HARMONICITY AND DIFFERENTIAL EQUATION OF INVOLUTE OF A CURVE IN E^3

by

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In this paper, we first give necessary conditions in which we can decide whether a given curve is biharmonic or 1-type harmonic and differential equations characterizing the regular curves. Then we research the Frenet formulas of involute of a unit speed curve by making use of the relations between the involute of a curve and the curve itself. In addition we apply these formulas to define the essential conditions by which one can determine whether the involute of a unit speed curve is biharmonic or 1-type harmonic and then we write differential equations characterizing the involute curve by means of Frenet apparatus of the unit speed curve. Finally we examined the helix as an example to illustrate how the given theorems work.

Key words: *Laplace operator, connection, differential equation, bihar-monic, involute curve, mean curvature*

Introduction

To establish a relationship between the curvatures and the characterizations of a given curve in Euclidean space and non-Euclidean spaces and then to expound it from the language of geometry has been the focus of interest for many researchers. Some curves are renowned by their explorers such as involute and evolute curves, [1]. Huygens [2] discovered the involute curves while trying to make a more accurate clock. Involute curves have lots of utilizations in so many areas apart from mathematics. One of the application of these areas is undoubtedly the thermal sciences. Therefore, many studies have been conducted in Euclidean and non-Euclidean spaces related to involute curves. To cite some examples Senyurt *et al*. [3], Bulca *et al.* [4], and Senyurt *et al*. [5] are the works that have received considerable attention. Thanks to meticulous studies, it has been revealed that curves can be classified [6]. After this classification, a great many number of articles have been written, [7, 8] and also [9]. In this paper, we first take a unit speed curve which we call through the work as main curve, then write the characterizations of an involute curve by means of Frenet apparatus of the main curve. Eventually we elucidate the characterizations of the involute curve as an example while assuming the helix as a main curve. Now we may look at the main concepts related to the curve theory. If any differentiable curve α, $\|\alpha'\| = \vartheta$ is given in E^3 then the relationship between the Frenet vector fields and its curvatures is stated as [10]:

$$T=\frac{\alpha'}{\|\alpha'\|},\quad N=B\times T,\quad B=\frac{\alpha'\times\alpha''}{\|\alpha'\times\alpha''\|},\quad \kappa=\frac{\|\alpha'\times\alpha''\|}{\|\alpha'\|^3},\quad \tau=\frac{\|\alpha'\times\alpha'',\alpha'''\|}{\|\alpha'\times\alpha''\|^2}$$

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Frenet vector fields can be expressed by means of covariant derivative of these vectors and this relation is known as Frenet formulas:

$$D_T T = \vartheta\kappa N, \quad D_T N = \vartheta(-\kappa T + \tau B), \quad D_T B = -\vartheta\tau N \tag{1}$$

Let α and β be two differentiable curves. If the tangent vector of α is perpendicular to the tangent vector of β, then we call β as the involute of α. According to this definition, following parametrization can be given:

$$\beta(s) = \alpha(s) + \lambda(s)T(s), \ \ \lambda(s) = c - s, \ c \in \mathbb{R} \tag{2}$$

When β is the involute of α, shown in fig. 1, we have such relation that $d[\alpha(s), \beta(s)] = | c - s | \ \forall s \in I$ and c = constant. The relationship between the curvatures of α and β is formulated:

$$\kappa_1(s) = \frac{\sqrt{\kappa^2(s) + \tau^2(s)}}{\kappa(s)\,|\,c - s\,|}, \quad \tau_1(s) = \frac{\kappa\tau' - \kappa'\tau}{\kappa(\kappa^2 + \tau^2)\,|\,c - s\,|} \tag{3}$$

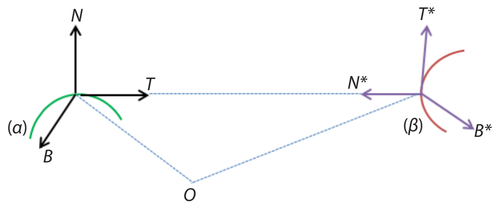

Figure 1. Evolute and involute curves

Also the relationship between the Frenet apparatus of α and β is given:

$$T_1 = N, \quad N_1 = \frac{-\kappa T + \tau B}{\sqrt{\kappa^2 + \tau^2}}, \quad B_1 = \frac{\tau T + \kappa B}{\sqrt{\kappa^2 + \tau^2}} \tag{4}$$

Here the set $\{T_1, N_1, B_1, \kappa_1, \tau_1\}$ denotes the Frenet apparatus of β [10] .

