

STUDY ON THE SOLITARY WAVE SOLUTIONS OF THE IONIC CURRENTS ON MICROTUBULES EQUATION BY USING THE MODIFIED KHATER METHOD

by

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Original scientific paper

<https://doi.org/10.2298/TSCI190722370L>

In this survey, the ionic current along microtubules equation is handled by applying the modified Khater method to get the solitary wave solutions that describe the ionic transport throughout the intracellular environment which describes the behavior of many applications in a biological non-linear dispatch line for ionic currents. The obtained solutions support many researchers who are concerned with the discussion of the physical properties of the ionic currents along microtubules. Microtubules are one of the main components of the cytoskeleton, and function in many operations, comprehensive constitutional backing, intracellular transmit, and DNA division. Moreover, we also study the stability property of our obtained solutions. All obtained solutions are verified by backing them into the original equation by using MAPLE 18 and MATHEMATICA 11.2. These solutions show the power and effective of the used method and its ability for applying to many other different forms of non-linear partial differential equations.

Key words: longitudinal wave equation, modified Khater method,
stability property, solitary wave solutions

Introduction

Microtubules this word consists of three syllables which are (micro + tube + ule). These three syllables are items of the cytoskeleton. The cytoskeleton existed orbit the cytoplasm. Tubular polymers of tubulin can regrow to 50 μm and also be highly dynamic. The external diameter of a microtubule is around 24 nm where the domestic diameter is around 12 nm. It existed in eukaryotic cells and some bacteria. It also formed by the polymerization of a dimer of two globular proteins, alpha, and beta-tubulin. Microtubules play an important and vital role in several cellular processes. These prevent architecture of the cell, microfilaments, and intermediate filaments. It also prevents the domestic architecture of cilia-and flagella. It is not only just that but also supply platforms for intracellular transport. Microtubules are implicated in miscellaneous of cellular processes and also inclusive the movement of secretory vesicles, organelles [1-10].

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Marc Kirschner and Tim Mitchison suggested in (1986s) that, the microtubules can employ its dynamic characteristics of growth and contraction at their plus ends to probe the 3-D space of the cell. In contrary to regular dynamism microtubules, that has a half-life of 5-10 minutes [11-15]. Electrostatics of nanosystems is considered as one of the applications of the microtubules. Electrostatics of nanosystems plays an important and vital role of morphogenesis [16-20].

As we see, many methods were discovered to serve this great idea and for example The (G'/G) -expansion method, extended (G'/G) -expansion method, generalized (G'/G) -expansion method, novel (G'/G) -expansion method, tanh-function method, extended tanh-function method, the modified simple equation method, the auxiliary equation method, the sine-Gordon expansion method, the hyperbolic function method, new auxiliary equation method, an algebraic method, the extended trial equation method, extended simplest equation method, an improved generalized Jacobi elliptic function method, extended coupled sub-equations expansion method, Khater method (a new auxiliary equation method), and so on [21-30].

The rest of this paper is systematized In *Section 1*, we apply the modified Khater method [31]-[35] to the longitudinal wave equation [36]-[39] to get the exact and solitary traveling wave solutions. Also, we study the stability property of the obtained solutions. In *Section 2*, conclusions are given.

Solitary wave solutions

This section carries out the modified Khater method regarding ionic currents along with microtubules equation after obtaining exact and solitary traveling solutions regarding the model that draws the ionic transport for the duration of the intracellular environment. We illustrate the fundamental non-linear mindset about ionic currents up to expectation is oriented using microtubules. Consider the Ionic currents along microtubules equation keep in the follows form [40]:

$$\wp_2 \Im \mathcal{L}^2 \Lambda_{xx} + \mathcal{L}^2 \Lambda_{xx} + 2\wp_1 \Im \aleph \Lambda_t - \wp_1 \Im \Lambda_t = 0 \quad (1)$$

where $\wp_1, \wp_1$ are the transverse or longitudinal element hence resistance regarding an element-tarrying (ER) such up to expectation $\wp_1 = 10^9 \Omega$, $\wp_2 = 7 \cdot 10^6 \Omega$, $\mathcal{L} = 8 \cdot 10^{-9} \Omega$, and durability $\Im, \aleph$ are, respectively representing, the amount greatest capacitance concerning the ER then equal $1.8 \cdot 10^{-15} F$, the non-linearity over an ER capacitor in an MT. Using the traveling wave transformation:

$$\Lambda = \Lambda(x, t) = \Lambda(\mathfrak{U}), \mathfrak{U} = x - ct$$

where is anxiety velocity:

$$c\wp_2 \Im \mathcal{L}^2 \Lambda''' + \mathcal{L}^2 \Lambda'' + 2c\wp_1 \Im \aleph \Lambda \Lambda' - c\wp_1 \Im \Lambda = 0 \quad (2)$$

