DIFFUSIVE BEJAN NUMBER AND SECOND LAW OF THERMODYNAMICS TOWARD A NEW DIMENSIONLESS FORMULATION OF FLUID DYNAMICS LAWS

by

Michele TRANCOSSI[a*] and Jose PASCOA[b]

[a] Henri Coanda Labs LLC, Delaware, Del., USA
Sheffield Hallam University, Sheffield, UK
[b] Universidade da Beira Interior, Covilla, Portugal, PT

In a recent paper, Liversage and Trancossi have defined a new formulation of drag as a function of the dimensionless Bejan and Reynolds numbers. Further analysis of this hypothesis has permitted to obtain a new dimensionless formulation of the fundamental equations of fluid dynamics in their integral form. The resulting equations have been deeply discussed for the thermodynamic definition of Bejan number evidencing that the proposed formulation allows solving fluid dynamic problems in terms of entropy generation, allowing an effective optimization of design in terms of the Second law of thermodynamics. Some samples are discussed evidencing how the new formulation can support the generation of an optimized configuration of fluidic devices and that the optimized configurations allow minimizing the entropy generation.

Key words: *fluid dynamic, conservation laws, Bejan number, drag, friction coefficient, entropy, exergy*

Introduction

All things are flowing. (Heraclitus)

Liversage and Trancossi [1] have produced a new definition of the drag force in fluid dynamics problems. They have rewritten the traditional expression of drag force:

$$D = \frac{1}{2} C_D A_f \rho u^2 \tag{1}$$

by determining that the drag coefficient C_D is based on Reynolds number and Bejan number according to eq. (2):

$$C_D = 2 \frac{A_w}{A_f} \frac{\mathrm{Be}_L}{\mathrm{Re}_L{}^2} \tag{2}$$

where A_w is the wet area, A_f – the front area, Re_L – the Reynold Number related to fluid path length, and Bejan number has been adopted in the diffusive definition by Bhattacharjee and Grosshandler [2], as formulated by Mahmud and Fraser [3, 4] and generalized by Awad and Lage [5] and Awad [6, 7]:

$$\mathrm{Be} = \frac{\Delta p l^2}{\mu v} = \frac{\Delta p l^2}{\rho v^2} = \frac{\rho \Delta p l^2}{\mu^2} \tag{3}$$

Trancossi and Sharma [8] have verified eq. (2) in the case of a low thickness high chamber wing profile with a good accuracy versus both experimental and CFD data. Trancossi and Pascoa [9] have produced an adequate formulation of the conservation laws in fluid dynamics, that takes advantage of new container terms, which are based on Bejan number, and a new derived term, which has been named Bejan number ξ, which has the same dimensional formulation and is obtained:

$$\mathrm{Be} = \frac{l^2 \Delta p}{\mu v} = \frac{l^2 \Delta p}{\rho v^2} = \frac{l^2}{\rho v^2}(p_m - p_n) = \xi_m - \xi_n \tag{4}$$

where m and n are a generic point of the domain

$$\xi_m = \frac{l^2}{\rho v^2} p_m \tag{5}$$

A more accurate analysis of the formulation in eq. (5), shows evidently that in fluid dynamics is equivalent to the Hagen number:

$$\mathrm{Hg} = \frac{\Delta P}{L} \frac{l^3}{\mu v} = \rho \frac{\Delta P}{L} \frac{l^3}{\mu^2} \tag{6}$$

Awad [10] have analyzed the Hagen number *vs.* the diffusive Bejan number. The first difference is that they have radically different physical meanings: Hagen number represents the dimensionless pressure gradient while the Bejan number represents the dimensionless pressure drop. In particular, they are coincident in the cases where the characteristic length, l, is equal to the flow length, L. The definition of Bejan energy relates to Hagen number by the following relation:

$$\mathrm{Hg} = \frac{L^3}{\rho v^2} \frac{\mathrm{d}p}{\mathrm{d}x} = L \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{L^2 p}{\rho v^2}\right) = L \frac{\mathrm{d}\xi}{\mathrm{d}x} \tag{7}$$

Consequently, Bejan energy can be defined as the rate of change of the Bejan energy over the flow path. Trancossi and Pascoa have also expressed a still preliminary formulation of conservation laws as a function of Bejan number and Bejan energy.

Awad [11] and Klein *et al.* [12] have remarked that also the second definition of Bejan number, a second dimensionless number is defined Bejan number in scientific literature as the efficiency of convective heat exchange according to the second law of thermodynamics by Sciubba [13]. Efficiency according to First law is the ratio of useful output and used input.

$$\eta = \frac{W_{\mathrm{out}}}{E_{\mathrm{in}}} \tag{8}$$

Energy efficiency is a fundamental parameter of energy analysis, but it does not seem adequate for the effective optimization of a physical system. For design analysis and optimization, effectiveness seems much more adequate. The processes that involve heat exchange allows assessing the heat-exchange effectiveness:

$$\varepsilon = \frac{Q_{\text{actual}}}{Q_{\text{ideal}}} \tag{9}$$

A more general definition applies to any physical phenomenon and is obtained in terms of the ratio of the efficiencies according to the First law of thermodynamics:

$$\varepsilon = \frac{\eta_{\text{actual}}}{\eta_{\text{ideal}}} = \frac{E_{\text{in,ideal}}}{E_{\text{in,actual}}} \frac{W_{\text{out,actual}}}{W_{\text{out,ideal}}} \tag{10}$$

Efficiency can also be defined according to the Second law of thermodynamics in terms of a ratio of entropy generations:

$$\varepsilon = \frac{\dot{S}_{\text{desired}}}{\dot{S}_{\text{total}}} = \frac{\dot{S}_{\text{desired}}}{\dot{S}_{\text{desired}} + \dot{S}_{\text{undesired}}} \tag{11}$$

In convective heat exchange and fluid dynamic processes can be stated that irreversibility arises because of the combination of heat transfer phenomena and viscous effects of the fluid [14, 15]. Similar considerations allowed Bejan and Sciubba [16] to define the efficiency of any process involving fluid flow and convective heat exchange according to the Second law of thermodynamics. In the case of heat exchange phenomena, effectiveness is the ratio of entropy generations in a convective exchange fluid flow and the total entropy generation by both the heat exchanged and the pressure drop which is proportional to the necessary pumping or propulsive power:

$$\varepsilon_{\text{local}} = \frac{\dot{s}_{\text{gen},\Delta T}}{\dot{s}_{\text{gen},\Delta T} + \dot{s}_{\text{gen},\Delta p}} \tag{12}$$

In particular, Sciubba defined such a magnitude local Bejan number is the dimensionless ratio of the entropy created by heat transfer and the total entropy. Sciubba has also provided the integral formulation at the domain level, with volume V. It can be defined as the global Bejan number and has the following formulation:

$$\overline{\text{Be}} = \frac{\dot{S}_{\text{gen},\Delta T}}{\dot{S}_{\text{gen},\Delta T} + \dot{S}_{\text{gen},\Delta p}} = \frac{\int\limits_V \dot{s}_{\text{gen},\Delta T} dV}{\int\limits_V \dot{s}_{\text{gen},\Delta T} dV + \int\limits_V \dot{s}_{\text{gen},\Delta p} dV} \tag{13}$$

It can also be expressed:

$$\overline{\text{Be}} = \frac{\dot{S}_{\text{gen},\Delta T}}{\dot{S}_{\text{gen},\Delta T} + \dot{S}_{\text{gen},\Delta p}} = \frac{1}{1 + \text{Br}} \tag{14}$$

where Br is Brinkman number, which is equal to:

$$\text{Br} = \frac{\mu u^2}{k \Delta T} \tag{15}$$

where u is the velocity of the fluid, and k is the conductivity. Consequently, it results:

$$\overline{\text{Be}} = \frac{1}{1 + \text{Br}} = \frac{1}{1 + \dfrac{\mu u^2}{k \Delta T}} = \frac{k \Delta T}{k \Delta T + \mu u^2} \tag{16}$$

It is evident that the Bejan number definition by Sciubba applies only to convective heat exchange. The diffusive definition by Bhattacharjee and Grosshandler is much more general because it can be applied to any fluid dynamic phenomenon, including convective heat exchange.