Let a differentiable curve α and a function $f \in C(E^3, R)$ be given. Then the operator D defined:

$$D : T_{E^3}[\alpha(t)] \times C(E^3, \mathbb{R}) \to \mathbb{R}, \ \ D[\alpha'(t), f] = D_{\alpha'(t)} f = \alpha'(t)(f) \tag{5}$$

is called a Levi-Civita connection and the value of $\alpha'(t)(f) \in \mathbb{R}$ is called as covariant derivative of the function f along the curve α. By this definition, following theorem can be given. Let two vector fields X and W defined on E^3 and another two vector fields Y, Z from C^2-class defined on E^3 be given. Then the following propositions are true:

$$\begin{aligned} & D_X(Y + Z) = D_X Y + D_X Z \\ & D_{X+W}(Y) = D_X Y + D_W Y \\ & D_{f(P)X}(Y) = f(P) D_X Y, f : E^3 \to \mathbb{R}, P \in E^3 \\ & D_X(fY) = X(f)Y + f D_X Y, f \in C(E^3, \mathbb{R}) \ [10] \end{aligned} \tag{6}$$

Let α be the unit speed curve, then the mean curvature vector field $\mathbb{H}$ along the curve α is defined:

$$\mathbb{H} = D_{\alpha'}\alpha' = \kappa N \tag{7}$$

Then the mapping defines:

$$\Delta : \chi(\alpha(I)) \to \chi(\alpha(I)), \quad \Delta\mathbb{H} = -D_T^2 \mathbb{H} \tag{8}$$

is called a Laplace operator [7, 8]. Let α be the unit speed curve and $\mathbb{H}$ be the mean curvature vector field along the curve α. Then we have:

– If $\Delta\mathbb{H} = \lambda\mathbb{H}$ then α is called a 1-type harmonic curve, $\lambda \in \mathbb{R}$,
– If $\Delta\mathbb{H} = 0$ then α is called a biharmonic curve [7, 8].

Let α be a non-unit speed curve, then the following propositions hold according to Levi-Civita connection.

– α is a biharmonic curve if and only if:

$$3(\vartheta\kappa)'\vartheta\kappa = 0, \quad (\vartheta\kappa)^3 + \vartheta\kappa(\vartheta\tau)^2 - (\vartheta\kappa)'' = 0, \quad -2(\vartheta\kappa)'\vartheta\tau - \vartheta\kappa(\vartheta\tau)' = 0$$

– α is a 1-type harmonic curve if and only if:

$$3(\vartheta\kappa)'\vartheta\kappa = 0, \quad (\vartheta\kappa)^3 + \vartheta\kappa(\vartheta\tau)^2 - (\vartheta\kappa)'' = \lambda\vartheta\kappa, \quad -2(\vartheta\kappa)'\vartheta\tau - \vartheta\kappa(\vartheta\tau)' = 0$$

