BAYESIAN ANALYSIS OF A CONSTANT-STRESS ACCELERATED LIFE TESTING WITH THERMAL AGING LIFE MODEL UNDER GENERAL PROGRESSIVE TYPE-II CENSORED DATA

by

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This paper discusses the parameter estimation by Bayesian method when the thermal aging lifetime follows the log-normal distribution and the sample is a general progressive type-II censoring from a constant-stress accelerated life test. The Bayes estimates cannot be obtained in an inexplicit form, and an approximate one is solved by the hybrid Markov chain Monte-Carlo method. The thermal aging life data are presented to illustrate proposed method.

Key words: *constant-stress, accelerated life test, thermal aging life, general progressive type-II censoring, Bayesian analysis*

Introduction

Reliability research is an important link in product design and production. In recent years, many scholars paid more and more attention to reliability research [1-3]. Bayesian statistical technique is much suitable to analyze the probability model in reliability engineering [4-6]. Long-life products are progressively increasing as the development of science and technology. Various accelerated life tests are widely used to study the product's lifetime with a long life in order to reduce the testing time and cost in the experiment [7-9]. In this paper, Bayesian method is used to analyze a thermal aging life model under a constant-stress accelerated life testing when data are general progressive type-II censoring.

Basic assumptions and life test procedure

Basic assumptions

The following assumptions are effective in this study.

Assumption 1. Under the normal stress S_0 and the accelerated stress S_i, $i = 1, 2, \dots, k$, the product's life follows log-normal distribution. The probability density function (PDF) and cumulative distribution function (CDF) of the log-normal model are given, respectively:

$$f(x_i;\mu_i,\sigma_i)=\frac{1}{x_i}\frac{1}{\sigma_i\sqrt{2\pi}}\exp\left[-\frac{(\ln x_i-\mu_i)^2}{2\sigma_i^2}\right] \tag{1}$$

$$F(x_i;\mu_i,\sigma_i)=\Phi\frac{\ln x_i-\mu_i}{\sigma_i} \tag{2}$$

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where σ_i is the shape parameter and μ_i – the scale parameter at stress S_i, $i = 1, 2, \ldots, k$.

Assumption 2. The failure mechanism of the product remains constant at each accelerated stress. Because the shape parameter reflects the failure mechanism, it implies that:

$$\sigma_0 = \sigma_1 = \cdots \sigma_k = \sigma \tag{3}$$

Assumption 3. The product accelerated model agrees with the linear Arrhenius equation. That is, the scale parameter μ_i and the accelerated stress S_i satisfy the following equation:

$$\ln \mu_i = a + b\phi(S_i), \quad i = 0,1,2,\cdots,k \tag{4}$$

According to eq. (4), we get that:

$$\mu_i = \mu_0 \theta^{\phi_i} = \mu \theta^{\phi_i} \tag{5}$$

where $\phi_i = \dfrac{\phi(S_i) - \phi(S_0)}{\phi(S_1) - \phi(S_0)}$, and θ is named as acceleration coefficient.

Life test procedure

Suppose the normal stress, S_0, and the k constant-stress accelerated stress levels $S_1 < S_2 < \ldots S_k$ are pre-fixed. The units with size n are simultaneously placed on the constant stress accelerated life test. The n units are divided into k groups with number n_i, $n_i > 1$, $n_1 + \ldots + n_i + \ldots + n_k = n$. Under the stress level, S_i, the n_i units are put into general progressive type-II censoring, it is conducted as the following.