Integral eq. (2) concerning $\mathfrak{U}$ with zero constant of integration:

$$\Lambda'' - a\Lambda' + b\Lambda^2 - d\Lambda = 0 \quad (3)$$

where

$$\left[a = \frac{1}{c\wp_0 \Im}, b = \frac{\wp_0 \aleph}{\wp_2 \mathcal{L}^2}, d = \frac{\wp_1}{\wp_1 \mathcal{L}^2} \right]$$

Balancing the highest order derivative term and the non-linear term in eq. (3), leads to According to the general solution of the suggested technique, we get the solution of eq. (3) be in the subsequent paradigm:

$$\Lambda(\mathfrak{U}) = a_0 + K^{f(\mathfrak{U})} a_1 + K^{2f(\mathfrak{U})} a_2 + K^{-f(\mathfrak{U})} b_1 + K^{-2f(\mathfrak{U})} b_2 \quad (4)$$

where a_0, a_1, a_2, b_1, b_2 and k are arbitrary constants while $f(\mathfrak{U})$ satisfies the following auxiliary equation:

$$\left[f'(\mathfrak{U}) = \frac{1}{\text{Ln}(K)} \left(\alpha K^{-f(\mathfrak{U})} + \beta + \sigma K^{f(\mathfrak{U})} \right) \right]$$

where α, β , and σ are arbitrary constants. Substituting eq. (4) and its derivatives into eq. (3). Collecting all terms of the same power of $K^{f(\mathfrak{U})}$. Solving the obtained algebraic system by any computer software program:

Family 1:

$$a_0 = \frac{\sigma b_1}{\beta}, \quad a_1 = a_2 = a = 0, \quad b_2 = \frac{\alpha b_1}{\beta}, \quad b = -\frac{6\alpha\beta}{b_1}, \quad d = \beta^2 - 4\alpha\sigma$$

Family 2:

$$a_0 = \frac{(\beta^2 + 2\alpha\sigma)b_1}{6\alpha\beta}, \quad a_1 = a_2 = a = 0, \quad b_2 = \frac{\alpha b_1}{\beta}, \quad b = -\frac{6\alpha\beta}{b_1}, \quad d = -\beta^2 + 4\alpha\sigma$$

Family 3:

$$\begin{aligned} a_0 &= \frac{1}{8\alpha^4\beta^2\sigma} \left\{ -\sqrt{2} \sqrt{-\alpha^3\beta^4(\beta^2 - 4\alpha\sigma)^2} b_1 \left[\alpha(-\beta^2 + 2\alpha\sigma)b_1 + \beta \sqrt{\alpha^2(\beta^2 - 4\alpha\sigma)b_1^2} \right] + \right. \\ &\quad \left. + \alpha\beta^2 \left(\alpha\beta(-\beta^2 + 6\alpha\sigma)b_1 + (\beta^2 - 2\alpha\sigma) \sqrt{\alpha^2(\beta^2 - 4\alpha\sigma)b_1^2} \right) \right\} \\ a_1 &= a_2 = 0, \quad b_2 = \frac{\alpha^2\beta^2b_1 - \alpha\beta\sqrt{-\alpha^2(-\beta^2 + 4\alpha\sigma)b_1^2}}{4\alpha^2\beta\sigma}, \quad b = \frac{6(-\alpha\beta b_1 - \sqrt{\alpha^2\beta^2b_1^2 - 4\alpha^3\sigma b_1^2})}{b_1^2} \\ a &= \frac{5\sqrt{-\alpha^2(-\beta^2 + 4\alpha\sigma)b_1^2}}{\alpha b_1} \\ d &= \frac{3 \left[\alpha\beta b_1 + \sqrt{\alpha^2(\beta^2 - 4\alpha\sigma)b_1^2} \right] \sqrt{\alpha^3\beta^4(\beta^2 - 4\alpha\sigma)^2} b_1 \left[\alpha(\beta^2 - 2\alpha\sigma)b_1 - \beta \sqrt{\alpha^2(\beta^2 - 4\alpha\sigma)b_1^2} \right]}{\sqrt{2}\alpha^4\beta^2\sigma b_1^2} \end{aligned}$$