Let α be a non-unit speed curve, then the differential equations characterizing the curve α according to unit tangent vector T is given:

$$D_T^3 T + \lambda_2 D_T^2 T + \lambda_1 D_T T + \lambda_0 T = 0$$

$$\lambda_0 = \vartheta^2\kappa\tau\left(\frac{\kappa}{\tau}\right)', \quad \lambda_1 = \vartheta^2\left(\kappa^2+\tau^2\right) - \frac{\vartheta''}{\vartheta} - \frac{\kappa''}{\kappa} + \left(\frac{\vartheta'}{\vartheta}+\frac{\kappa'}{\kappa}\right)\left(3\frac{\vartheta'}{\vartheta}+\frac{\tau'}{\tau}\right) + 2\left(\frac{\kappa'}{\kappa}\right)^2 \tag{9}$$

$$\lambda_2 = -\left(3\frac{\vartheta'}{\vartheta} + 2\frac{\kappa'}{\kappa} + \frac{\tau'}{\tau}\right) \text{ [11]}$$

Characterizations of involute of a curve

Frenet formulas for involute of a curve

When we say α, unless we stated otherwise, we mean a unit speed curve in Euclidean 3-space with the Frenet apparatus of $\{T, N, B, \kappa, \tau\}$ and when we mention β, it stands for the involute of the curve α in the same space with the Frenet apparatus of $\{T_1, N_1, B_1, \kappa_1, \tau_1\}$, $\vartheta = \| \mathrm{d}/\mathrm{d}s\, \beta(s) \|$.

Theorem 1. Let β be the involute of a differentiable curve α. Then the Frenet formulas for the curve β, with respect to Levi-Civita connection D is given:

$$D_N T = \kappa N$$
$$D_N N = -\kappa T + \tau B \tag{10}$$
$$D_N B = -\tau N$$

Proof: Since $D_{T_1} T_1 = \vartheta\kappa_1 N_1$ and also we have $\vartheta = \|\beta'(s)\| = (c-s)\kappa$. Putting the equivalent of T_1, κ_1, and N_1 from eq. (4), we get:

$$D_{T_1} T_1 = \vartheta\kappa_1 N_1 = \vartheta\frac{\sqrt{\kappa^2+\tau^2}}{(c-s)\kappa}\left(\frac{-\kappa}{\sqrt{\kappa^2+\tau^2}}T + \frac{\tau}{\sqrt{\kappa^2+\tau^2}}B\right)$$

$$D_{T_1} T_1 = D_N N = -\kappa T + \tau B$$

By the similar way τ_1, B_1 can be replaced by their equivalents and it follows from eq. (6):

$$\frac{1}{\sqrt{\kappa^2+\tau^2}}(-\kappa D_N T + \tau D_N B) = \left[\frac{(\kappa\tau' - \kappa'\tau)\tau}{(\kappa^2+\tau^2)^{3/2}} + \left(\frac{\kappa}{\sqrt{\kappa^2+\tau^2}}\right)'\right]T -$$
$$-\sqrt{\kappa^2+\tau^2}N + \left[\frac{(\kappa\tau'-\kappa'\tau)\kappa}{(\kappa^2+\tau^2)^{3/2}} - \left(\frac{\tau}{\sqrt{\kappa^2+\tau^2}}\right)'\right]B \tag{11}$$

In a similar case:

$$D_{T_1}B_1 = -\vartheta\tau_1 N_1$$

$$D_N\left(\frac{\tau}{\sqrt{\kappa^2+\tau^2}}T+\frac{\kappa}{\sqrt{\kappa^2+\tau^2}}B\right)=\vartheta\left[-\frac{\kappa\tau'-\kappa'\tau}{(c-s)\kappa(\kappa^2+\tau^2)^{\frac{3}{2}}}(-\kappa T+\tau B)\right]$$

From eq. (6), we achieve:

$$\frac{1}{\sqrt{\kappa^2+\tau^2}}\left(\tau D_N T+\kappa D_N B\right)=\left[\frac{(\kappa\tau'-\kappa'\tau)\kappa}{(\kappa^2+\tau^2)^{\frac{3}{2}}}-\left(\frac{\tau}{\sqrt{\kappa^2+\tau^2}}\right)'\right]T-\left[\frac{(\kappa\tau'-\kappa'\tau)\tau}{(\kappa^2+\tau^2)^{\frac{3}{2}}}+\left(\frac{\kappa}{\sqrt{\kappa^2+\tau^2}}\right)'\right]B \quad (12)$$