Suppose that first failure times with size r_i units are not observed, and then at the time of the $(r_i + 1)^{\text{th}}$ failure, the surviving units with size $R_{i:r_i+1}$ are withdrawn from the test randomly. At the time of the $(r_i + 2)^{\text{th}}$ failure, $R_{i:r_i+2}$ surviving units are withdrawn from the test randomly. Finally, at the time of the $(m_i)^{\text{th}}$ failure, the rest $R_{i:m_i} = n_i - m_i - R_{i:r_i+1} - R_{i:r_i+2} - \cdots R_{i:m_i-1}$ units are withdrawn from the experiment, where the failure number m_i is prefixed. The $m_i - r_i$ failure times:

$$x_i = (x_{i:r_i+1}, x_{i:r_i+2}, \cdots, x_{i:m_i}) \tag{6}$$

are named as general progressive censored data with censoring scheme $R_i = (R_{i:r_i+1}, R_{i:r_i+2}, \cdots, R_{i:m_i})$, $i = 1, 2, \ldots, k$, where $x_{i:r_i+1} \le x_{i:r_i+2} \le \cdots \le x_{i:m_i}$. In this paper, we suppose the tests are independent of each other under the different stress levels, and $m_i - r_i > 0$ such that at least one unit failure time is observed, $i = 1, 2, \ldots, k$.

Maximum likelihood estimation and Bayesian estimation of the unknown parameters

At the stress level S_i, $i = 1, 2, \ldots, k$, the likelihood function is given by:

$$L(\sigma_i, \mu_i \mid x_i) \propto [F(x_{i:r_i+1}; \sigma_i, \mu_i)]^{r_i} \prod_{j=r_i+1}^{m_i} f(x_{i:j}; \sigma_i, \mu_i)[1 - F(x_{i:j}; \sigma_i, \mu_i)]^{R_{i:j}} \tag{7}$$

Then, by *Assumption 3* the likelihood function of the general progressive type-II censored sample $x = (x_1, x_2, \cdots, x_k)$, , $x_i = (x_{i:r_i+1}, x_{i:r_i+2}, \cdots, x_{i:m_i})$, is given by:

$$L(\sigma,\mu,\theta\,|\,x)\propto\prod_{i=1}^{k}\,[F(x_{i:r_i+1};\sigma,\mu,\theta)]^{r_i}\prod_{j=r_i+1}^{m_i}f(x_{i:j};\sigma,\mu,\theta)[1-F(x_{i:j};\sigma,\mu,\theta)]^{R_{i:j}}$$

$$\propto\prod_{i=1}^{k}\left(\Phi\frac{\ln x_{i:r_i+1}-\mu\theta^{\phi_i}}{\sigma}\right)^{r_i}\prod_{j=r_i+1}^{m_i}\frac{1}{x_{i:j}}\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(\ln x_{i:j}-\mu\theta^{\phi_i})^2}{2\sigma^2}\right]$$

$$\left(1-\Phi\frac{\ln x_{i:r_i+1}-\mu\theta^{\phi_i}}{\sigma}\right)^{R_{i:j}} \tag{8}$$

The log-likelihood function is given by:

$$\ln L(\sigma,\mu,\theta\,|\,x)=\sum_{i=1}^{k}r_i\log\left(\Phi\frac{\ln x_{i:r_i+1}-\mu\theta^{\phi_i}}{\sigma}\right)-\sum_{i=1}^{k}\sum_{j=r_i+1}^{m_i}\log(\sqrt{2\pi}\sigma x_{i:j})-$$

$$-\sum_{i=1}^{k}\sum_{j=r_i+1}^{m_i}\left[-\frac{(\ln x_{i:j}-\mu\theta^{\phi_i})^2}{2\sigma^2}\right]+\sum_{i=1}^{k}\sum_{j=r_i+1}^{m_i}R_{i:j}\log\left[1-\Phi\frac{\ln x_{i:j}-\mu\theta^{\phi_i}}{2\sigma^2}\right] \tag{9}$$

Maximum likelihood estimation

In order to compare with Bayesian estimation, we provide a brief discussion on the maximum likelihood estimation (MLE) of the unknown parameters as follows.

