A FRACTAL DERIVATIVE MODEL FOR SNOW'S THERMAL INSULATION PROPERTY

by

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Snow is of porous structure and good thermal insulation property. A fractal derivative model is established to reveal its thermal property, it is extremely high thermal-stable, the whole snow will not be affected much by the sudden environmental temperature change. A simple experiment is carried out to verify the theoretical finding, and the result is helpful to design advanced materials mimicking the snow structure.

Keywords: *fractal calculus, thermal insulation, porous materials*

Introduction

As everyone knows that snow is of porous structure, and it has an excellent insulation property. Porous materials have been widely used in engineering to absorb energy and to protect thermal lose. For example, the cocoon or other hierarchical structures have excellent thermal property [1-3]. Snow has at least a three cascade hierarchy making it as an excellent natural material for thermal insulation with high air permeability, and it becomes the most economic way to study the thermal properties of the porous materials. Nanotechnology has the ability to fabricate nanoscale porous hierarchy now [4-6], and the hierarchy can begin with molecular level by the macromolecular electrospinning [7-9]. In this paper, we will establish a fractal derivative model to show the mechanism of the snow's insulation property.

Snow's fractal dimensions

This paper suggests a simple way to calculation of the fractal dimensions of a porous medium. Assuming that the snow weighs w_{snow} with density ρ_{snow} volume V_0, we can calculate its porosity volume:

$$V = V_0 - \frac{w_{\text{snow}}}{\rho_{\text{snow}}} \tag{1}$$

We choose two scales r_0 and r_1 defined, respectively:

$$r_0 = (V_0)^{1/3} \tag{2}$$

and

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$$r_1 = \left(V_0 - \frac{w_{\text{snow}}}{\rho_{\text{snow}}} \right)^{1/3} \tag{3}$$

The self-similarity number at the scale of r_1 is:

$$N = \frac{\dfrac{w_{\text{snow}}}{\rho_{\text{snow}}}}{V_0 - \dfrac{w_{\text{snow}}}{\rho_{\text{snow}}}} \tag{4}$$

The fractal dimensions can be calculated:

$$\alpha = \frac{\ln N}{\ln \dfrac{r_1}{r_0}} \tag{5}$$

Fractal derivative model

In our study, a snow sphere was randomly taken outside, and a hot metal rod with a temperature of about 100 °C was suddenly inserted into the sphere center as illustrated in fig. 1 to see the melting property of the snow under a sudden thermal impact.

Figure 1. Snow sample. A rod with high temperature was inserted into the center of the snow

In this paper we adopt Ji-Huan He's fractal derivative, which is defined as [2,10]:

$$\frac{DT}{Dx^\alpha} = \Gamma(1+\alpha) \lim_{\Delta x \to L_0} \frac{T(A) - T(B)}{(x_A - x_B)^\alpha} \tag{6}$$

where T is the temperature, A and B are two adjacent points with distance of L_0, which can be considered as the average diameter of porous size of snow.

The fractal calculus was proposed in [9] and developed in [10], the fractal derivative is local and can be used to deal with phenomena arising in porous media.

The 1-D heat conduction in the fractal snow can be written:

$$\frac{D}{Dx^\alpha} \left(k \frac{DT}{Dx^\alpha} \right) = 0 \tag{7}$$

Using He-Li fractional transform [10,11]:

$$s = x^\alpha \tag{8}$$

The geometrical explanation of eq. (8) is given in [12-16]. Equation (7) becomes:

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(k\frac{\mathrm{d}T}{\mathrm{d}s}\right)=0 \tag{9}$$

The solution of eq. (9) is:

$$kT = as + b \tag{10}$$

or

$$T=\frac{a}{k}x^{\alpha}+\frac{b}{k} \tag{11}$$

Considering the hot rod as a point heat source, we have the following boundary conditions:

$$T(0) = T_0 \tag{12}$$

$$T(R) = T_R \tag{13}$$

We finally obtain the following solution:

$$T=\frac{T_R-T_0}{R^{\alpha}}x^{\alpha}+T_0 \tag{14}$$

We consider the temperature change at $x = 0$:

$$\frac{\mathrm{d}T}{\mathrm{d}x}(x=0)=\lim_{x\to 0}\frac{\alpha(T_R-T_0)}{R^{\alpha}}x^{\alpha-1}=\begin{cases}0, & \alpha>1\\ \frac{\alpha(T_R-T_0)}{R^{\alpha}}, & \alpha=1\\ \infty, & \alpha<1\end{cases} \tag{15}$$

As the snow is porous structure, so the fractal dimensions are less than 3, and its fractal dimensions along radical direction is less than 1, that is $\alpha<1$, $\mathrm{d}T/\mathrm{d}x(x=0)\to\infty$, the temperature change is extremely high. Consider a special case when T_0 = 100 °C, $T_R=-10$ °C, R = 1 m, we have:

$$\frac{\mathrm{d}T}{\mathrm{d}x}=-110\alpha x^{\alpha-1} \tag{16}$$

It can be seen from fig. 2, the temperature change rockets at x = 0, that means the temperature decreases dramatically from a high temperature at x = 0 to its original temperature near the heat source point.

We can also assume the radius of the hot rod is r, and the boundary conditions are:

$$T(r) = T_0 \tag{17}$$

$$T(R) = T_R \tag{18}$$

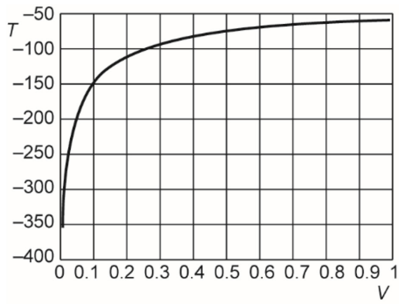

Figure 2. Temperature change from the center to the outmost of the snow

By a similar way as above, we can obtain the following solution:

$$T=\frac{T_R-T_0}{(R-r)^{\alpha}}(x-r)^{\alpha}+T_0 \tag{19}$$

We have the same the same property at $x = r$ as that for the previous.

$$\frac{\mathrm{d}T}{\mathrm{d}x}(x=r)=\lim_{x\to 0}\frac{\alpha(T_R-T_0)}{(R-r)^\alpha}(x-r)^{\alpha-1}=\begin{cases}0, & \alpha>1\\ \dfrac{\alpha(T_R-T_0)}{(R-r)^\alpha}, & \alpha=1\\ \infty, & \alpha<1\end{cases} \tag{20}$$

Discussion and conclusion

The value of the fractal dimensions given eq. (5) can be replaced by the two-scale dimension [17], and eq.(8) is also called the two-scale transform [17]. Equation (9) can be also solved effectively by Taylor method as illustrated in [18].

For the first time ever, we reveal the thermal property of porous medium using a fractal derivative model, any energy from the outside of the porous medium is extremely difficult to be converted into any part of the porous medium except the boundary in a short time.

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