Finally by making use of the eqs. (11) and (12) we obtain:

$$D_N T=\frac{-1}{\sqrt{\kappa^2+\tau^2}}\left[\kappa\left(\frac{\kappa}{\sqrt{\kappa^2+\tau^2}}\right)'+\tau\left(\frac{\tau}{\sqrt{\kappa^2+\tau^2}}\right)'\right]T+\kappa N+$$

$$+\frac{1}{\sqrt{\kappa^2+\tau^2}}\left[\kappa\left(\frac{\tau}{\sqrt{\kappa^2+\tau^2}}\right)'-\tau\left(\frac{\kappa}{\sqrt{\kappa^2+\tau^2}}\right)'-\frac{\kappa\tau'-\kappa'\tau}{\sqrt{\kappa^2+\tau^2}}\right]B \quad (13)$$

$$D_N T=\kappa N$$

and

$$D_N B=\frac{1}{\sqrt{\kappa^2+\tau^2}}\left[\frac{\kappa\tau'-\kappa'\tau}{\sqrt{\kappa^2+\tau^2}}+\tau\left(\frac{\kappa}{\sqrt{\kappa^2+\tau^2}}\right)'-\kappa\left(\frac{\tau}{\sqrt{\kappa^2+\tau^2}}\right)'\right]T-$$

$$-\tau N-\frac{1}{\sqrt{\kappa^2+\tau^2}}\left[\kappa\left(\frac{\kappa}{\sqrt{\kappa^2+\tau^2}}\right)'+\tau\left(\frac{\tau}{\sqrt{\kappa^2+\tau^2}}\right)'\right]B \quad (14)$$

$$D_N B=-\tau N$$

hence proof is completed.

Harmonicity of involute of a curve

Theorem 2. Let β be the involute of a differentiable curve α with the mean curvature vector field $\mathbb{H}_1$, then the followings are true with respect to connection.

– β is a biharmonic curve if and only if:

$$\kappa''-\kappa^3-\kappa\tau^2=0,\quad \kappa\kappa'+\tau\tau'=0,\quad \kappa^2\tau+\tau^3-\tau''=0 \quad (15)$$

– β is a 1-type harmonic curve, $\lambda\in\mathbb{R}$, if and only if:

$$\kappa''-\kappa^3-\kappa\tau^2=-\lambda\kappa,\ \kappa\kappa'+\tau\tau'=0,\ \kappa^2\tau+\tau^3-\tau''=\lambda\tau \quad (16)$$

Proof: Since β is the involute of α, by the eq. (8), we can write:

$$\mathbb{H}_1=D_N N=-\kappa T+\tau B$$

If we write the image of mean curvature $\mathbb{H}_1$ under the mapping of Laplace operator Δ:

$$\Delta:\chi^{\perp}(\beta(I))\to\chi(\beta(I))\quad \Delta\mathbb{H}^{*}=-D_N^2(D_N N)$$
$$=-D_N^2(-\kappa T+\tau B)$$
$$=-D_N D_N(-\kappa T+\tau B)$$
$$\Delta(D_N N)=-D_N(-\kappa' T-\kappa D_N T+\tau' B+\tau D_N B)$$

After setting the counterparts of $D_N T$, $D_N N$, and $D_N B$ from the eq. (10), we arrange the last equality with regarding to T, N, and B:

$$\Delta(D_N N)=(\kappa''-\kappa^3-\kappa\tau^2)T+3(\kappa\kappa'+\tau\tau')N+(\kappa^2\tau+\tau^3-\tau'')B$$

If we take the condition that eq. (8) of b, that is, $\Delta\mathbb{H}1 = 0$ into account, then 1. proposition holds and if we take the condition that eq. (8) of a, that is, $\Delta\mathbb{H}1 = \lambda\mathbb{H}1$ into account, then 2. proposition holds. This yields the required result and completes the proof.