Friction losses in fluids

Herwig and Schmandt [17] have studied the possibility of more effective modeling of losses in terms of drag for external flows and head loss for internal flows in terms of entropy generation or exergy disruption rate.

Fluid flows are treated according to two different categories. External flows can be expressed in terms of drag force [18-21]:

$$F_D = C_D A_f \frac{\rho}{2} u_\infty^2 \tag{17}$$

where C_D is the drag coefficient, and A_f is front section area.

Two different formulations of drag are present in literature. The first one is a function of head loss [21-25] coefficient:

$$\Delta P = K \frac{\rho}{2} u_{av}^2 \tag{18}$$

where K is the head loss coefficient. In both cases, a second pressure loss formulation can be adopted. It can be expressed as a function of friction coefficient f:

$$\frac{dp}{dx} = -\frac{f}{D} \frac{\rho}{2} u_{av}^2 \rightarrow \Delta P \simeq f \frac{L}{D} \frac{\rho}{2} u_{av}^2 \tag{19}$$

where f is the friction factor, L – the flow path length, and D – the hydraulic diameter. Friction factor $f = \tau/(\rho u^2/2)$ represents the friction effect of shear stress. It is not fully representative of friction phenomena. Drela [26] has observed that the friction factor represents only what happens on the surface being defined by eq. (19).

$$D_f = \iint \left[\tau_{xy} \right]_{y=0} dx\, dz = \iint \rho_e u_\infty^3 f\, dx\, dz \tag{20}$$

Head loss or dissipative terms consider all the losses in the boundary layer along its development and after the detachment.

$$\Phi = \iint \left[\int_0^\delta \tau_{xy} \frac{\partial u}{\partial y} dy \right] dx\, dz = \iint \rho_e u_\infty^3 C_D\, dx\, dz \tag{21}$$

The meaning of the two terms can be explained at the domain level by fig. 1, in a case of external fluid dynamics, and fig. 2, in the case of internal fluid dynamics.

Consequently, this paper accounts only the losses according to the dissipative model, because they give an exhaustive answer to the problem of the determination of losses.

Any real process in the domain of fluid dynamics generates losses of mechanical energy that increases the internal energy of the fluid. Thus, the total energy is maintained. It means that energetic analysis in fluid dynamics deals with energy availability [27] and usefulness [28] and then with the second law of thermodynamics [29]. It can be expressed as the entropy generation or degradation of exergy or reduction of available work, and degradation of available energy in the flow field.
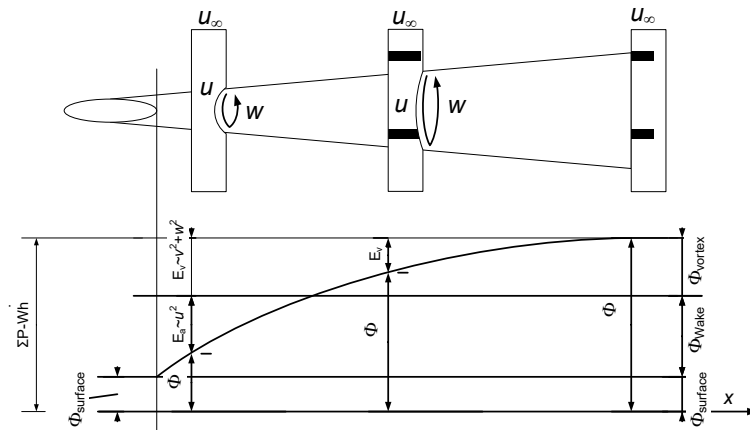
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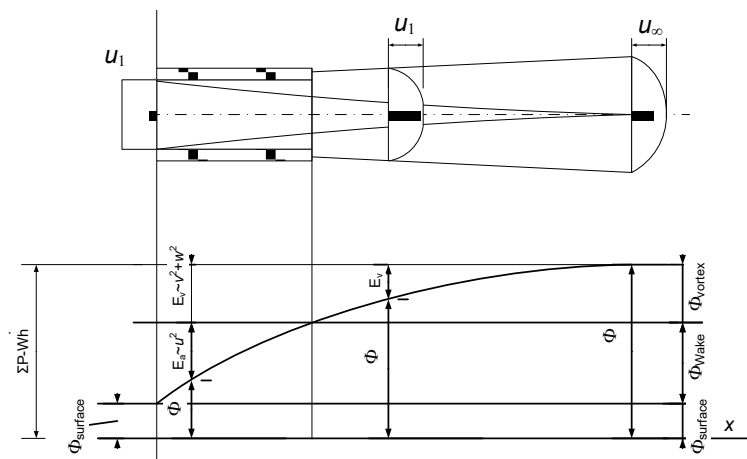**Figure 1. Dissipative terms and their graphical representation in external fluid dynamics**

Figure 2. Dissipative terms and their graphical representation in internal fluid dynamics

If $\dot{S}$ it is the rate of entropy generation, the processes can be analyzed in terms of exergy destruction rate:

$$\dot{X}_{\text{loss}} = T_0 \dot{S}_{\text{gen}} \tag{22}$$

The dissipated mechanical power is equal to:

$$\dot{E}_L = F_D u_{\text{ref}} = \Delta P A u_{\text{ref}} = \dot{X}_{\text{loss}} = T_0 \dot{S}_{\text{gen},f} \tag{23}$$

The case of an internal and an external fluid dynamic problem are reported in tab. 1.

Bejan number and the Second law of thermodynamics

Assuming u_{ref} the reference velocity for the specific problem, is evident that the general expression of pressure losses is:

Table 1. Fluid-dynamic friction coefficient, force loss and pressure loss as a function of entropy generation and exergy dissipations

External fluid-flow		Friction coefficient	Force loss	Pressure loss
$\dot{S}_{\text{gen}}$	Entropy generation	$C_D = \dfrac{2T_0\dot{S}_{\text{gen}}}{A_f\rho u_\infty^{\,3}}$	$F_D = \dfrac{T_0\dot{S}_{\text{gen}}}{u_\infty}$	$\Delta p_D = \dfrac{T_0\dot{S}_{\text{gen}}}{A_w u_\infty}$
$\dot{X}$	Exergy disruption	$C_D = \dfrac{2\dot{X}}{A_f\rho u_\infty^{\,3}}$	$F_D = \dfrac{\dot{X}}{u_\infty}$	$\Delta p_D = \dfrac{\Delta\dot{X}}{A_w u_\infty}$
Internal fluid-flow		Friction coefficient	Force loss	Pressure loss
$\dot{S}_{\text{gen}}$	Entropy generation	$K_0 = \dfrac{2T_0\dot{S}_{\text{gen}}}{A_w\rho u_{av}^{\,3}}$	$F_K = \dfrac{T_0\dot{S}_{\text{gen}}}{u_{av}}$	$\Delta p_K = \dfrac{T_0\dot{S}_{\text{gen}}}{A_w u_{av}}$
$\dot{X}$	Exergy disruption	$K_0 = \dfrac{2\dot{X}}{A_w\rho u_{av}^{3}}$	$F_K = \dfrac{\dot{X}}{u_{av}}$	$\Delta p_K = \dfrac{\Delta\dot{X}}{A_w u_{av}}$

$$\Delta p_{\text{loss}} = \frac{T}{A_w u_{\text{ref}}}\dot{S}_{\text{gen,loss}} = \frac{T}{TA_w u_{\text{ref}}}\Delta\dot{X}_{\text{loss}} \tag{24a}$$

Thus, the Bejan number related to losses can be expressed:

$$\text{Be}_L = \frac{l^2}{\rho v^2}\Delta p = \begin{matrix} \nearrow \dfrac{1}{\rho A_w u_{\text{ref}}}\dfrac{l^2}{v^2}T\Delta\dot{S}_{\text{gen}} = \dfrac{l^2}{\dot{m}v^2}T\Delta\dot{S}_{\text{gen}} \\[2em] \searrow \dfrac{1}{\rho A_w u_{\text{ref}}}\dfrac{l^2}{v^2}\Delta\dot{X}_{\text{loss}} = \dfrac{l^2}{\dot{m}v^2}\Delta\dot{X}_{\text{loss}} \end{matrix} \tag{24b}$$

Equations (24a) and (24b) allows demonstrating that Bejan number related to fluid-dynamic problems is a thermodynamic related magnitude that refers to entropy generation and exergy dissipation rate.