The likelihood equations for parameters μ, σ, θ are given by:

$$\frac{\partial\ln[L(\mu,\sigma,\theta\,|\,x)]}{\partial\mu}=0,\quad\frac{\partial\ln[L(\mu,\sigma,\theta\,|\,x)]}{\partial\sigma}=0,\quad\frac{\partial\ln[L(\mu,\sigma,\theta\,|\,x)]}{\partial\theta}=0 \tag{10}$$

The MLE of (μ, σ, θ), denoted as $(\hat{\mu},\hat{\sigma},\hat{\theta})$, can be obtained by eq. (10). However, eq. (10) is very complex and difficult to be solved analytically. Some famous analytical methods are the homotopy perturbation method [10-14] and the variational iteration method [15-18], in this paper we will use Newton-Raphson method to compute $(\hat{\mu},\hat{\sigma},\hat{\theta})$. We then assess the accuracy of MLE by inverting the observed Fisher information matrix $\mathrm{I}(\mu,\sigma,\theta)$ in $(\hat{\mu},\hat{\sigma},\hat{\theta})$ as follows:

$$\mathrm{I}^{-1}(\hat{\mu},\hat{\sigma},\hat{\theta})=\begin{bmatrix}\dfrac{\partial^2\ln L}{\partial\mu^2} & \dfrac{\partial^2\ln L}{\partial\mu\partial\sigma} & \dfrac{\partial^2\ln L}{\partial\mu\partial\theta}\\[2ex] \dfrac{\partial^2\ln L}{\partial\sigma\partial\mu} & \dfrac{\partial^2\ln L}{\partial\sigma^2} & \dfrac{\partial^2\ln L}{\partial\sigma\partial\theta}\\[2ex] \dfrac{\partial^2\ln L}{\partial\theta\partial\mu} & \dfrac{\partial^2\ln L}{\partial\theta\partial\sigma} & \dfrac{\partial^2\ln L}{\partial\theta^2}\end{bmatrix}^{-1}_{(\hat{\mu},\hat{\sigma},\hat{\theta})}\approx\begin{bmatrix}Var(\mu) & Cov(\mu,\sigma) & Cov(\mu,\theta)\\ & Var(\sigma) & Cov(\sigma,\theta)\\ & & Var(\theta)\end{bmatrix}^{-1}_{(\hat{\mu},\hat{\sigma},\hat{\theta})} \tag{11}$$

Therefore, the approximate 100(1 – τ)% confidence intervals of the parameters of $P_{\mathrm{MLE}}=(\mu,\sigma,\theta)$ are:

$$\left[\hat{P}_{\text{MLE}} - z_{\tau/2}\sqrt{Var(\hat{P}_{\text{MLE}})},\quad \hat{P}_{\text{MLE}} + z_{\tau/2}\sqrt{Var(\hat{P}_{\text{MLE}})}\right] \tag{12}$$

where $z_{\tau/2}$ is the upper $(\tau/2)^{\text{th}}$ percentile of the standard normal distribution.

Bayesian estimation

The parameters are derived by Bayesian method in this section. Firstly, the PDF of the Gamma distribution with shape and scale parameters $a > 0$ and b>0, respectively, is given by:

$$g(t;a,b) = \frac{b^a}{\Gamma(a)} t^{a-1} \mathrm{e}^{-b\,t} \tag{13}$$

In this paper, we suppose that μ, σ, and θ are independent and μ has gamma prior with PDF $g(\mu;a_1,b_1)$, σ has gamma prior with PDF $g(\sigma;a_2,b_2)$, and θ has uniform prior with PDF $u(\theta;\theta_1,\theta_2)=1$. Then, the joint posterior density of (μ,σ,θ) given x is derived by:

$$\pi(\mu,\sigma,\theta \mid x) = \frac{L(\mu,\sigma,\theta \mid x) g(\mu;a_1,b_1) g(\sigma;a_2,b_2) u(\theta;\theta_1,\theta_2)}{\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} L(\mu,\sigma,\theta \mid x) g(\mu;a_1,b_1) g(\sigma;a_2,b_2) u(\theta;\theta_1,\theta_2) \mathrm{d}\mu \mathrm{d}\sigma \mathrm{d}\theta} \tag{14}$$

