DETERMINATION OF CONVECTIVE HEAT TRANSFER PARAMETERS BY THE CALORIMETRIC METHOD OF A THIN WALL

by

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The paper proposes a procedure of experimental data processing by the thin wall method for the diagnostics of stationary gas-flows. The procedure is based on solving an inverse convective heat transfer problem. It does not use smoothing and numerical differentiation of experimental data, and is resistant to measurement errors. Sensitivity coefficients are analyzed to find the most informative times for the determination of convective heat transfer parameters with the least influence of the measurement error. The numerical examples given show the effectiveness of the calculation procedure for the heat transfer coefficient and the gas-flow temperature determined from discrete measurements of the sensor temperature.

Key words: *gas-flow, heat transfer coefficient, calorimetric sensor, thin wall method, inverse heat transfer problem*

Introduction

The measurement of the heat flux density is an important task in the study of heat transfer processes in various science and engineering applications. It constitutes a significant aspect of the problem of aerodynamic heating of structures at supersonic flow velocities [1, 2]. Modeling of the effect of high-enthalpy gas-flows on heat-shielding materials requires the knowledge of the convective heat transfer parameters, such as the flow temperature, T_e, and the heat transfer coefficient, α_e.

Direct instrumental measurements are possible only for the flow temperature, T_e, but only in the low temperature region. The heat transfer coefficient, α_e, is the proportionality factor in the expression for the convective heat flux. It is not a thermodynamic quantity and therefore cannot be measured directly. The heat transfer coefficient, as well as the heat flux density, is determined by solving inverse heat transfer problems [3-5].

Much experience has been gained in studying the convective heat transfer of high-enthalpy gas-flows with the use of various sensors and experimental data processing techniques [6-12]. The simplest of these are calorimetric heat flux sensors operating by the method of a thermally thin wall with uniform through-thickness temperature. The schematic view of such a sensor is shown in fig. 1. The diameter and thickness of the sensor are selected in accordance with the experimental conditions, and are usually a few millimeters. The sensing element of the sensor is a thin disk – 1 (plate) of metal with high thermal conductivity (Cu), with

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an embedded thermocouple – 3. The disk is surrounded by thermal insulation – 2 on the lateral and rear sides. There can be a thin air gap – 4 between the disk and insulation to reduce the heat loss from the rear side. The heat flux density is determined by the heating dynamics of the sensitive element [11]. The determination procedure can be the following. The time derivative of the temperature is graphically calculated in the initial period by the heating curve of the sensor, which involves an element of subjectivity due to statistical measurement error. Or, the derivative is calculated numerically, which requires preliminary smoothing of the experimental temperature values. After the convective heat flux is calculated at a known gas temperature, the heat transfer coefficient is found. Its estimation accuracy is entirely related to the accuracy of measuring the rate of change in the average integral temperature of the sensitive element of the sensor. The main assumption of this method is that the time of measurement must be short in order to neglect the heat sink to the insulating layer.

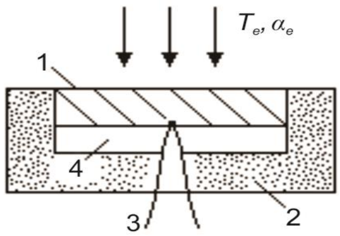

Figure 1. Schematic view of a calorimetric heat flux sensor:
1 – thin copper disk, 2 – insulation, 3 – thermocouple, 4 – air gap

The aim of this paper is to develop a procedure for the determination of heat transfer parameters by the thin wall method, based on the solution of an inverse problem resistant to measurement errors without smoothing and numerical differentiation of experimental data.

Mathematical formulation of the problem and analysis of sensitivity coefficients

The differential equation for the heating of a thermally insulated sensing element in the approximation of a thermally thin body suddenly heated by a high-enthalpy gas-flow can be written:

$$\rho c V \frac{\mathrm{d}T}{\mathrm{d}t} = \alpha_\mathrm{e} S \left(T_\mathrm{e} - T\right), \qquad T(0) = T_0 \tag{1}$$

where T is the (average integral) temperature, t – the time, ρ – the density, c – the specific heat, V and S are the material volume and the heating surface area of the sensing element, respectively, and α_e is the heat transfer coefficient. The subscript e stands for the gas-flow, and 0 is for the initial conditions.