Integral equations of fluid dynamics

The integral equations of fluid dynamics can be expressed as follows.

Conservation of mass

The equation of conservation of mass is:

$$\dot{m}_i = \dot{m}_e = \dot{m} \rightarrow \rho_i A_i u_i = \rho_e A_e u_e = \dot{m} \tag{25}$$

and being:

$$\rho_i A_i u_{i,x} = \dot{m}_{i,x}; \rho_i A_i u_{i,y} = \dot{m}_{i,y}; \rho_e A_e u_{e,x} = \dot{m}_{e,x}; \rho_e A_e u_{e,y} = \dot{m}_{e,y}$$

it results

$$\dot{m}_i = \dot{m}_e = \dot{m} \rightarrow \dot{m}_i^2 = \dot{m}_e^2 \rightarrow \dot{m}_{i,x}^2 + \dot{m}_{i,y}^2 = \dot{m}_{e,x}^2 + \dot{m}_{e,y}^2$$

where

$$\dot{m}_i = \dot{m}_e = \dot{m} \rightarrow \rho_i A_i u_i = \rho_e A_e u_e = \dot{m} \tag{26}$$

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And, according to fig. 3.

$$\begin{cases} \dot{m}_{i,x} = \dot{m}_i \cos\phi_i \\ \dot{m}_{i,y} = \dot{m}_i \sin\phi_i \end{cases}; \begin{cases} \dot{m}_{e,x} = \dot{m}_e \cos\phi_e \\ \dot{m}_{e,y} = \dot{m}_e \sin\phi_e \end{cases} \tag{27}$$

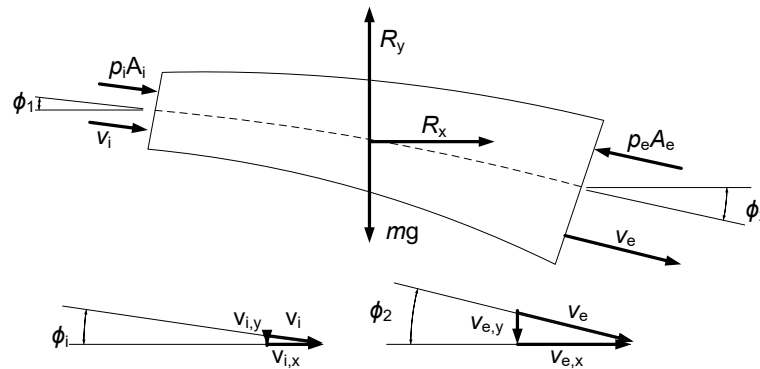**Figure 3. Vector forces and velocity**

Conservation of momentum

$$\begin{cases} p_{i,x}A_i - p_{e,x}A_e + R_x = -\rho A_i u_i u_{i,x} + \rho A_e u_e u_{e,x} \\ p_{i,y}A_i - p_{e,y}A_e + R_y - W = -\rho A_i u_i u_{i,y} + \rho A_e u_e u_{e,y} \end{cases} \tag{28}$$

$$\begin{cases} p_{i,x}A_i - p_{e,x}A_e + R_x = -\dot{m}_i u_{i,x} + \dot{m}_e u_{e,x} \\ p_{i,y}A_i - p_{e,y}A_e + R_y - W = -\dot{m}_i u_{i,y} + \dot{m}_e u_{e,y} \end{cases} \tag{29}$$

$$\begin{cases} p_i A_i \cos\phi_i - p_e A_e \cos\phi_e + R_x = -\rho_i A_i u_i u_{i,x} + \rho_e A_e u_e u_{e,x} = \\ = -\rho_i A_i \cos\phi_i u_i^2 + \rho_e A_e \cos\phi_e u_e^2 \\ p_i A_i \sin\phi_i - p_e A_e \sin\phi_e + R_y - W = -\rho_i A_i u_{i,y}^2 + \rho_e A_e u_{e,y}^2 = \\ = \rho_i A_i \sin\phi_i u_i^2 + \rho_e A_e \sin\phi_e u_e^2 \end{cases} \tag{30}$$

$$\begin{cases} p_i A_i \cos\phi_i - p_e A_e \cos\phi_e + R_x = -\dot{m}_i u_{i,x} + \dot{m}_e u_{e,x} = \\ = -\dot{m}\dfrac{\dot{m}}{\rho_i A_i}\cos\phi_i + \dot{m}\dfrac{\dot{m}}{\rho_e A_e}\cos\phi_e = -\dfrac{\dot{m}^2}{\rho_i A_i}\cos\phi_i + \dfrac{\dot{m}^2}{\rho_e A_e}\cos\phi_e \\ p_i A_i \sin\phi_i - p_e A_e \sin\phi_e + R_y - W = -\dot{m}_i u_{i,y} + \dot{m}_e u_{e,y} = \\ = -\dot{m}\dfrac{\dot{m}}{\rho_i A_i}\cos\phi_i + \dot{m}\dfrac{\dot{m}}{\rho_e A_e}\cos\phi_e = -\dfrac{\dot{m}^2}{\rho_i A_i}\cos\phi_i + \dfrac{\dot{m}^2}{\rho_e A_e}\cos\phi_e \end{cases} \tag{31}$$

If the considered system can be considered horizontal or at large curvature, eq. (30) becomes:

$$
\begin{cases}
p_i A_i - p_e A_e + R_x = -\dfrac{\dot{m}^2}{\rho_i A_i} + \dfrac{\dot{m}^2}{\rho_e A_e} \\[3mm]
R_y - W = -\dfrac{\dot{m}^2}{\rho_i A_i} + \dfrac{\dot{m}^2}{\rho_e A_e}
\end{cases}
\tag{32}
$$

$$
\begin{cases}
p_i A_i - p_e A_e + R_x = -\rho_i A_i u_i^2 + \rho_e A_e u_e^2 \\[2mm]
R_y - W = -\rho_i A_i u_i^2 + \rho_e A_e u_e^2
\end{cases}
\tag{33}
$$

Conservation of energy

The general expression of the conservation in a fluid can be expressed in both differential and integral form. The integral form has been reported in eqs. (34) and (35).

$$
\underbrace{\dot{m}\left(\frac{u_i^2}{2} + gz_i + \frac{p_i}{\rho_i}\right)}_{\text{Energy In}} + \underbrace{\dot{W}_{\text{in}}}_{\text{Work In}} + \underbrace{\dot{Q}_{\text{in}}}_{\text{Heat In}} =
$$
$$
= \underbrace{\dot{m}\left(\frac{u_e^2}{2} + z_e + \frac{p_e}{\rho_e}\right)}_{\text{Energy In}} + \underbrace{\dot{W}_{\text{out}}}_{\text{Work Out}} + \underbrace{\dot{Q}_{\text{in}}}_{\text{Heat Out}} + \underbrace{\dot{m}\frac{\Delta p_L}{\rho_{ib}}}_{\text{Losses}}
\tag{34}
$$

where W_{in} is the work input, Q_{in} – the heat input, W_{out} – the work output, and Q_{out} – the heat output.