According to the value of parameters in *Family 1*, we get the solitary wave solutions of eq. (1):

– when $[\beta^2 - 4\alpha\sigma < 0, \sigma \neq 0]$ we get:

$$\Lambda_1(x, t) = \frac{-\sigma(\beta^2 - 4\alpha\sigma)b_1}{\beta \left\{ \beta \cos \left[\frac{1}{2}(-ct + x) \sqrt{-\beta^2 + 4\alpha\sigma} \right] - \sqrt{-\beta^2 + 4\alpha\sigma} \sin \left[\frac{1}{2}(-ct + x) \sqrt{-\beta^2 + 4\alpha\sigma} \right] \right\}^2} \quad (5)$$

$$\Lambda_2(x, t) = \frac{-\sigma(\beta^2 - 4\alpha\sigma)b_1}{\beta \left\{ \sqrt{-\beta^2 + 4\alpha\sigma} \cos \left[\frac{1}{2}(-ct + x) \sqrt{-\beta^2 + 4\alpha\sigma} \right] - \beta \sin \left[\frac{1}{2}(-ct + x) \sqrt{-\beta^2 + 4\alpha\sigma} \right] \right\}^2} \quad (6)$$

– when $[\beta^2 - 4\alpha\sigma > 0, \sigma \neq 0]$ we get:

$$\Lambda_3(x, t) = - \frac{\sigma(\beta^2 - 4\alpha\sigma)b_1}{\beta \left\{ \beta \cosh \left[\frac{1}{2}(-ct + x)\sqrt{\beta^2 - 4\alpha\sigma} \right] + \sqrt{\beta^2 - 4\alpha\sigma} \sinh \left[\frac{1}{2}(-ct + x)\sqrt{\beta^2 - 4\alpha\sigma} \right] \right\}^2} \quad (7)$$

$$\Lambda_4(x, t) = - \frac{\sigma(\beta^2 - 4\alpha\sigma)b_1}{\beta \left\{ \sqrt{\beta^2 - 4\alpha\sigma} \cosh \left[\frac{1}{2}(-ct + x)\sqrt{\beta^2 - 4\alpha\sigma} \right] + \beta \sinh \left[\frac{1}{2}(-ct + x)\sqrt{\beta^2 - 4\alpha\sigma} \right] \right\}^2} \quad (8)$$

– when $[\beta = \alpha/2 = \kappa \text{ and } \sigma = 0]$ we get:

$$\Lambda_5(x, t) = \frac{e^{(-ct+x)\kappa} b_1}{\left(-2 + e^{(-ct+x)\kappa}\right)^2} \quad (9)$$

– when $[\alpha = 0, \beta \neq 0, \text{ and } \sigma \neq 0]$ we get:

$$\Lambda_6(x, t) = \frac{2e^{ct\beta-x\beta} b_1}{\beta} \quad (10)$$

– when $[\sigma = 0, \beta \neq 0, \text{ and } \alpha \neq 0]$ we get:

$$\Lambda_7(x, t) = \frac{e^{(-ct+x)\beta} \beta^2 b_1}{\left(\alpha - e^{(-ct+x)\beta} \beta\right)^2} \quad (11)$$

– when $[\beta^2 - 4\alpha\sigma = 0]$ we get:

$$\Lambda_8(x, t) = \frac{4\sigma b_1}{\beta(2 - ct\beta + x\beta)^2} \quad (12)$$

According to the value of parameters in *Family 2*, we get the solitary wave solutions of eq. (1):