The solution of eq. (1) for stationary values of α_e and T_e reads:

$$\theta = \frac{T - T_0}{T_\mathrm{e} - T_0} = 1 - \exp(-\tau), \quad \tau = \frac{\alpha_\mathrm{e} t}{\rho c L}, \quad L = \frac{V}{S} \tag{2}$$

where θ and τ are the dimensionless temperature and time, respectively, and L is the length scale equal to the thickness of the (sensing element) plate. The heat flux $q_\mathrm{e}(t)$ measured by the sensor obeys the equation:

$$q = \frac{q_\mathrm{e}(t)}{\alpha_\mathrm{e}\left(T_\mathrm{e} - T_0\right)} = \exp(-\tau) \tag{3}$$

where $q(\tau)$ is the dimensionless heat flux. Dependences (2) and (3) are represented by curves 1 and 2 in fig. 2.

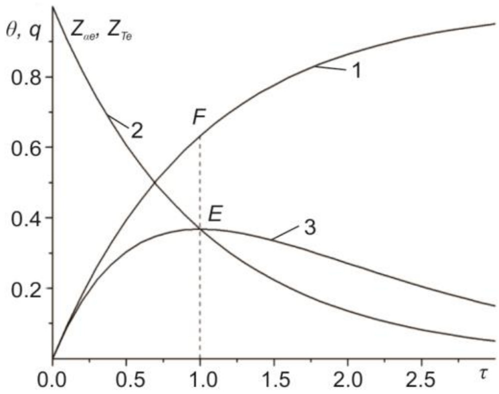

Figure 2. Temperature, θ, heat flux, q, and sensitivity coefficients, $Z_{\alpha e}$, Z_{Te}, for convective heat transfer of an infinitely thin body;
$1 - \theta$, Z_{Te}; $2 - q$; $3 - Z_{\alpha e}$

An important issue in measurement data processing is to know the most informative time points for the calculation of the sought parameters α_e and T_e, in which the influence of measurement errors on the parameters is minimal. It is solved by analyzing the sensitivity coefficients of the solution to the change of these parameters [3, 4]. The higher are the absolute values of the coefficients, the more sensitive is the solution to the change of the parameters, and the less is the measurement error influence on their calculation. This is a necessary condition for a good inverse problem solution. We analyze the response of solution (2) to the changes in the parameters α_e and T_e.

We take the derivative of temperature (2) with respect to the sought parameter α_e, reduce the resulting expression to a dimensionless form, and hence obtain a sensitivity coefficient in the form:

$$Z_{\alpha \mathrm{e}} = \frac{\alpha_\mathrm{e}}{T_\mathrm{e} - T_0} \frac{\partial T}{\partial \alpha_\mathrm{e}} = \exp(-\tau)\tau \tag{4}$$

Analysis of eq. (4) shows that the function $Z_{\alpha e}(\tau)$ is non-monotonic (see curve 3 in fig. 2). Its maximum is found from the condition:

$$\frac{\partial Z_{\alpha \mathrm{e}}}{\partial \tau} = \exp(-\tau)(1-\tau) = 0$$

It follows that the co-ordinate of the maximum $\tau_{\max} = 1$. Thus, it is worthwhile to determine the parameter α_e at low τ in the vicinity of $\tau_{\max} = 1$. Substituting this value in eqs. (2) and (4), we obtain the values of the temperature $\theta(\tau_{\max}) = 1 - \exp(-1) \approx 0.632$ and the sensitivity coefficient $\exp(-1) \approx 0.368$ (points F and E in fig. 2).

Similar calculations for the sensitivity coefficient $Z_{T\mathrm{e}}$ of solution (2) with respect to the gas flow temperature T_e yield:

$$Z_{T\mathrm{e}} = \frac{\partial T}{\partial T_\mathrm{e}} = 1 - \exp(-\tau) = \theta \tag{5}$$

The maximum values of $Z_{T\mathrm{e}}$ are obtained here at large values of the dimensionless time $\tau > 1$. The values of $Z_{T\mathrm{e}}$ are much higher than the values of $Z_{\alpha\mathrm{e}}$, so T_e can be estimated with greater accuracy than α_e and with less sensitivity to measurement errors.

Calculation procedure for convective heat transfer parameters

Case 1. The gas-flow temperature T_e is known. This is the most common case of estimating the heat transfer coefficient α_e in practice. Let the i measurements of temperature T_i ($1 \le i \le N$) be known. Then, we can derive:

$$\theta_i = \frac{T_i - T_0}{T_\mathrm{e} - T_0} = 1 - \exp(-\tau_i) \tag{6}$$

Taking the logarithm of eq. (6) to straighten the experimental data, transforming and applying the averaging formula for N measurements, we have:

$$\bar{\alpha}_{\mathrm{e}} = \rho cL \left\{ \frac{1}{N} \sum_{i=1}^{N} \left[-\frac{\ln(1-\theta_i)}{t_i} \right] \right\} \tag{7}$$

where $\bar{\alpha}_{\mathrm{e}}$ is the average value of the heat transfer coefficient.

