)^2 \rightarrow \min$$

The advantage of this approach in comparison with the artificial neural networks is that the same parameters are chosen as for the exact solution of the eq. (8).

On the figs. 3 and 4, we present the results of the calculations for $N = 2$.

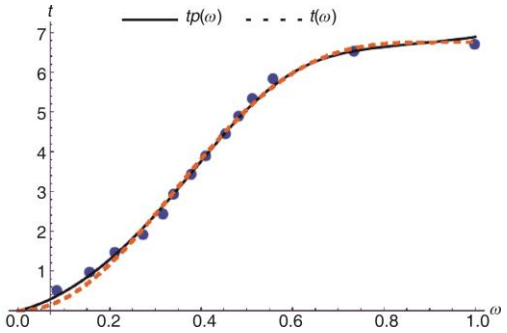

Figure 3. A graph of the exact solution of the Cauchy problem *t*(*ω*) and an approximate neural network solution (15) *tp*(*ω*); material constants obtained by the ordinary least squares method; circles correspond to the experimental values

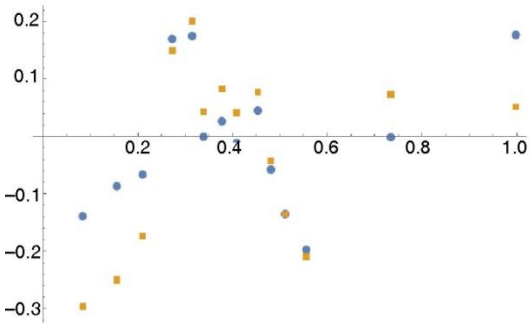

Figure 4. A matching error between the experimental data and the exact solution *t*(*ω*) of the Cauchy problem (squares), as well as between the experimental data an approximate solution (15) *tp*(*ω*) (circles) in the case *N* = 2; neural network weights obtained by the ordinary least squares method

For this case, the value of error functional for the approximate solution is equal to 0.185, which is significantly less than for the exact solution. From the analysis of the graphs, it can be seen that the exact and approximate solutions practically coincide on the whole interval, except for the initial area, on which the approximate solution is much better corresponds to the experimental data.

On the figs. 5 and 6, we present the results of the calculations for $N = 3$.

For this case, the value of error functional for the approximate solution is equal to 0.197. The analysis and results in this case are similar to those indicated in the previous paragraph.

An increase in the number of neurons does not lead to an increase in the accuracy of the correspondence to the experimental data.

Neural network approach

The first method will compare to a neural network approach [10, 11]. We will find a solution in the form of a neural network approximation:

$$u_N(\omega) = \sum_{i=1}^{N} c_i \nu(\omega, a_i)$$

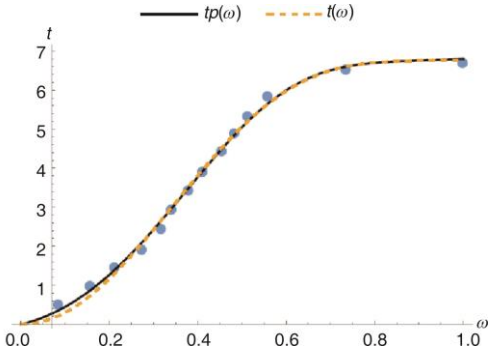

Figure 5. A graph of the exact solution of the Cauchy problem *t*(*ω*) and an approximate neural network solution (15) *tp*(*ω*); material constants obtained by the ordinary least squares method; circles correspond to the experimental values

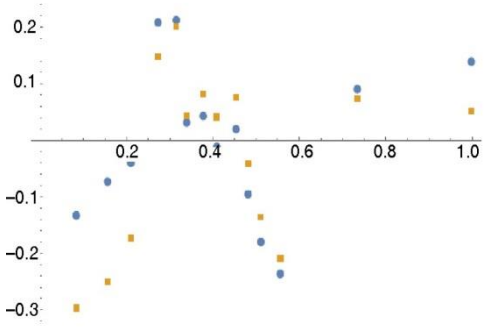

Figure 6. A matching error between the experimental data and the exact solution *t*(*ω*) of the Cauchy problem (squares), as well as between the experimental data an approximate solution (15) *tp*(*ω*) (circles) in the case *N* = 3; neural network weights obtained by the ordinary least squares method

where neural network weights are linear input parameters c_i and non-linear input parameters $\mathbf{a}_i$ as well as material constants α, m, and a. These parameters will obtain in the process of learning the network as minimizing the error functional:

$$J = \sum_{q=1}^{l}\left[t_q - u_N\left(\omega_q\right)\right]^2 + \sum_{j=1}^{M}\left[\frac{\mathrm{d}u_N}{\mathrm{d}\omega}(\tilde{\omega}_j) - \frac{\tilde{\omega}_j^{\alpha}(1-\tilde{\omega}_j^{\alpha+1})^m}{f_2(\sigma)}\right]^2 \to \min$$

We choose a function of the sigmoid type $\nu(\omega, \mathbf{a}_i) = th(a\omega + b)$ as a neural network activation function.

On the figs. 7 and 8, we present the results of the calculations for $N = 1$.