– when $[\beta^2 - 4\alpha\sigma < 0, \sigma \neq 0]$ we get:

$$\Lambda_9(x, t) = \frac{1}{6} b_1 \left\{ \frac{\beta}{\alpha} + \frac{24\alpha\sigma^2}{\beta \left\{ \beta - \sqrt{-\beta^2 + 4\alpha\sigma} \tan \left[\frac{1}{2}(-ct + x)\sqrt{-\beta^2 + 4\alpha\sigma} \right] \right\}^2} + \right. \\ \left. + 2\sigma \left\{ \frac{1}{\beta} - \frac{6}{\beta - \sqrt{-\beta^2 + 4\alpha\sigma} \tan \left[\frac{1}{2}(-ct + x)\sqrt{-\beta^2 + 4\alpha\sigma} \right]} \right\} \right\} \quad (13)$$

$$\Lambda_{10}(x, t) = \frac{1}{6} \left\{ \frac{\beta}{\alpha} + \frac{24\alpha\sigma^2}{\beta \left\{ \beta - \sqrt{-\beta^2 + 4\alpha\sigma} \cot \left[\frac{1}{2}(-ct + x)\sqrt{-\beta^2 + 4\alpha\sigma} \right] \right\}^2} + \right. \quad (14)$$

$$+ 2\sigma \left\{ \frac{1}{\beta} - \frac{6}{\beta - \sqrt{-\beta^2 + 4\alpha\sigma} \cot \left[\frac{1}{2}(-ct+x)\sqrt{-\beta^2 + 4\alpha\sigma} \right]} \right\} b_1 \quad (14)$$

– when $[\beta^2 - 4\alpha\sigma > 0, \sigma \neq 0]$ we get:

$$\Lambda_{11}(x,t) = \frac{1}{6\alpha\beta \left\{ \beta + \sqrt{\beta^2 - 4\alpha\sigma} \tanh \left[\frac{1}{2}(-ct+x)\sqrt{\beta^2 - 4\alpha\sigma} \right] \right\}^2} \cdot \left((\beta^2 - 4\alpha\sigma) b_1 \left\{ \beta^2 - 6\alpha\sigma + 2\beta\sqrt{\beta^2 - 4\alpha\sigma} \tanh \left[\frac{1}{2}(-ct+x)\sqrt{\beta^2 - 4\alpha\sigma} \right] + (\beta^2 + 2\alpha\sigma) \tanh \left[\frac{1}{2}(-ct+x)\sqrt{\beta^2 - 4\alpha\sigma} \right]^2 \right\} \right) \quad (15)$$

$$\Lambda_{12}(x,t) = \frac{1}{6\alpha\beta \left\{ \beta + \sqrt{\beta^2 - 4\alpha\sigma} \coth \left[\frac{1}{2}(-ct+x)\sqrt{\beta^2 - 4\alpha\sigma} \right] \right\}^2} \cdot \left[(\beta^2 - 4\alpha\sigma) \left\{ \beta^2 - 6\alpha\sigma + 2\beta\sqrt{\beta^2 - 4\alpha\sigma} \coth \left[\frac{1}{2}(-ct+x)\sqrt{\beta^2 - 4\alpha\sigma} \right] + (\beta^2 + 2\alpha\sigma) \coth \left[\frac{1}{2}(-ct+x)\sqrt{\beta^2 - 4\alpha\sigma} \right]^2 \right\} b_1 \right] \quad (16)$$

– when $[\beta = \alpha/2 = \kappa, \text{ and } \sigma = 0]$ we get:

$$\Lambda_{13}(x,t) = \frac{1}{6} \left[\frac{1}{2} + \frac{12}{(-2 + e^{(-ct+x)\kappa})^2} + \frac{6}{-2 + e^{(-ct+x)\kappa}} \right] b_1 \quad (17)$$

– when $[\sigma = 0, \beta \neq 0, \text{ and } \alpha \neq 0]$ we get:

$$\Lambda_{14}(x,t) = \frac{1}{6} \beta \left\{ \frac{1}{\alpha} + \frac{6\alpha}{[\alpha - e^{(-ct+x)\beta} \beta]^2} - \frac{6}{\alpha - e^{(-ct+x)\beta} \beta} \right\} b_1 \quad (18)$$

– when $[\beta^2 - 4\alpha\sigma = 0]$ we get:

$$\Lambda_{15}(x,t) = \frac{\left(\frac{\beta^2 [8 - (ct-x)\beta(-4 + ct\beta - x\beta)]}{\alpha(2 - ct\beta + x\beta)^2} + 4\sigma \right) b_1}{12\beta} \quad (19)$$

According to the value of parameters in *Family 3*, we get the solitary wave solutions of eq. (1):