Differential equation of involute of a curve

Theorem 3. Let β be the involute of the unit speed curve α. Then the differential equation characterizing the curve β with respect to principal normal N:

$$c_3 D_N^3 N+c_2 D_N^2 N+c_1 D_N N+c_0 N=0$$

Here c_0, c_1, c_2, c_3 are the coefficients:

$$c_0=\Big[3(\kappa\kappa'+\tau\tau')(\kappa\tau'-\kappa'\tau)+(\kappa^2+\tau^2)(\kappa''\tau-\kappa\tau'')\Big],\; c_2=\kappa''\tau-\kappa\tau''$$
$$c_1=\Big[\kappa'\tau''-\kappa''\tau'+(\kappa^2+\tau^2)(\kappa\tau'-\kappa'\tau)\Big],\; c_3=\kappa\tau'-\kappa'\tau$$

Proof: From eq. (10), we obviously have $D_N N = -\kappa T + \tau B$. Taking the covariant derivative of this phrase with respect to N two times:

$$\begin{aligned}D_N(D_N N)&=D_N(-\kappa T+\tau B)\\&=-\kappa' T-\kappa D_N T+\tau' B+\tau D_N B\\&=-\kappa' T-\kappa^2 N+\tau' B-\tau^2 N\\D_N^2 N&=-\kappa' T-(\kappa^2+\tau^2)N+\tau' B\end{aligned}\tag{17}$$

and again from eq. (10):

$$\begin{aligned}D_N(D_N^2 N)&=D_N(-\kappa' T-(\kappa^2+\tau^2)N+\tau' B)\\&=-\kappa'' T-\kappa' D_N T-(\kappa^2+\tau^2)' N-(\kappa^2+\tau^2)D_N N+\tau'' B+\tau' D_N B\\&=-\kappa'' T-3(\kappa\kappa'+\tau\tau')N+(\kappa^2+\tau^2)\kappa T-(\kappa^2+\tau^2)\tau B+\tau'' B\\D_N^3 N&=(\kappa(\kappa^2+\tau^2)-\kappa'')T-3(\kappa\kappa'+\tau\tau')N+(\tau''-(\kappa^2+\tau^2)\tau)B\end{aligned}\tag{18}$$

By taking advantage of the following system we can evaluate the equivalents of T and B:

$$\begin{cases}D_N N=-\kappa T+\tau B\\D_N^2 N=-\kappa' T-\left(\kappa^2+\tau^2\right)N+\tau' B\end{cases}$$

Now we first multiply both sides of the first equality by $-\tau'$ and second equality by τ, after adding both sides of the equalities, we divide both sides by $\kappa\tau'-\kappa'\tau$:

$$T=\frac{\tau}{\kappa\tau'-\kappa'\tau}D_N^2N-\frac{\tau'}{\kappa\tau'-\kappa'\tau}D_NN+\frac{\tau\left(\kappa^2+\tau^2\right)}{\kappa\tau'-\kappa'\tau}N$$

By the same way, we first multiply both sides of the first equality by $-\kappa'$ and second equality by κ, after adding both sides of the equalities, we divide both sides by $\kappa\tau' - \kappa'\tau$:

$$B=\frac{\kappa}{\kappa\tau'-\kappa'\tau}D_N^2N-\frac{\kappa'}{\kappa\tau'-\kappa'\tau}D_NN+\frac{\kappa\left(\kappa^2+\tau^2\right)}{\kappa\tau'-\kappa'\tau}N$$

Finally setting the counterparts of T and B into the eq. (18):

$$D_N^3N=\frac{\kappa\tau''-\kappa''\tau}{\kappa\tau'-\kappa'\tau}D_N^2N+\frac{\left(\kappa^2+\tau^2\right)\left(\kappa'\tau-\kappa\tau'\right)+\kappa''\tau'-\kappa'\tau''}{\kappa\tau'-\kappa'\tau}D_NN$$
$$\left[\frac{\left(\kappa^2+\tau^2\right)\left(\kappa\tau''-\kappa''\tau\right)}{\kappa\tau'-\kappa'\tau}-3\left(\kappa\kappa'+\tau\tau'\right)\right]N$$