$$
\frac{l^2}{v^2}\frac{u_i^2}{2} + \frac{l^2}{v^2}gz_i + \frac{l^2}{v^2}\frac{p_i}{\rho_i} + \frac{l^2}{v^2}\frac{\dot{W}_{\text{in}}}{\dot{m}} + \frac{l^2}{v^2}\frac{\dot{Q}_{\text{in}}}{\dot{m}} =
$$
$$
= \frac{l^2}{v^2}\frac{u_e^2}{2} + \frac{l^2}{v^2}gz_e + \frac{l^2}{v^2}\frac{p_e}{\rho_e} + \frac{l^2}{v^2}\frac{\dot{W}_{\text{out}}}{\dot{m}} + \frac{l^2}{v^2}\frac{\dot{Q}_{\text{out}}}{\dot{m}} + \frac{l^2}{v^2}\frac{\Delta p_L}{\rho_{av}}
\tag{35}
$$

If the fluid is compressible it can be assumed the ideal gas law, it results:

$$
\rho = \frac{p}{RT}
$$

and then

$$
\frac{l^2}{v^2}\frac{u_i^2}{2} + \frac{l^2}{v^2}gy_i + \frac{l^2}{v^2}RT_i + \frac{l^2}{v^2}\frac{\dot{W}_{\text{in}}}{\dot{m}} + \frac{l^2}{v^2}\frac{\dot{Q}_{\text{in}}}{\dot{m}} =
$$
$$
= \frac{l^2}{v^2}\frac{u_e^2}{2} + \frac{l^2}{v^2}gy_e + \frac{l^2}{v^2}RT_e + \frac{l^2}{v^2}\frac{\dot{W}_{\text{out}}}{\dot{m}} + \frac{l^2}{v^2}\frac{\dot{Q}_{\text{out}}}{\dot{m}} + \frac{l^2}{v^2}\frac{RT_i}{p_{av}}\Delta p_L
$$

If $\Delta p = p_i - p_e$, $\Delta u = u_i - u_e$, $\Delta z = z_i - z_e$, it becomes:

$$
\left(\frac{\Delta u^2}{2g} + \Delta z + \frac{\Delta p}{\rho_{av}g}\right) + \frac{\Delta \dot{W}}{\dot{m}g} + \frac{\Delta \dot{Q}}{\dot{m}g} = \frac{\Delta p_L}{\rho_i g}
\tag{36}
$$

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A new expression of fluid dynamics laws

If we consider a 2-D flow in a 2-D closed domain, fig. 1, the steady-state of any fluid system [30, 31] can be described by conservation laws for the control volume with inlets and outlets of mass, energy, work together with internal dissipations, fig.1.

$$\Delta p = \rho g \Delta y = \frac{\rho}{2} \Delta u^2 \tag{37}$$

which represent Archimedes law ($p=\rho g y$) [32], Torricelli's law ($gz=0.5u^2$) [33] and dynamic pressure ($p = 0.5\rho u^2$) [34]. Equation (37) can be expressed:

$$\left(\frac{\Delta u^2}{2} \frac{l^2}{v^2} + \frac{l^2}{v^2} g \Delta y + \frac{l^2}{v^2} \frac{\Delta p}{\rho_{av}} \right) + \frac{l^2}{v^2} \frac{\Delta \dot{W}}{\dot{m}} + \frac{l^2}{v^2} \frac{\Delta \dot{Q}}{\dot{m}} = \frac{l^2}{v^2} \frac{\Delta p_L}{\rho_{av}} \tag{38}$$

By considering the definition of Bejan Number by Mahmud and Fraser, which is expressed by equation (3) it is possible to define the following terms.

Definition of Bejan numbers and Bejan energies

Pressure Bejan number

$$\text{Be}_P = \frac{l^2}{v^2} \frac{\Delta p}{\rho_{av}} = \frac{l^2}{v^2} \frac{p_i - p_e}{\rho_{av}} = \frac{l^2}{v^2} \left(\frac{p_i}{\rho_i} - \frac{p_e}{\rho_e} \right) = \xi_{P,i} - \xi_{P,e} \tag{39}$$

or

$$\text{Be}_P = \frac{l^2}{v^2} \frac{\Delta p}{\rho_{av}} = \frac{l^2}{v^2} \left(RT_i - RT_e \right) = \xi_{P,i} - \xi_{P,e} \tag{40}$$

where $\rho_{av} = (\rho_i + \rho_e)/2$. Hence, if ρ = constant it simplifies as:

$$\text{Be}_P = \frac{l^2}{v^2} \frac{\Delta p}{\rho} = \frac{l^2}{v^2} \frac{p_i - p_e}{\rho} = \xi_{P,i} - \xi_{P,e} \tag{41}$$

Kinetic Bejan number

$$\text{Be}_K = \frac{l^2}{v^2} \frac{\Delta u^2}{2} = \frac{l^2}{v^2} \frac{\Delta p_K}{\rho_{av}} = \frac{l^2}{v^2} \left(\frac{u_i^2}{2} - \frac{u_e^2}{2} \right) = \xi_{K,i} - \xi_{K,e} \tag{42}$$

$$\rho = \text{constant} \rightarrow \text{Be}_K = \frac{l^2}{\rho v^2} \left(\frac{\rho u_i^2}{2} - \frac{\rho u_e^2}{2} \right) = \xi_{K,i} - \xi_{K,e}$$

Hydrostatic (potential) Bejan number

$$\text{Be}_H = \frac{l^2}{v^2} g \Delta y = \frac{l^2}{v^2} \frac{\Delta P_H}{\rho_{av}} = \frac{l^2}{v^2} g \left(y_i - y_e \right) = \xi_{H,i} - \xi_{H,e} \tag{43}$$

$$\rho = \text{constant} \rightarrow \text{Be}_H = \frac{l^2}{\rho v^2} \left[\rho g \left(y_i - y_e \right) \right] = \xi_{H,i} - \xi_{H,e} \tag{44}$$

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Work Bejan number

$$\mathrm{Be}_W = \frac{l^2}{v^2}\frac{\Delta \dot{W}}{\dot{m}} = \frac{l^2}{v^2}\frac{\dot{W}_i - \dot{W}_e}{\dot{m}} = \frac{l^2}{v^2}\frac{\dot{W}_i - \dot{W}_e}{\dot{m}} = \xi_{W,i} - \xi_{W,e} \tag{45}$$

Heat Bejan number

$$\mathrm{Be}_Q = \frac{l^2}{v^2}\frac{\Delta \dot{Q}}{\dot{m}} = \frac{l^2}{v^2}\frac{\dot{Q}_i - \dot{Q}_e}{\dot{m}} = \xi_{Q,i} - \xi_{Q,e} \tag{46}$$

Loss Bejan number

$$\mathrm{Be}_L = \frac{l^2}{v^2}\frac{\Delta p_L}{\rho_{av}} = 0 - \frac{l^2}{v^2}\frac{\Delta p_L}{\rho_{av}} = 0 - \xi_L \tag{47}$$

Reaction Bejan number

$$\xi_R = \frac{l^2}{\rho_{av}v^2}\frac{\mathrm{R}}{A_w} \tag{48}$$

Weight Bejan number

$$\xi_G = \frac{l^2}{\rho_{av}v^2}\frac{m}{A_w}\mathrm{g} \tag{49}$$

Conservation equations

It is then possible to determine the different equations of the conservation laws.