– when $[\beta^2 - 4\alpha\sigma < 0, \sigma \neq 0]$ we get:

$$\begin{aligned} \Lambda_{16}(x, t) = & \frac{1}{8\alpha^4\beta^2\sigma} \left\{ -\sqrt{2} \sqrt{-\alpha^3\beta^4(\beta^2 - 4\alpha\sigma)^2} b_1 \left[\alpha(-\beta^2 + 2\alpha\sigma)b_1 + \beta\sqrt{\alpha^2(\beta^2 - 4\alpha\sigma)b_1^2} \right] + \right. \\ & + \alpha\beta^2 \left[\alpha\beta(-\beta^2 + 6\alpha\sigma)b_1 + (\beta^2 - 2\alpha\sigma)\sqrt{\alpha^2(\beta^2 - 4\alpha\sigma)b_1^2} \right] \Big\} + \\ & + \frac{\sigma \left[\alpha\beta b_1 - \sqrt{\alpha^2(\beta^2 - 4\alpha\sigma)b_1^2} \right]}{\alpha \left\{ \beta - \sqrt{-\beta^2 + 4\alpha\sigma} \tan \left[\frac{1}{2}(-ct + x) \sqrt{-\beta^2 + 4\alpha\sigma} \right] \right\}^2} - \\ & - \frac{2\sigma b_1}{\beta - \sqrt{-\beta^2 + 4\alpha\sigma} \tan \left[\frac{1}{2}(-ct + x) \sqrt{-\beta^2 + 4\alpha\sigma} \right]} \end{aligned} \quad (20)$$

$$\begin{aligned} \Lambda_{17}(x, t) = & \frac{1}{8\alpha^4\beta^2\sigma} \left\{ -\sqrt{2} \sqrt{-\alpha^3\beta^4(\beta^2 - 4\alpha\sigma)^2} b_1 \left[\alpha(-\beta^2 + 2\alpha\sigma)b_1 + \beta\sqrt{\alpha^2(\beta^2 - 4\alpha\sigma)b_1^2} \right] + \right. \\ & + \alpha\beta^2 \left[\alpha\beta(-\beta^2 + 6\alpha\sigma)b_1 + (\beta^2 - 2\alpha\sigma)\sqrt{\alpha^2(\beta^2 - 4\alpha\sigma)b_1^2} \right] \Big\} - \\ & - \frac{2\sigma b_1}{\beta - \sqrt{-\beta^2 + 4\alpha\sigma} \cot \left[\frac{1}{2}(-ct + x) \sqrt{-\beta^2 + 4\alpha\sigma} \right]} + \\ & + \frac{\sigma \left[\alpha\beta b_1 - \sqrt{\alpha^2(\beta^2 - 4\alpha\sigma)b_1^2} \right]}{\alpha \left\{ \beta - \sqrt{-\beta^2 + 4\alpha\sigma} \cot \left[\frac{1}{2}(-ct + x) \sqrt{-\beta^2 + 4\alpha\sigma} \right] \right\}^2} \end{aligned} \quad (21)$$

– when $[\beta^2 - 4\alpha\sigma > 0, \sigma \neq 0]$ we get:

$$\begin{aligned} \Lambda_{18}(x, t) = & \frac{1}{8\alpha^4\beta^2\sigma} \left\{ -\sqrt{2} \sqrt{-\alpha^3\beta^4(\beta^2 - 4\alpha\sigma)^2} b_1 \left[\alpha(-\beta^2 + 2\alpha\sigma)b_1 + \beta\sqrt{\alpha^2(\beta^2 - 4\alpha\sigma)b_1^2} \right] + \right. \\ & + \alpha\beta^2 \left[\alpha\beta(-\beta^2 + 6\alpha\sigma)b_1 + (\beta^2 - 2\alpha\sigma)\sqrt{\alpha^2(\beta^2 - 4\alpha\sigma)b_1^2} \right] \Big\} + \\ & + \frac{\sigma \left[\alpha\beta b_1 - \sqrt{\alpha^2(\beta^2 - 4\alpha\sigma)b_1^2} \right]}{\alpha \left\{ \beta + \sqrt{\beta^2 - 4\alpha\sigma} \tanh \left[\frac{1}{2}(-ct + x) \sqrt{\beta^2 - 4\alpha\sigma} \right] \right\}^2} - \\ & - \frac{2\sigma b_1}{\beta + \sqrt{\beta^2 - 4\alpha\sigma} \tanh \left[\frac{1}{2}(-ct + x) \sqrt{\beta^2 - 4\alpha\sigma} \right]} \end{aligned} \quad (22)$$