Equation (14) cannot reduce to a closed form. We use MCMC method to estimate the parameters. The $\pi(\mu,\sigma,\theta|x)$ is proportional to:

$$\pi(\mu,\sigma,\theta \mid x) \propto L(\mu,\sigma,\theta \mid x) g(\mu;a_1,b_1) g(\sigma;a_2,b_2) u(\theta;\theta_1,\theta_2) \tag{15}$$

Substituting eqs. (8), (13), and $u(\theta,\theta_1,\theta_2)$ into eq. (15), the conditional PDF $\pi(\mu \mid \sigma,\theta,x)$ of μ given σ, θ, and x is proportional to:

$$\pi(\mu \mid \sigma,\theta,x) \propto$$

$$\prod_{i=1}^{k}\left(\Phi\frac{\ln x_{i:r_i+1} - \mu\theta^{\phi_i}}{\sigma}\right)^{r_i} \prod_{j=r_i+1}^{m_i} \exp\left[-\frac{(\ln x_{i:j} - \mu\theta^{\phi_i})^2}{2\sigma^2}\right]\left(1-\Phi\frac{\ln x_{i:r_i+1} - \mu\theta^{\phi_i}}{\sigma}\right)^{R_{i:j}} \cdot$$

$$\cdot \mu^{a_1-1}\exp(-b_1\mu) \tag{16}$$

and it is log-concave since $\partial^2\pi(\mu \mid \sigma,\theta,x)/\partial\mu^2 \le 0$.

The conditional PDF of σ given μ, θ and x is proportional to:

$$\pi(\sigma \mid \mu,\theta,x) \propto$$

$$\prod_{i=1}^{k}\left(\Phi\frac{\ln x_{i:r_i+1} - \mu\theta^{\phi_i}}{\sigma}\right)^{r_i} \prod_{j=r_i+1}^{m_i} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln x_{i:j} - \mu\theta^{\phi_i})^2}{2\sigma^2}\right]\left(1-\Phi\frac{\ln x_{i:r_i+1} - \mu\theta^{\phi_i}}{\sigma}\right)^{R_{i:j}} \cdot$$

$$\cdot \sigma^{a_2-1}\exp(-b_2\sigma) \tag{17}$$

and it is log-concave since $\partial^2\pi(\sigma \mid \mu,\theta,x)/\partial\sigma^2 \le 0$.

The conditional PDF of θ given μ, σ, and x is proportional to:

$$\pi(\theta \mid \mu,\sigma,x) \propto$$

$$\prod_{i=1}^{k}\left(\Phi\frac{\ln x_{i:r_i+1}-\mu\theta^{\phi_i}}{\sigma}\right)^{r_i}\prod_{j=r_i+1}^{m_i}\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(\ln x_{i:j}-\mu\theta^{\phi_i})^2}{2\sigma^2}\right]$$

$$\left(1-\Phi\frac{\ln x_{i:r_i+1}-\mu\theta^{\phi_i}}{\sigma}\right)^{R_{i:j}} \tag{18}$$

Based on eqs. (16)-(18), a hybrid MCMC algorithm that combines Metropolis steps within the Gibbs sampler is implemented to estimate parameters as follows.

Step 1. Give initial values $\mu^{(0)},\sigma^{(0)},\theta^{(0)}$, and $i=1$.

Step2. Sample $\mu^{(i)}$ is generated from the log-concave conditional PDF $\pi(\mu \mid \sigma^{(i-1)},\theta^{(i-1)},x)$ using adaptive rejection sampling (ARS) algorithm.

Step 3. Sample $\sigma^{(i)}$ from the log-concave conditional PDF $\pi(\sigma \mid \mu^{(i)},\theta^{(i-1)},x)$ using ARS algorithm.

Step 4. Sample $\theta^{(i)}$ is generated from the conditional PDF $\pi(\theta \mid \mu^{(i)},\sigma^{(i)},x)$ using Metropolis algorithm.

Step 5. Set $i = i + 1$.