It should be noted that eq. (7) follows exactly from the least square method when experimental data are approximated by a constant. A disadvantage of eq. (7) is that its application is limited at small $t_i \to 0$.

This can be avoided by choosing a different approximation of experimental data. Consider a time-linear function $f(t_i, a)$ with a slope angle equal to the sought constant:

$$f(t_i, a) = at_i, \quad a = \frac{\alpha_{\mathrm{e}}}{\rho cL}, \quad F_i = -\ln(1-\theta_i) \tag{8}$$

The sum of squared deviations of the function $f(t_i, a)$ from the experimental values of F_i for all $1 \le i \le N$ measurements must be minimum:

$$J(a) = \sum_{i=1}^{N} (at_i - F_i)^2 \to \min$$

From the condition for the minimum of the functional with respect to the sought parameter, we have the equation:

$$\frac{\partial J}{\partial a} = 2\sum_{i=1}^{N} (at_i - F_i) t_i = 0$$

The solution of the equation yields an explicit formula for the calculation of the heat transfer coefficient α_e:

$$\alpha_{\mathrm{e}} = \rho cL \frac{\left\{ \sum_{i=1}^{N} F_i t_i \right\}}{\left\{ \sum_{i=1}^{N} t_i^2 \right\}}, \quad F_i = -\ln(1-\theta_i), \quad \theta_i = \frac{T_i - T_0}{T_{\mathrm{e}} - T_0} \tag{9}$$

Finally, let us consider the general case of experimental data approximation on the basis of dependence (6). The residual functional in this case reads:

$$J(a) = \sum_{i=1}^{N} \left\{ \left[1 - \exp(-at_i) \right] - F_i \right\}^2 \to \min, \quad F_i = \theta_i, \quad a = \frac{\alpha_{\mathrm{e}}}{\rho cL} \tag{10}$$

The condition for the minimum of functional (10) being met, we obtain a non-linear equation to determine the sought heat transfer coefficient α_{e} (through a):

$$\frac{\partial J}{\partial a} = 2\sum_{i=1}^{N} \left\{ \left[1 - \exp(-at_i) \right] - F_i \right\} \exp(-at_i) t_i = 0 \tag{11}$$

its solution requires the use of numerical methods.

Case 2. The heat transfer coefficient α_{e} is known, while the gas-flow temperature, T_{e}, is unknown. This is a purely theoretical case. We choose the temperature scale T_* close to T_{e}. Solution (2) is written:

$$\theta = \frac{T - T_0}{T_* - T_0} = \theta_e \left[1 - \exp(-at)\right], \quad a = \frac{\alpha_e}{\rho c L}, \quad \theta_e = \frac{T_e - T_0}{T_* - T_0} \tag{12}$$

The unknown quantity is θ_e. From eq. (12), the averaging for N measurements, we obtain:

$$\bar{\theta}_e = \frac{1}{N} \sum_{i=1}^{N} \frac{\theta_i}{1 - \exp(-at_i)} \tag{13}$$

where $\bar{\theta}_e$ is the average value of the dimensionless gas-flow temperature. A disadvantage of eq. (13) is that its application is limited at $t_i \to 0$.

Case 3. The flow temperature T_e and the heat transfer coefficient α_e are unknown. This is a common and the most interesting case for practice. The temperature scale T_* is also chosen to be close to T_e. Solution (2), is written:

$$\theta = \theta_e \left[1 - \exp(-at)\right], \quad a = \frac{\alpha_e}{\rho c L}, \quad \theta = \frac{T - T_0}{T_* - T_0} \tag{14}$$

The unknown quantities are θ_e and a. Now compose the functional of deviations of the experimental dimensionless temperature values F_i from the calculated values of θ_i (a, θ_e):

$$J(a, \theta_e) = \sum_{i=1}^{N} \left[\theta_i(a, \theta_e) - F_i\right]^2 \to \min \tag{15}$$