$$\Lambda_{19}(x,t) = \frac{1}{8\alpha^4\beta^2\sigma} \left\{ -\sqrt{2} \sqrt{-\alpha^3\beta^4(\beta^2-4\alpha\sigma)^2} b_1 \left[\alpha(-\beta^2+2\alpha\sigma)b_1 + \beta\sqrt{\alpha^2(\beta^2-4\alpha\sigma)b_1^2} \right] + \right. \\ \left. + \alpha\beta^2 \left[\alpha\beta(-\beta^2+6\alpha\sigma)b_1 + (\beta^2-2\alpha\sigma)\sqrt{\alpha^2(\beta^2-4\alpha\sigma)b_1^2} \right] \right\} - \\ - \frac{2\sigma b_1}{\beta + \sqrt{\beta^2-4\alpha\sigma} \coth \left[\frac{1}{2}(-ct+x)\sqrt{\beta^2-4\alpha\sigma} \right]} + \\ + \frac{\sigma \left[\alpha\beta b_1 - \sqrt{\alpha^2(\beta^2-4\alpha\sigma)b_1^2} \right]}{\alpha \left\{ \beta + \sqrt{\beta^2-4\alpha\sigma} \coth \left[\frac{1}{2}(-ct+x)\sqrt{\beta^2-4\alpha\sigma} \right] \right\}^2} \quad (23)$$

– when $[\beta^2 - 4\alpha\sigma = 0]$ we get:

$$\Lambda_{20}(x,t) = \frac{\beta \left[12\alpha + 8\alpha \left(-1 + \frac{2}{2-ct\beta+x\beta} \right) - \frac{2\beta^2}{\sigma} + \frac{(-ct+x)^2\beta^4}{(2-ct\beta+x\beta)^2\sigma} \right] b_1}{16\alpha^2} \quad (24)$$

Stability property

This section of our research paper investigates one of the basic properties of any model. It examines the stability property for the ionic current along with microtubules equation by using a Hamiltonian system. The momentum in the Hamiltonian system given by the following formula:

$$M = \frac{1}{2} \int_{-\mathfrak{J}}^{\mathfrak{J}} \Lambda^2(\mathfrak{U}) d\mathfrak{U} \quad (25)$$

where $\mathfrak{J}$ is arbitrary constant. Consequently, the condition for stability of solutions:

$$\frac{\partial M}{\partial c} \Big|_{c=\mathfrak{c}} > 0 \quad (26)$$

where $\mathfrak{c}$ is arbitrary constant.

For an example of studying the stability property for eq. (7), we get:

$$M = \frac{4}{27c} \left\{ \frac{2e^{5+5c}(-1+e^{10})(-4+e^{10c})}{(2e^5+e^{5c})^2(2+e^{5+5c})^2} + \frac{2e^{5+5c}(-1+e^{10})(-1+4e^{10c})}{(e^5+2e^{5c})^2(1+2e^{5+5c})^2} - \right. \\ \left. - \log[2+e^{5-5c}] + \log[2e^5+e^{-5c}] - \log[2e^5+e^{5c}] + \log[2+e^{5+5c}] \right\} \quad (27)$$

Thus, we obtain:

$$\frac{\partial M}{\partial c} \Big|_{c=5} = 0.2962962962962963 > 0 \quad (28)$$

Figure

The 3- and 2-D plots of the eqs. (7), (8), and eq. (15)