Step 6. Repeat Steps 2-6 M times and the results denoted as:

$$P_B^{(1)}=(\mu^{(1)},\sigma^{(1)},\theta^{(1)}),\cdots,P_B^{(M)}=(\mu^{(M)},\sigma^{(M)},\theta^{(M)})$$

Step 7. Eliminate the first K samples, the remaining $N = M - K$ samples are denoted by $P_B^{(1)}=[\mu^{(1)},\sigma^{(1)},\theta^{(1)}],\cdots,P_B^{(N)}=[\mu^{(N)},\sigma^{(N)},\theta^{(N)}]$ for convenience.

The Bayesian estimates and the variances of μ, σ, and θ are obtained by:

$$\hat{P}_B=\frac{1}{N}\sum_{i=1}^{N}P_B{}^{(i)},\quad \hat{P}_B=(\hat{\mu},\hat{\sigma},\hat{\theta})$$

$$Var(\hat{P}_B)=\frac{1}{N}\sum_{i=1}^{N}(P_B^{(i)}-\hat{P}_B)^2$$

Then the approximate $100(1-\tau)\%$ credible intervals of the parameters are obtained by:

$$\hat{P}_B-z_{\tau/2}\sqrt{Var(\hat{P}_B)},\quad \hat{P}_B+z_{\tau/2}\sqrt{Var(\hat{P}_B)}$$

Thermal aging life model

Consider a real accelerated life test data reported in [19]. The data refer to failure times of thermal aging life for enamelled wires with composite insulation layers under three different temperatures S_1 = 210 °C, S_2 = 230 °C, S_3 = 250 °C. At each test temperature, a number of failure times were observed with number n_1 = 21, n_2 = 21, n_3 = 21. In this study, we first use Kolmogorov-Smirnov (K-S) test technique to verify that the data under different temperature obey log-normal distribution, where the K-S values are shown in tab. 1. The main purpose of the experiment is to estimate the parameters at normal temperature when data are general progressively censoring.

We set $S_0 = 180$; $r_1 = 1$, $r_2 = 0$, $r_3 = 0$; $m_1 = 18$, $m_2 = 19$, $m_3 = 19$; $R_1 = (0,1,0,0,1,0,\cdots,0)$, $R_2 = (0,0,2,0,\cdots,0)$ $R_3 = (0,1,0,0,0,0,1,\cdots,0)$, to get a general progressive type-II censored data, it is shown in tab. 2. We use the non-informative gamma priors for μ and σ, that is, the hyper-parameters are $a_i = 0, b_i = 0, i = 1,2.$ Using the algorithm proposed in section *Bayesian estimation*, MCMC samples of μ,σ, and θ with size 4000 are generated under the general progressive type-II censored data in tab. 2, and the first 500 samples are eliminated. The sample path maps and histograms of the remaining MCMC samples are listed in fig. 1. From fig. 1, we can see the MCMC is convergent. Based on the MCMC samples, the Bayesian estimates (BE) of μ,σ, and θ, the mean squared errors (MSE), and 95% credible intervals (CI) are calculated and listed in tab. 3. For comparison, MLE of μ,σ, and θ, the MSE, and 95% CI are also listed in tab. 3.

From tab. 3, we can see that the BE of μ,σ, and θ are similar to corresponding MLE. However, MLE are complex, and it is often influenced by the initial value. And it is difficult to prove the uniqueness of the MLE. But the Bayesian method does not need to prove the uniqueness of the solution, and the convergence is not affected by the initial value.

Table 1. **Failure data of thermal aging life for enameled wires with composite insulation layers**

Temperature	Failure times [min]	K-S
210 °C	(3853, 3853, 4523, 5193, 5193, 5193, 5863, 5863, 5863, 5863, 5863, 5863, 5863, 5863, 5863, 6871, 6871, 7015, 7375, 8023, 8023)	0.2382
230 °C	(1086, 1420, 1587, 1754, 1921, 2255, 2255, 2422, 2422, 2422, 2589, 2589, 2589, 2589, 3093, 3237, 3237, 3237, 4101, 4101, 4101	0.7227
250 °C	(245.5, 268.5, 268.5, 268.5, 268.5, 291.5, 291.5, 314.5, 383.5, 383.5, 406.5, 406.5, 406.5, 429.5, 452.5, 475.5, 475.5, 475.5, 475.5, 521.5, 567.5)	0.6118