Conservation of mass

$$\dot{m}_i = \dot{m}_e \rightarrow A_i \rho_i u_i = A_e \rho_e u_e \rightarrow A_i \frac{\rho_i u_i l}{\mu} = A_e \frac{\rho_e u_e l}{\mu} \rightarrow A_i \,\mathrm{Re}_i = A_e \,\mathrm{Re}_e$$

$$\dot{m}_i = \dot{m}_e \rightarrow A_i \rho_i u_i = A_e \rho_e u_e \rightarrow A_i \sqrt{\rho_i} \sqrt{\rho_i \left(\frac{u_i l}{\mu}\right)^2} = A_e \sqrt{\rho_e} \sqrt{\rho_e \left(\frac{u_e l}{\mu}\right)^2}$$

$$A_i \sqrt{\rho_i}\sqrt{\xi_{K,i}} = A_e \sqrt{\rho_e}\sqrt{\xi_{K,e}} \tag{50}$$

Relation between Bejan energy and Reynolds number

$$\mathrm{Re} = \sqrt{\rho}\sqrt{\xi_K} \rightarrow \xi_K = \frac{\mathrm{Re}^2}{\rho} \tag{51}$$

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Conservation of momentum

$$
\begin{cases}
\dfrac{l^2}{\rho_{av}v^2}\left(A_i\cos\phi_i p_i - A_e\cos\phi_e p_e\right) + \dfrac{l^2}{\rho_{av}v^2}R_x = \\[2ex]
= -A_i\cos\phi_i\dfrac{l^2}{\rho_{av}v^2}\rho_i u_i^2 + A_e\cos\phi_e\dfrac{l^2}{\rho_{av}v^2}\rho_e u_e^2 \\[2ex]
\dfrac{l^2}{\rho_{av}v^2}\left(A_i\sin\phi_i p_i - A_e p_e\sin\phi_e\right) + \dfrac{l^2}{\rho_{av}v^2}R_y - \dfrac{l^2}{\rho_{av}v^2}mg = \\[2ex]
= A_i\sin\phi_i\dfrac{l^2}{\rho_{av}v^2}\rho_i u_i^2 + A_e\sin\phi_e\dfrac{l^2}{\rho_{av}v^2}\rho_e u_e^2
\end{cases}
$$

$$
\begin{cases}
A_i\cos\phi_i\left(\xi_{P,i} - \xi_{K,i}\right) - A_e\cos\phi_e\left(\xi_{P,e} + \xi_{K,e}\right) = -A_w\xi_{R,x} \\[1ex]
A_i\sin\phi_i\left(\xi_{P,i} + \xi_{K,i}\right) - A_e\sin\phi_e\left(\xi_{P,e} + \xi_{K,e}\right) = -A_w\xi_{R,y} + A_w\xi_G
\end{cases}
\tag{52}
$$

Bernoulli theorem

Bernoulli theorem equation is obtained by the law of conservation of momentum expressed on a flow line and can be expressed:

$$
\frac{1}{2}\rho u^2 + \rho gz + p = \text{constant}
\tag{53}
$$

It can be expressed utilizing Bejan energy by multiplying both terms for $l^2/\rho v^2$ and becomes:

$$
\xi_{k,i} + \xi_{z,i} + \xi_{p,i} = \xi_{k,e} + \xi_{z,e} + \xi_{p,e}
\tag{54a}
$$

Assuming that Bejan energies could be summed it is possible to write a condensed form:

$$
\xi_{K+H+P,i} = \xi_{K+H+P,e}
\tag{54b}
$$

or in function of Bejan number:

$$
\left(\text{Be}_K + \text{Be}_H + \text{Be}_P\right) = \left(\text{Be}_{K+H+P}\right) = 0
\tag{55}
$$

Conservation of energy

Conservation of energy equation is expressed:

$$
\begin{aligned}
\xi_{k,i} + \xi_{z,i} + \xi_{p,i} + \xi_{W,in} + \xi_{Q,in} = \\
= \xi_{k,e} + \xi_{z,e} + \xi_{p,e} + \xi_{W,out} + \xi_{Q,out} + \text{Be}_L
\end{aligned}
\tag{56}
$$

$$
\xi_{K+H+P,i} + \xi_{W,in} + \xi_{Q,in} = \xi_{K+H+P,e} + \xi_{W,out} + \xi_{Q,out} + \text{Be}_L
\tag{57}
$$

$$
\left(\text{Be}_K + \text{Be}_H + \text{Be}_P\right) + \text{Be}_W + \text{Be}_Q = \text{Be}_L
\tag{58}
$$

$$
\left(\text{Be}_{K+H+P}\right) + \text{Be}_W + \text{Be}_Q = \text{Be}_L
$$

The Second law of thermodynamics and fluid laws

Being expressed in terms of the pressure difference (3), Bejan number caused by fluid dynamic losses has been referred to both exergy dissipation and entropy generation (23). Natterer and Camberos [35] and Arntz *et al.* [36, 37] evidences the correspondence between exergy states and fluid dynamic energy states. As it will be easy to show the importance of the use of Bejan number and/or Bejan energy in the above equations is that they can be directly related to second law of thermodynamics.

Second law expression of Bejan number and Bejan energy

It is consequently possible to express all the terms in the conservation equations in thermodynamic terms according to second law in terms of both entropy created or exergy destroyed:

$$\mathrm{Be}_L = \frac{l^2}{\rho v^2}\Delta p =
\begin{cases}
\nearrow \dfrac{1}{\rho A_w u_{\mathrm{ref}}}\dfrac{l^2}{v^2}T\Delta\dot{S}_{\mathrm{gen}} = \dfrac{l^2}{\dot{m}v^2}T\Delta\dot{S}_{\mathrm{gen}}\\[2ex]
\searrow \dfrac{1}{\rho A_w u_{\mathrm{ref}}}\dfrac{l^2}{v^2}\Delta\dot{X}_{\mathrm{loss}} = \dfrac{l^2}{\dot{m}v^2}\Delta\dot{X}_{\mathrm{loss}}
\end{cases}$$

Pressure Bejan number

$$\mathrm{Be}_P = \frac{l^2}{v^2}\frac{\Delta p}{\rho_{av}} = \frac{l^2}{\dot{m}v^2}T\Delta\dot{S}_P = \frac{l^2}{\dot{m}v^2}\Delta\dot{X}_P \tag{59}$$

or

$$\mathrm{Be}_P = \xi_{P,i} - \xi_{P,\mathrm{e}} = \frac{l^2}{\dot{m}v^2}\dot{X}_{P,i} - \frac{l^2}{\dot{m}v^2}\dot{X}_{P,\mathrm{e}} = \frac{l^2}{\dot{m}v^2}\left(\dot{X}_{P,i} - \dot{X}_{P,\mathrm{e}}\right) \tag{60}$$

Kinetic Bejan number

Being exergetic state correspondent to kinetic energy:

$$\dot{X}_K = \frac{1}{2}\rho u^2$$

it results:

$$\mathrm{Be}_K = \frac{l^2}{v^2}\frac{\Delta p_K}{\rho} = \frac{l^2}{\rho v^2}\frac{1}{2}\rho\left(u_i^2 - u_e^2\right) = \frac{l^2 T}{\dot{m}v^2}\Delta\dot{S}_{K,\mathrm{gen}} = \frac{l^2}{\dot{m}v^2}\Delta\dot{X}_{K,\mathrm{loss}} \tag{61}$$

where Be_K

$$\mathrm{Be}_K = \xi_{K,i} - \xi_{K,\mathrm{e}} = \frac{l^2\dot{X}_{K,i}}{\dot{m}v^2} - \frac{l^2\dot{X}_{K,\mathrm{e}}}{\dot{m}v^2} = \frac{l^2}{\dot{m}v^2}\left(\dot{X}_{K,i} - \dot{X}_{K,\mathrm{e}}\right) \tag{62}$$

Hydrostatic (potential) Bejan number

It is:

$$\mathrm{Be}_H = \frac{l^2}{v^2}\mathrm{g}\Delta y = \frac{l^2}{v^2}\frac{\Delta P_H}{\rho_{av}} = \frac{l^2}{v^2}\mathrm{g}\left(y_i - y_e\right) = \frac{l^2 T}{\dot{m}v^2}\Delta\dot{S}_H = \frac{l^2}{\dot{m}v^2}\Delta\dot{X}_H \tag{63}$$

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and consequently:

$$\mathrm{Be}_H = \xi_{H,i} - \xi_{H,e} = \frac{l^2}{\dot{m}v^2}\dot{X}_{H,i} - \frac{l^2}{\dot{m}v^2}\dot{X}_{H,e} \tag{64}$$