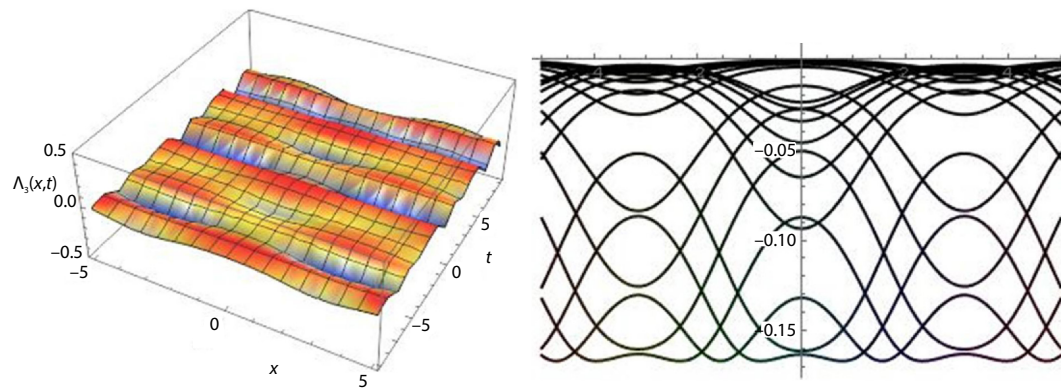


Figure 1. Periodic solitary wave solution in 3- and 2-D plots of eq. (7) when $\beta = 3$, $\alpha = 1$, $\sigma = 1$, $b_1 = 4$, and $c = 5$

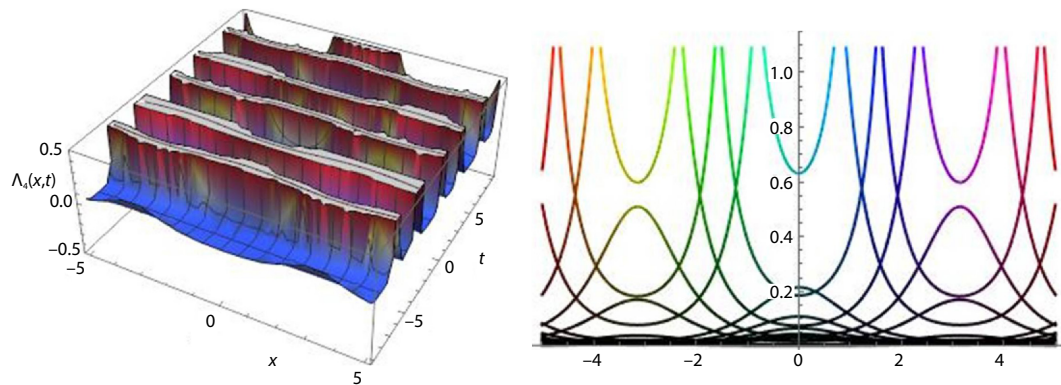


Figure 2: Periodic cuspon wave in 3- and 2-D plot of eq. (8) when $\beta = 3$, $\alpha = 1$, $\sigma = 1$, $b_1 = 4$, and $c = 5$

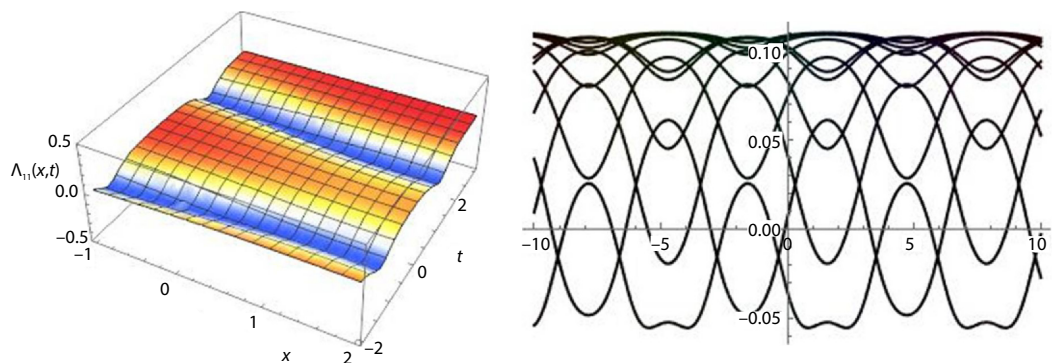


Figure 3: Periodic bright wave in 3- and 2-D plot of eq. (15) when $\beta = 3$, $\alpha = 1$, $\sigma = 1$, $b_1 = 4$, and $c = 5$

Conclusion

In this paper, we obtain solitary wave solutions of the longitudinal wave equation by using the modified Khater method (the modified auxiliary equation method). These solutions are novel and different solitary wave solutions of that obtained by using different schemes. Moreover, the stability property of solutions is tested by using the properties of the Hamiltonian system. The performance of the used technique shows useful and powerful in studying many of non-linear PDE. This performance shows its superiority and generalization of the used method in this research paper over some previous method. Some sketches are given to explain more physical properties of the ionic transport throughout the intracellular environment which describes the behavior of many applications in a biological non-linear dispatch line for ionic currents.

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