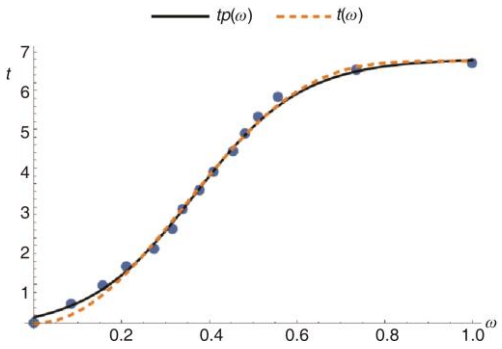

Figure 7. A graph of the exact solution of the Cauchy problem *t*(*ω*) and an approximate neural network solution (one neuron and neural network weights obtained by the ordinary least squares method); circles correspond to the experimental values

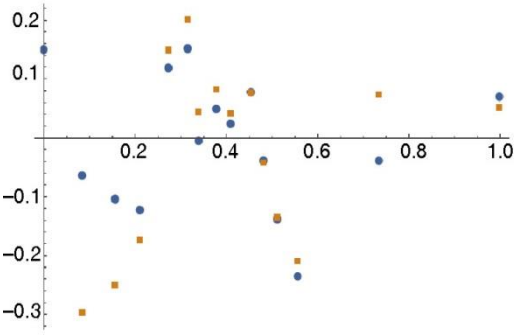

Figure 8. A matching error between the experimental data and the exact solution of the Cauchy problem (squares), as well as between the experimental data an approximate neural network solution (circles) in the case of one neuron (neural network weights obtained by the ordinary least squares method)

The value of error functional for the approximate solution is equal to 0.181, which is significantly less than for the exact solution.

On the figs. 9 and 10, we present the results of the calculations $N = 2$.

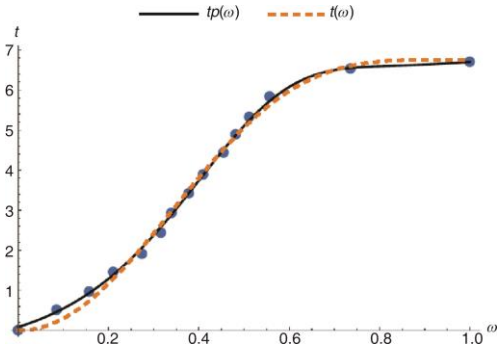

Figure 9. A graph of the exact solution of the Cauchy problem *t*(*ω*) and an approximate neural network solution (two neurons and neural network weights obtained by the ordinary least squares method); circles correspond to the experimental values

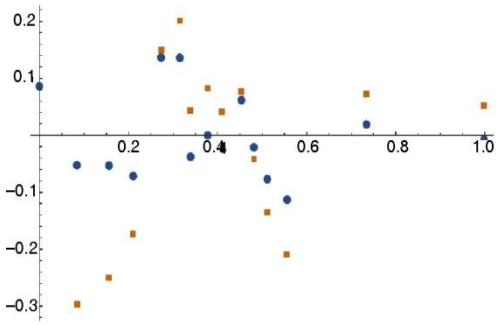

Figure 10. A matching error between the experimental data and the exact solution of the Cauchy problem (squares), as well as between the experimental data an approximate neural network solution (circles) in the case of two neurons (neural network weights obtained by ordinary least squares method)

For this case, the value of error functional for the approximate solution is equal to 0.0575, which is significantly less than for the exact solution. From the analysis of graphs, it can be seen that at the entire interval the approximate neural network solution is much better corresponds to the experimental data.

A further increase in the number of neurons does not lead to an increase in the accuracy of the correspondence to the experimental data.

Conclusion

Comparative testing of two methods for constructing approximate solutions concerning differential equations and experimental data is carried out. The first method (multilayer approach) allows building approximate solutions that better satisfy to experimental data than the exact solution with the same set of material constants. The second method (neural network approach) allows building approximate solutions which even better corresponding to the experimental data, that the first method, but requires the definition of a larger number of constants.

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References

[1] Rabotnov, Yu. N., *Creep Problems in Structural Members*, North-Holland Publishing Company, Amsterdam/London, 1969
[2] Gorev, B. V., *et al*., On Description of Creep and Fracture of the Materials with Non-Monotonic Variation of Deformation-Strength Properties, *Physical Mesomechanics Journal*, *5* (2002), 2, pp. 17-22
[3] Gorev, B. V., *et al*., Description of Creep and Fracture of Modern Construction Materials Using Kinetic Equations in Energy Form, *J. Appl. Mech. Tech. Phys.*, *55* (2014), 6, pp. 1020-1030
[4] Burden, R. L., *et al*., *Numerical Analysis*, Cengage Learning, Boston, Mass., USA, 2016

[5] Lazovskaya, T., Tarkhov, D., Multilayer Neural Network Models Based on Grid Methods, *Proceedings*, IOP Conf. Series: Materials Science and Engineering, Kazan, Russia, 2016, Vol. 158
[6] Lazovskaya, T., *et al*., Multi-Layer Solution of Heat Equation, in: *Advances in Neural Computation, Machine Learning, and Cognitive Research*, (Ed. Kyzanovski, B., *et al*.,), Moscow, Russia, 2017, pp. 17-22
[7] Vasilyev, A. N., *et al.*, Semi-Empirical Neural Network Model of Real Thread Sagging, *Studies in Computational Intelligence, 736* (2018), Oct., pp.138-146
[8] Zulkarnay, I. U., *et al*., A Two-Layer Semi-Empirical Model of Nonlinear Bending of the Cantilevered Beam, *Journal of Physics: Conference Series*, *1044* (2018), Conference 1, 012005
[9] Bortkovskaya, M. R., *et al*., Modeling of the Membrane Bending with Multilayer Semi-Empirical Models Based on Experimental Data, *Proceedings*, II International Scientific Conference "Convergent Cognitive Information Technologies" (Convergent'2017), Moscow, Russia, 2017
[10] Lazovskaya, T. V., *et al*., Parametric Neural Network Modeling in Engineering, *Recent Patents on Engineering, 11* (2017), 1, pp. 10-15
[11] Budkina, E. M., Neural Network Approach to Intricate Problems Solving for Ordinary Differential Equations, *Optical Memory and Neural Networks, 26* (2017), 2, pp. 96-109

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