We require the minimum of the two-variable functional eq. (15). As a result, we come to a non-linear system of equations for the calculation of a and θ_e:

$$\begin{cases} \dfrac{\partial J}{\partial a} = 2\sum_{i=1}^{N}(\theta_i - F_i)\dfrac{\partial \theta}{\partial a} = 2\sum_{i=1}^{N}\left\{\theta_e\left[1 - \exp(-at_i)\right] - F_i\right\}\theta_e \exp(-at_i) t_i = 0 \\ \dfrac{\partial J}{\partial \theta_e} = 2\sum_{i=1}^{N}(\theta_i - F_i)\dfrac{\partial \theta}{\partial \theta_e} = 2\sum_{i=1}^{N}\left\{\theta_e\left[1 - \exp(-at_i)\right] - F_i\right\}\left[1 - \exp(-at_i)\right] = 0 \end{cases} \tag{16}$$

The factors in eq. (16) have the meaning of the weight factors in the case of discrepancy between the experimental and calculated values.

The system must be solved using numerical methods. It should be kept in mind that the determination of the unknown parameters requires a wider measurement range with respect to the dimensionless time, τ, because θ_e is more reliably determined for $\tau > 1$.

Examples of applying the calculation procedure

Let us consider a model thermophysical experiment. Let the gas-flow parameters are: T_e = 1000 °C, T_0 = 0 °C, and α_e = 100 [Wm^{-2}K^{-1}]. The thermophysical complex of the sensing element material is ρcL = 6000 [Jm^{-2}K^{-1}]. With these values, the time scale of the sensor is $t_* = \rho cL/\alpha_e$ = 60 seconds.

The exact solution of problem (2) is shown by curve 1 in fig. 3. *Experimental* data (symbols) for the solution of an inverse heat transfer problem will be the perturbed values of the exact solution $\theta_i(1 \pm \delta)$ with the relative error, δ, in several arbitrarily chosen time points t_i = [6; 9; 15; 18] seconds, which is τ_i = [0.1; 0.15; 0.25; 0.3] for the dimensionless time.

The results of determining the heat transfer coefficient α_e at a known gas temperature T_e on the basis of eqs. (7), (9), and the solution of eq. (11) are given in tab. 1. It is seen that in

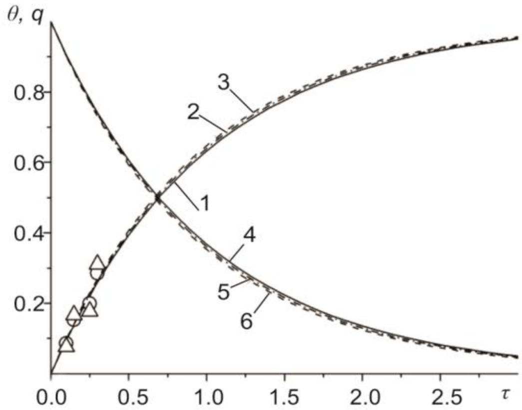

Figure 3. Temperature, *θ*, and heat flux, *q*, *vs*. time, *τ*; ○, and Δ – experimental data with error *δ* = 0.1 and 0.2; *1, 4 – exact; 2, 3 and 5, 6 – restored values of θ and q at δ = 0.1; 0.2*

Table 1. Restored values of heat transfer coefficient, α_e, at a known gas-flow temperature, T_e

	$\alpha_e \cdot 10^{-2}$, eq. (7)	$\alpha_e \cdot 10^{-2}$, eq. (9)	$\alpha_e \cdot 10^{-2}$, eq. (11)
$\delta = 0$	1.0	1.0	1.0
$\delta = 0.05$	1.001	1.013	1.011
$\delta = 0.1$	1.002	1.027	1.021
$\delta = 0.2$	1.008	1.058	1.043

the absence of errors in the initial data ($\delta = 0$), the parameter α_e is restored exactly. The solution of the inverse heat transfer problem is stable with increasing amplitude of error in the data. Usually, the actual measurement error in experiments does not exceed 5% ($\delta < 0.05$). As can be seen, the most accurate results are obtained with eq. (7), which is directly related to the positive effect of experimental data straightening. Figure 3 shows that the exact solution almost coincides with the temperature and heat flux curves reconstructed by the value of α_e found from eq. (11) with erroneous experimental data, which confirms the good accuracy of the inverse heat transfer problem solution. It should be noted that the conventional graphical calculation of the derivative with respect to discrete perturbed temperature values is associated with a large error.