Table 2. **General progressive type-II censored data**

Temperature	Failure times [min]
210 °C	(–, 3853, 4523, 5193, 5193, 5193, 5863, 5863, 5863, 5863, 5863, 5863, 5863, 6871, 6871, 7015, 7375, 8023, 8023)
230 °C	(1086, 1420, 1587, 1754, 1921, 2255, 2255, 2422, 2422, 2589, 2589, 2589, 2589, 3093, 3237, 3237, 3237, 4101, 4101)
250 °C	(245.5, 268.5, 268.5, 268.5, 268.5, 291.5, 291.5, 314.5, 383.5, 406.5, 406.5, 429.5, 452.5, 475.5, 475.5, 475.5, 475.5, 521.5, 567.5)

Table 3. The MLE, BE, MSE and 95% confidence (credible) intervals of parameters

Parameters	MLE	MSE	95% CI	BE	MSE	95% CI
μ	12.698	0.3652	(11.983, 13.414)	12.678	0.3843	(11.925, 13.431)
σ	0.4452	0.0422	(0.3625, 0.5278)	0.4756	0.0424	(0.3925, 1.1104)
θ	0.6969	0.0138	(0.6698, 0.7241)	0.6981	0.0146	(0.6979, 0.7241)

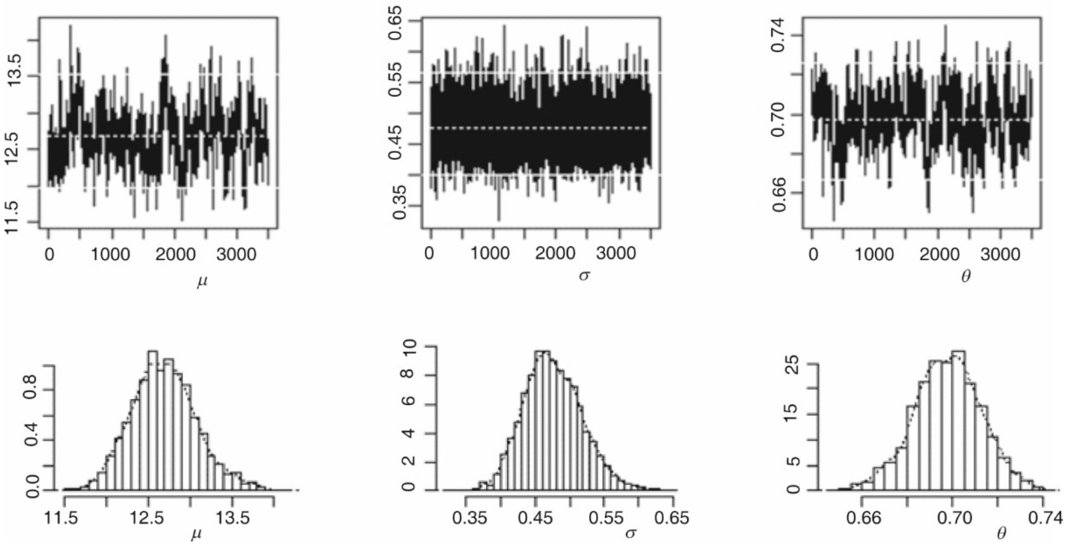

Figure 1. The sample path maps and histograms of MCMC samples of the parameters based on the general progressive type-II censored data

Conclusion

The log-normal distribution parameters of thermal aging life data was discussed for the constant-stress accelerated life test under the general progressive type-II censoring scheme. The point and interval estimates of the unknown parameters were derived by using Bayes method. We use the MCMC method to compute the BE because they cannot be obtained in the explicit form when data are general progressive type-II censoring. The real thermal aging life data under accelerated life test was presented to illustrate the application proposed method in practice.

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