After multiplying both sides of previous equality by $\kappa\tau' - \kappa'\tau$ we can rearrange them:

$$(\kappa\tau'-\kappa'\tau)D_N^3N+(\kappa''\tau-\kappa\tau'')D_N^2N+$$
$$+\left[(\kappa^2+\tau^2)(\kappa\tau'-\kappa'\tau)+\kappa'\tau''-\kappa''\tau'\right]D_NN$$
$$\left[(\kappa^2+\tau^2)(\kappa''\tau-\kappa\tau'')+3(\kappa\kappa'+\tau\tau')(\kappa\tau'-\kappa'\tau)\right]N=0$$

Putting the coefficients as c_3, c_2, c_1, c_0, respectively, we obtain desired result which completes the proof.

Numerical example

Let $\alpha(s) = 1/(2)^{1/2}(\cos s, \sin s, s)$ be a curve with unit speed tangent vector. Then the Frenet apparatus of α are:

$$T=\frac{1}{\sqrt{2}}(-\sin s,\cos s,1),\ \ N=(-\cos s,-\sin s,0),\ \ B=\frac{1}{\sqrt{2}}(\sin s,-\cos s,1)$$

$\kappa = 1 / 2^{1/2}$ and $\tau = 1 / 2^{1/2}$. The Frenet formulas can be evaluated:

$$D_TT=\frac{-1}{\sqrt{2}}(\cos s,\sin s,0),\ \ D_TN=(\sin s,-\cos s,0),\ \ D_TB=\frac{1}{\sqrt{2}}(\cos s,\sin s,0)$$

According to these traditional Frenet apparatus, we have the involute of α, which we call here as β:

$$\beta(s)=\frac{1}{\sqrt{2}}(\cos s-(c-s)\sin s,\quad \sin s+(c-s)\cos s,c)$$

where s is the arc length parameter of α, $c \in \mathbb{R}$.

After having done simple calculations we can write the Frenet apparatus of β:

$$T_1=(-\cos s,-\sin s,0),\ \ N_1=(\sin s,-\cos s,0),\ \ B_1=(0,0,1)\ \text{ and }\ \kappa_1=\frac{\sqrt{2}}{c-s},\ \ \tau_1=0$$

Now we can clarify the characterizations of β in two distinct cases.

According to *Theorem 2*, $\Delta\mathbb{H}_1 = N1 = D_N N = \mathbb{H}_1$, that is, β is of 1-type harmonic. According to *Theorem 3*, differential equation characterizing β is given as $D_N^2 N + N = 0$.

Main results

Corollary 1. Let β be the involute of a curve α. If α is a circular helix, then β is of 1-type harmonic.

Proof: Suppose that α is a circular helix, from eq. (16) we have κ = constant and τ = constant. It follows that Laplace image of mean curvature vector $D_N N$ of α is equal to $\Delta(D_N N) = (\kappa^2 + \tau^2)(-\kappa T + \tau B)$.

Hence we obtain that $\lambda = \kappa^2 + \tau^2$, so the proof is completed.

Corollary 2. Let β be the involute of a curve α. If α is a general helix, then β is biharmonic.

Proof: Assume that α is a general helix, so we have κ/τ = sbt. It follows that Laplace image of mean curvature vector $D_N N$ of α gives us $\kappa/\tau = -\tau'/\kappa'$ and $\kappa''/\tau'' = \kappa'/\tau'$, that is, we obtain $\kappa/\tau = sbt$.

This verifies the eq. (15) hence the proof is completed.

Conclusion

We define a new way of covariant derivative and then derive the Frenet formulas by making use of the relations between the involute of a curve and the curve itself. In addition we apply these formulas to define the essential conditions by which one can determine whether the involute of a curve is biharmonic or 1-type harmonic and then we write differential equations characterizing the involute of the curve.

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