The calculation results for the flow temperature T_e given in fig. 4 and in tab. 2 were obtained with eq. (13) at an exact value of α_e. The set of experimental values was the same as in fig. 3. As follows from the table, the temperature T_e is exactly restored due to symmetrical oscillations of the experimental results about the (exact) mean value. When using the first three experimental points (asymmetric error), the error of restoring T_e is quite acceptable. This is evidenced by the temperature and heat flux curves in fig. 4 with the use of the restored T_e value.

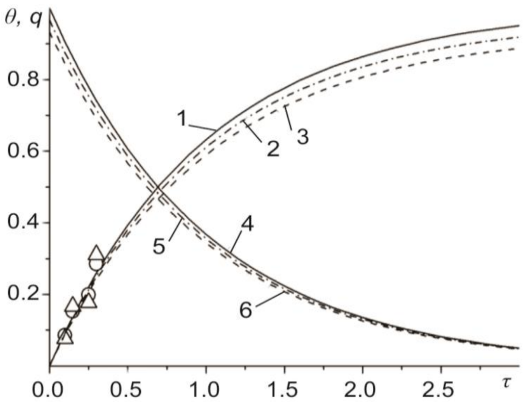

Figure 4. Temperature, *θ*, and heat flux, *q*, *vs*. time, *τ*; ○, and Δ – experimental data with error *δ* = 0.1 and 0.2; *1, 4 – exact; 2, 3 and 5, 6 – restored values of θ and q at δ = 0.1; 0.2 (by 3 points)*

Table 2. Restored values of gas-flow temperature, T_e

	$T_e \cdot 10^{-3}$, 4 points	$T_e \cdot 10^{-3}$, 3 points
$\delta = 0$	1.0	1.0
$\delta = 0.05$	1.0	0.983
$\delta = 0.1$	1.0	0.966
$\delta = 0.2$	1.0	0.933

Figure 5 and tab. 3 show the results of solving the inverse heat transfer problem by finding two parameters $\{\alpha_e, T_e\}$ on the basis of discrete temperature values in the dimensionless time range $\tau_i = [0.2; 0.5; 1.0; 1.1]$. It

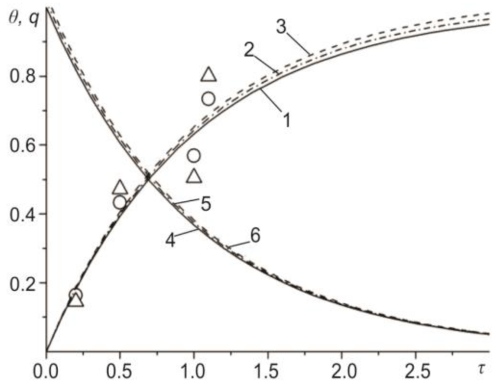

Figure 5. Temperature, θ, and heat flux, q, *vs.* time, τ; ○, and Δ – experimental data with error δ = 0.1 and 0.2; *1, 4 – exact; 2, 3 and 5, 6 – restored values of θ and q at δ = 0.1; 0.2*

Table 3. Restored values of gas-flow temperature, T_e, and heat transfer coefficient, α_e

	$\{T_e \cdot 10^{-3}, \alpha_e \cdot 10^{-2}\}$
$\delta = 0$	{1.0; 1.0}
$\delta = 0.05$	{1.009; 0.999}
$\delta = 0.1$	{1.017; 0.998}
$\delta = 0.2$	{1.034; 0.997}

can be seen that there is a large scatter in the experimental data due to errors in the *measurement*s. The time range is extended due to the need for a more accurate determination of T_e. It is seen from the table that $\{\alpha_e, T_e\}$ are restored quite accurately despite the large error in the initial data (see fig. 5). The use of the restored $\{\alpha_e, T_e\}$ values confirms the good convergence of the temperature and the heat flux to their exact solutions (curves 1 and 4 in fig. 5).

The performed calculations show the good accuracy and practical reliability of the proposed procedure for calculating the temperature T_e and the heat transfer coefficient α_e of stationary gas-flow from discrete measurements of the sensing element temperature of the heat flux sensor.

Conclusions

A procedure for calculating convective heat transfer parameters was proposed for the diagnostics of stationary gas-flows by the method of a thermally thin wall. The procedure does not use smoothing and numerical differentiation of experimental data, and is resistant to measurement errors.

The coefficients of solution sensitivity to convective heat transfer parameters were analyzed to determine the most informative measurement times with the least influence of the measurement error.

The proposed procedure allows one to restore the heat transfer coefficient and the temperature of stationary gas-flow from several discrete temperature values in the presence of errors in the initial data.

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