Work Bejan number

It is evidently related to second law:

$$\mathrm{Be}_W = \frac{l^2}{v^2}\frac{\Delta\dot{W}}{\dot{m}} = \frac{l^2}{\dot{m}v^2}\left(\dot{W}_{\mathrm{in}} - \dot{W}_{\mathrm{out}}\right) = \frac{l^2 T}{\dot{m}v^2}\Delta\dot{S}_W = \frac{l^2}{\dot{m}v^2}\Delta\dot{X}_W \tag{65}$$

Hence, assuming that,

$$\dot{X}_K = \frac{1}{2}\rho u^2,$$

It results:

$$\mathrm{Be}_W = \xi_{W,in} - \xi_{W,\mathrm{out}} = \frac{l^2}{\dot{m}v^2}\left(\dot{W}_{\mathrm{in}} - \dot{W}_{\mathrm{out}}\right) = \frac{l^2}{\dot{m}v^2}\dot{X}_{W,\mathrm{in}} - \frac{l^2}{\dot{m}v^2}\dot{X}_{W,\mathrm{out}} \tag{66}$$

Heat Bejan number

It is also related to second law:

$$\mathrm{Be}_Q = \frac{l^2}{v^2}\frac{\Delta\dot{Q}}{\dot{m}} = \frac{l^2}{\dot{m}v^2}\left(\dot{Q}_{\mathrm{in}} - \dot{Q}_{\mathrm{out}}\right) = \frac{l^2 T}{\dot{m}v^2}\Delta\dot{S}_Q = \frac{l^2}{\dot{m}v^2}\Delta\dot{X}_Q \tag{67}$$

and results:

$$\mathrm{Be}_W = \xi_{Q,\mathrm{in}} - \xi_{Q,\mathrm{out}} = \frac{l^2}{\dot{m}v^2}\left(\dot{Q}_{\mathrm{in}} - \dot{Q}_{\mathrm{out}}\right) = \frac{l^2}{\dot{m}v^2}\dot{X}_{Q,\mathrm{in}} - \frac{l^2}{\dot{m}v^2}\dot{X}_{Q,\mathrm{out}}$$

Loss Bejan number

$$\mathrm{Be}_L = \frac{l^2}{v^2}\frac{\Delta p_L}{\rho_{av}} = 0 - \frac{l^2}{v^2}\frac{\Delta p_L}{\rho_{av}} = \frac{l^2}{\dot{m}v^2}T\Delta\dot{S}_{\mathrm{gen}} = \frac{l^2}{\dot{m}v^2}\Delta\dot{X}_{\mathrm{loss}} \tag{68}$$

Reaction Bejan number

$$\xi_R = \frac{l^2}{\rho_{av}v^2}\frac{R}{A_w} \tag{69}$$

Weight Bejan number

$$\xi_G = \frac{l^2}{\rho_{av}v^2}\frac{m}{A_w}\mathrm{g} \tag{70}$$

Second law expression of conservation laws

It is then possible to determine the different equations of the conservation law of thermodynamics.

Conservation of mass

$$A_i \sqrt{\rho_i} \sqrt{\frac{l^2 T\dot{S}_{K,i}}{\dot{m}v^2}} = A_i \sqrt{\rho_i} \sqrt{\frac{l^2 \dot{X}_{K,i}}{\dot{m}v^2}} = A_e \sqrt{\rho_e} \sqrt{\frac{l^2 T\dot{S}_{K,e}}{\dot{m}v^2}} = A_e \sqrt{\rho_e} \sqrt{\frac{l^2 \dot{X}_{K,e}}{\dot{m}v^2}} \tag{71}$$

which is

$$A_i \sqrt{\frac{T\dot{S}_{K,i}}{A_i u_i}} = A_i \sqrt{\frac{\dot{X}_{K,i}}{A_i u_i}} = A_e \sqrt{\frac{T\dot{S}_{K,e}}{A_e u_e}} = A_e \sqrt{\frac{\dot{X}_{K,e}}{A_e u_e}} \tag{72}$$

Conservation of momentum

$$\begin{cases} A_i \dfrac{\cos\phi_i l^2}{\dot{m}v^2}\left(\dot{X}_{P,i} - \dot{X}_{K,i}\right) - A_e \dfrac{\cos\phi_e l^2}{\dot{m}v^2}\left(\dot{X}_{P,e} + \dot{X}_{K,e}\right) = -A_w \xi_{R,x} \\[3mm] A_i \dfrac{\sin\phi_i l^2}{\dot{m}v^2}\left(\dot{X}_{P,i} + \dot{X}_{K,i}\right) - A_e \dfrac{\sin\phi_e l^2}{\dot{m}v^2}\left(\dot{X}_{P,e} + \dot{X}_{K,e}\right) = -A_w \xi_{R,y} + A_w \xi_G \end{cases} \tag{73}$$

Bernoulli theorem

$$\frac{l^2 \dot{X}_{K,i}}{\dot{m}v^2} + \frac{l^2 \dot{X}_{H,i}}{\dot{m}v^2} + \frac{l^2 \dot{X}_{P,i}}{\dot{m}v^2} = \frac{l^2 \dot{X}_{K,e}}{\dot{m}v^2} + \frac{l^2 \dot{X}_{H,e}}{\dot{m}v^2} + \frac{l^2 \dot{X}_{P,e}}{\dot{m}v^2} \tag{74}$$

and become:

$$\xi_{K+H+P,i} = \frac{l^2}{\dot{m}v^2}\left(\dot{X}_{K,i} + \dot{X}_{H,i} + \dot{X}_{P,i}\right) = \xi_{K+H+P,e} = \frac{l^2}{\dot{m}v^2}\left(\dot{X}_{K,e} + \dot{X}_{H,e} + \dot{X}_{P,e}\right) \tag{75}$$

or

$$Be_{K+H+P,i} = \frac{l^2}{\dot{m}v^2}\left[\left(\dot{X}_{K,i} - \dot{X}_{K,e}\right) + \left(\dot{X}_{H,i} - \dot{X}_{H,e}\right) + \left(\dot{X}_{P,i} - \dot{X}_{P,e}\right)\right] \tag{76}$$

Conservation of energy

Conservation of energy equation can be expressed:

$$\xi_{k,i} + \xi_{z,i} + \xi_{p,i} + \xi_{W,\text{in}} + \xi_{Q,\text{in}} = \xi_{k,e} + \xi_{z,e} + \xi_{p,e} + \xi_{W,\text{out}} + \xi_{Q,\text{out}} + Be_L \tag{77}$$

$$\frac{l^2}{\dot{m}v^2}\left(\dot{X}_{K,i} + \dot{X}_{H,i} + \dot{X}_{P,i} + \dot{X}_{W,\text{in}} + \dot{X}_{Q,\text{in}}\right) =$$
$$= \frac{l^2}{\dot{m}v^2}\left(\dot{X}_{K,e} + \dot{X}_{H,e} + \dot{X}_{P,e} + \dot{X}_{W,\text{out}} + \dot{X}_{Q,\text{out}}\right) + \frac{l^2}{\dot{m}v^2}\dot{X}_L \tag{78}$$

$$\left(\dot{X}_{K,i} + \dot{X}_{H,i} + \dot{X}_{P,i}\right) + \dot{X}_{W,\text{in}} + \dot{X}_{Q,\text{in}} = \left(\dot{X}_{K,e} + \dot{X}_{H,e} + \dot{X}_{P,e}\right) + \dot{X}_{W,\text{out}} + \dot{X}_{Q,\text{out}} + \dot{X}_L \tag{79}$$

Analysis of a specific sample case

The effects of compressibility can be evaluated by considering the ideal gas law:

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$$pV = \mathrm{R}T \rightarrow \frac{p}{\rho} = \mathrm{R}T \rightarrow p = \rho\mathrm{R}T \tag{80}$$

If a certain length, l, of the pipe is considered, the loss of pressure can be expressed as Be_L and is:

$$\mathrm{Be}_L = \frac{l^2}{\rho v^2} \Delta p_{\mathrm{loss}} = \frac{1}{\rho A_w u_{\mathrm{ref}}} \frac{l^2}{v^2} T \Delta \dot{S}_{\mathrm{gen}} = \frac{l^2}{\dot{m} v^2} T \Delta \dot{S}_{\mathrm{gen}} \tag{81}$$

It is then possible to evaluate:

$$\xi_{P,i} - \xi_{P,e} = \mathrm{Be}_L \rightarrow \frac{l^2 \dot{X}_{P,i}}{\dot{m} v^2} - \frac{l^2 \dot{X}_{P,e}}{\dot{m} v^2} = \frac{1}{\rho A_w u} \frac{l^2}{v^2} T \Delta \dot{S}_{\mathrm{gen}} = \mathrm{Be}_L \tag{82}$$

from which it result:

$$\frac{l^2}{\rho v^2}(P_i - P_e) = \frac{1}{\dot{m}} \rho \frac{l^2}{\rho v^2} T \Delta \dot{S}_{\mathrm{gen}} \rightarrow (P_i - P_e) = \frac{1}{\dot{m}} \rho T \Delta \dot{S}_{\mathrm{gen}} \tag{83}$$

and then:

$$P_i - P_e = \frac{1}{\dot{m}} \rho T \Delta \dot{S}_{\mathrm{gen}} = \frac{1}{\dot{m}} \frac{P_i}{R} \Delta \dot{S}_{\mathrm{gen}} \rightarrow \Delta \dot{S}_{\mathrm{gen}} = \dot{m} R \frac{P_i - P_e}{P}$$

An accurate verification of the proposed equations allows to consider some samples, fig. 4.
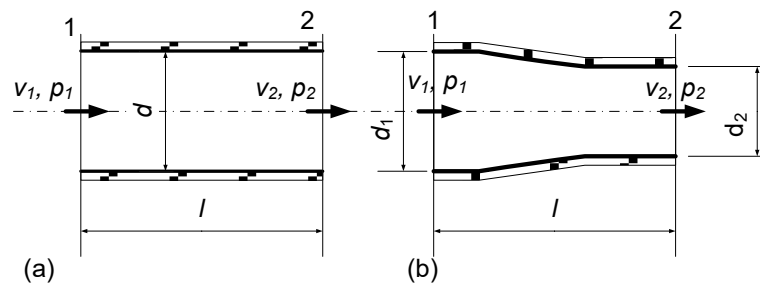

Figure 4. Two samples of flow in ducts

Throttle flow

An ideal gas is considered. It can be considered the ideal gas law $pV = \mathrm{R}T$ and the constancy of entropy $h_1 = h_2$. By applying the previous method, it can be easily obtained the entropy generation during the flux along a horizontal pipe without friction between two sections 1 and 2.

If the total loss of entropy is Δp

$$\Delta S_{1-2} = -R \int_1^2 \frac{\mathrm{d}p}{p} = -R \ln \frac{p_2}{p_1} = R \ln \left(1 - \frac{\Delta p}{p_1}\right) \tag{84}$$

$$\dot{S}_{\mathrm{gen}} = \dot{m} R \frac{\Delta p}{p_1} \tag{85}$$

Moreover, the result which has been obtained in eq. (22) is verified according to scientific literature.

Flow with friction

It can be possible to analyze the steady and adiabatic flow of an ideal gas through the segment of a pipe. In this case, the same result of eq. (22) can be expressed:

$$\dot{S}_{gen} = m\text{R} \ln \frac{\Delta p}{p_1} \rightarrow \dot{S}_{gen} = m\text{R} \frac{\Delta p}{p_1} \tag{86}$$

The work lost because of irreversibility is:

$$\dot{W}_{lost} = \dot{m}\left[(h_1 - T_0 s_1) - (h_2 - T_0 s_2)\right] = \dot{m}T_0 \Delta s_{1-2} = TS_{gen} \tag{87}$$

$$\dot{W}_{lost} = T\dot{S}_{gen} = \dot{m}\text{R}T_0 \frac{\Delta p}{p_1} \tag{88}$$

It can be remarked that the decrease in exergy is proportional to the pressure drop as well as the mass flow rate.

Conclusions

In a recent paper, Liversage and Trancossi have defined a new formulation of drag as a function of the dimensionless Bejan and Reynolds numbers. Further analysis of this hypothesis has permitted to obtain a new dimensionless formulation of the fundamental equations of fluid dynamics in their integral form. This activity shows that fluid mechanics problems can be modeled by mean of Bejan number, which allows correlating the fluid dynamic problems to entropy generation.

In particular, it demonstrates that fluid dynamic phenomena can be described by new sets of dimensionless equations that account both first and second law of thermodynamics. This direct correlation which currently under further investigation, allows describing fluid phenomena as function of dimensionless exergy dissipation rate or entropy generation rate.

The resulting equations have been deeply discussed for the thermodynamic definition of Bejan number evidencing that the proposed formulation allows solving fluid dynamic problems in terms of entropy generation, allowing an effective optimization of design in terms of the second law of thermodynamics.

A final sample has allowed verifying that in simple cases results coincides with the ones obtained by traditional methods. Further activity is planned in the direction of defining an effective method for solving fluid dynamic problems in the domain of second law of thermodynamics.

Acknowledgment

Nomenclature

A	– area, [m²]		C_D	– drag coefficient
A_f	– front Area, [m²]		g	– acceleration of gravity, [ms⁻²]
A_w	– wet Area, [m²]		K	– pressure loss coefficient
Be	– Bejan number		k	– heat conduction coefficient, [Wm⁻²K⁻¹]
Be_H	– hydrostatic Bejan number		l	– characteristic length, [m]
Be_K	– kinetic Bejan number		$\dot{m}$	– mass flow, [kgs⁻¹]
$6e_L$	– Bejan number caused by losses		p	– pressure, [Pa]
Be_P	– pressure Bejan number		Δp	– pressure difference [Pa]
Be_Q	– Bejan number by Heat transfer		Δp_H	– hydrostatic pressure [Pa]
Be_W	– Bejan number caused by Work		Δp_L	– pressure losses [Pa]

$\dot{Q}$ – heat flux, [W]
Q_{in} – inlet heat transferred, [J]
Q_{out} – outlet heat transferred, [J]
Re – Reynolds number, $[= (\rho u l)/\mu = u l/\nu]$
$\dot{S}$ – integral entropy rate, [WK^{-1}]
$\dot{s}$ – local entropy rate, [Wm^{-2}K^{-1}]
$\dot{s}_{gen,\Delta P}$ – local entropy rate by dissipation, [Wm^{-2}K^{-1}]
$\dot{s}_{gen,\Delta T}$ – local entropy generation by heat exchange, [Wm^{-2}K^{-1}]
T – temperature, [K]
u – velocity, [ms^{-1}]
u_e – outlet velocity, [ms^{-1}]
u_i – inlet velocity, [ms^{-1}]
$\dot{W}_{in}$ – power input, [W]
$\dot{W}_{out}$ – power output, [W]
W_{out} – work output, [J]
$\dot{W}_P$ – pressure power, [W]
$\dot{X}$ – exergy dissipation flux, [W]

$\dot{X}_H$ – hydrostatic exergy dissipation flux, [W]
$\dot{X}_K$ – kinetic exergy dissipation flux, [W]
$\dot{X}_{loss}$ – exergy dissipation flux by loss, [W]
$\dot{X}_P$ – pressure exergy dissipation flux, [W]
$\dot{X}_Q$ – exergy dissipation flux by heat, [W]
$\dot{X}_W$ – exergy dissipation flux by work, [W]

Greek symbols

ε – heat-exchange effectiveness
η – efficiency
μ – dynamic viscosity [Pa s]
ν – kinematic viscosity [m^2/s]
ρ – density, [kgm^{-3}]
ξ_H – hydrostatic Bejan energy
ξ_K – kinetic Bejan energy
ξ_L – Bejan energy caused by losses
ξ_P – pressure Bejan Energy
ξ_Q – Bejan energy by Heat transfer
ξ_W – Bejan Energy caused by Work

References

[1] Liversage, P., Trancossi M., Analysis of Triangular Sharkskin Profiles According to the Second Law, *IIE-TA Modelling, Measurement and Control B (MMC_B), 87* (2018), 3, pp. 188-196

[2] Bhattacharjee, S., Grosshandler, W. L., The Formation of a Wall Jet Near a High-Temperature Wall under Microgravity Environment, *Proceedings,* ASME 1988 National Heat Transfer Conference, Houston, Tex., USA, 1988, Vol. 1, pp. 711-716

[3] Mahmud, S., Fraser, R. A., Thermodynamic Analysis of Flow and Heat Transfer Inside Channel with Two Parallel Plates, *Exergy an International Journal, 2* (2002), 3, pp.140-146

[4] Mahmud, S., Fraser, R. A., The Second Law Analysis in Fundamental Convective Heat Transfer Problems, *Intl. J. of Th. Sci., 42* (2002), 2, pp. 177-186

[5] Awad, M. M., Lage, J. L., Extending the Bejan Number to a General Form, *Thermal Science, 17* (2013), 2, pp. 631-633

[6] Awad, M. M., A New Definition of Bejan Number, *Thermal Science, 16* (2012) 4, pp. 1251-1253

[7] Awad, M. M., An Alternative Form of the Darcy Equation, *Thermal Science, 18* (2014), Suppl. 2, pp. S617-S619

[8] Trancossi, M., Sharma, S., Numerical and Experimental Second Law Analysis of a Low Thickness High Chamber Wing Profile, SAE Technical Paper 2018-01-1955, 2018

[9] Trancossi, M., Pascoa, J., A New Dimensionless Approach to General Fluid Dynamics Problems that Accounts Both the First and The Second Law of Thermodynamics" *Mathematical Modelling of Engineering Problems, 5*, (2018), 4, pp. 331-340

[10] Awad, M. M., Hagen Number Versus Bejan Number, *Thermal Science, 17* (2013), 4, pp. 1245-1250

[11] Awad, M. M., The Science and the History of the Two Bejan Numbers, *International Journal of Heat and Mass Transfer, 94* (2016), Mar., pp. 101-103

[12] Sciubba, E., A Minimum Entropy Generation Procedure for the Discrete Pseudo-Optimization of Finned-Tube Heat Exchangers, *Revue Générale de Thermique, 35* (1996), 416, pp. 517-525

[13] Klein, R. J., *et al.,* Dimensionless Pressure Drop Number for Non-Newtonian Fluids Applied to Constructal Design of Heat Exchangers, *International Journal of Heat and Mass Transfer, 115* (2017), Part A, pp. 910-914

[14] Bejan, A., General Criterion for Rating Heat-Exchanger Performance, *International Journal of Heat and Mass Transfer, 21* (1992), 5, pp. 655-658

[15] Xie, G., *et al.*, Optimization of Pin-Fins for a Heat Exchanger by Entropy Generation Minimization and Constructal Law, *Journal of Heat Transfer, 137* (2015), 6, 061901

[16] Bejan, A., Sciubba, E., The Optimal Spacing of Parallel Plates Cooled by Forced Convection, *International Journal of Heat and Mass Transfer, 35* (1992), 12, pp. 3259-3264

[17] Herwig, H., Schmandt, B., How to Determine Losses in a Flow Field: A Paradigm Shift Towards the Second Law Analysis, *Entropy, 16* (2014), 6, pp. 2959-2989

[18] Borisov, A. V., *et al.,* Describing the Motion of a Body with an Elliptical Cross Section in a Viscous Un-compressible Fluid by Model Equations Reconstructed from Data Processing, *Technical Physics Letters, 42* (2016), 9, pp. 886-890, 2016

[19] Chernov, N. N., *et al.,* A Method of Body Shape Optimization for Decreasing the Aerodynamic Drag Force in Gas Flow, *Technical Physics Letters, 44* (2018), 4, pp. 328-330

[20] Rastan, M. R., *et al.,* High-Reynolds number Flow around Coated Symmetrical Hydrofoil: Effect of Streamwise Slip on Drag Force and Vortex Structures, *Journal of Marine Science and Technology, 24* (2019), 2, pp. 500-511

[21] Czyz, Z., *et al.,* The Air Flow Influence on the Drag Force of a Sports Car, *Advances in Science and Technology, Research Journal, 24* (2018), 2, pp. 121-127

[22] Giese, M., *et al.*, Turbulent and Laminar Flow in Karst Conduits under Unsteady Flow Conditions: Interpretation of Pumping Tests by Discrete Conduit-Continuum Modeling, *Water Resources Research, 54* (2018), 3, pp. 1918-1933

[23] Pimenta, B. D., *et al.,* Performance of Explicit Approximations of the Coefficient of Head Loss for Pressurized Conduits, *Revista Brasileira de Engenharia Agrícola e Ambiental, 22* (2018), 5, pp. 301-307

[24] Paez, D., *et al.,* Method for Extended Period Simulation of Water Distribution Networks with Pressure Driven Demands, *Water Resources Management, 32* (2018), 8, pp. 2837-2846

[25] Fontana, N., *et al.,* Hydraulic and Electric Regulation of a Prototype for Real-Time Control of Pressure and Hydropower Generation in a Water Distribution Network, Journal of Water Resources Planning and Management, 144 (2018), 11, 04018072

[26] Drela, M., Power Balance in Aerodynamic Flows, *AIAA Journal, 47* (2009), 7, pp. 1761-1771

[27] Li, K. W., Applied Thermodynamics: Availability Method and Energy Conversion, CRC Press, Boka Raton, Fla., USA, 1995

[28] Hayes, D., *et al.* Entropy Generation Minimisation and Exergy Analysis Approaches for Aerospace Applications – A Review, *Proceedings*, 54th AIAA Aerospace Sciences Meeting, San Diego, Cal., USA, 2016

[29] Tuck, A. F., Proposed Empirical Entropy and Gibbs Energy Based on Observations of Scale Invariance in Open Nonequilibrium Systems, *The Journal of Physical Chemistry A, 121* (2017), 35, pp. 6620-6629

[30] Ward-Smith, A. J., Internal Fluid Flow – The Fluid Dynamics of Flow in Pipes and Ducts, Nasa Sti/recon Technical Report A, 81, 1980

[31] Johnson, R. W., *Handbook of Fluid Dynamics*, CRC Press, Boka Raton, Fla., USA, 2016

[32] Trancossi, M., *et al.*, Safety Analysis of an Airship which Loses Lifting Gas from the Hull, SAE Technical Paper No. 2018-01-1954, 2018

[33] Driver, R. D., Torricelli's Law – An Ideal Example of an Elementary ODE, *The American Mathematical Monthly, 105* (1998), 5, pp. 453-455

[34] Shin, J. H., Computational Study on Dynamic Pressure in a Swash-Plate Axial Piston Pump Connected to a Hydraulic Line with an End Resistance, *Journal of Mechanical Science and Technology, 29* (2015), 6, pp. 2381-2390

[35] Naterer, G. F., Camberos, J. A., Entropy Based Design and Analysis of Fluids Engineering Systems, CRC Press, Boka Raton, Fla., USA, 2008

[36] Arntz, A., *et al.,* Numerical Airframe Aerodynamic Performance Prediction: An Exergy Point of View, *Proceedings,* 49th International Symposium of Applied Aerodynamics, Lille, France, 2014

[37] Arntz, A., *et al.*, Exergy-Based Formulation for Aircraft Aeropropulsive Performance Assessment: Theoretical Development, *AIAA Journal, 53,* (2014), 6, pp.1627-1639