FRACTAL STUDY IN SOIL SPATIAL VARIABILITY AND THERMAL CONDUCTIVITY

by

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This paper takes advantage of fractal method to research some soil characteristics through case analysis. The multifractal spectrum and random Sierpinski carpet are used to describe the spatial variability and thermal conductivity of soil quantitatively. On the basis of predecessors, the scatter plots of various types of data have been used to supplement the multifractal results in a more detailed way. It turns out that the content of clay, silt, and coarse sand could reflect the degree of spatial variability of soil. Then based on this case, the effect of porosity on soil thermal conductivity is discussed by using random Sierpinski carpet. The result shows that the effective thermal conductivity of the clay, silt and coarse sand decreases linearly with the increase of porosity, but the degree of reduction is different. Moreover, when the porosity is definite, the effective thermal conductivity of the coarse sand is the largest, that of the clay with the highest thermal conductivity is second, and that of the silt is the smallest.

Key words: multi-fractal, soil spatial variability, random Sierpinski carpet, porosity, soil thermal conductivity

Introduction

Fractals are not only geometric figures, but also some self similar performance in form, function and information. In order to better understand fractal and distinguish it from fractal geometry, the fractal formed by such properties is called fractal behaviour [1].

From the 1980s, soil scholars have realized that fractal theory could be introduced to the study of soil spatial variability. The fractal theory can be used in the study of soil properties for three reasons. First, the object of study in fractal theory applies soil particle well, because both are irregular geometrical form and using the fractal theory to explain the impact of the spatial variation of soil can receive significant effect. Second, some studies have shown that the particle volume, porosity, particle distribution and the particle surface area in soil have self-similar characteristics [2, 3]. Third, the fractal dimension of fractal parameter can be used to reflect the degree of soil spatial variability, as the higher fractal dimension of soil, suggests that the rougher spatial distribution of soil and the more uniform soil texture [4]. Therefore, we can make use of fractal theory for studying the spatial variation of soil.

In 2017, Xiao *et al*. [5] by using the volume fractal dimension of soil particles compared the relationship between the soil particle content of different forest age in Xishuangbanna

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city, China and got the soil particle volume distribution characteristics of different forest age. Dai and Li [6], discussed the relationship between the soil particle of different bush and the content of soil organic carbon in Wulan Bush desert through the multi-fractal theory. In fact, there have been a lot of research on soil spatial variability and other domain using fractal theory in our country in recent years, and one of them will be the key case to analyse.

Material and methods

When the main factors affecting the distribution of soil spatial structure have the same influence basically, this effect will be repeated in time and space to produce an approximate single fractal soil attribute space, but actually the role of the factors affecting the soil distribution is often not the same. The interaction between them is more complex and more varied. In addition the influence of numerous random factors, a single fractal is not sufficient to quantify the complex system which is generated by iterating in different time and space continuously. Also there may be multiple or even large number of single fractal objects changing and winding in space. At this time, it is necessary to optimize the single fractal of soil spatial variability into the multi-fractal of soil spatial variability.

Definition of multi-fractal analysis

Multi-fractal has also been translated into multi-scale fractal and complex fractal*, etc.* The problem involved is a parametric probability distribution. It is the form of the multi-fractal spectrum to replace with a fractal dimension describe overall characteristic parameters of the object, or using a spectral function describes the growth characteristics of fractals at different structural levels, linking local characteristic parameters to study the overall characteristics.

The model of multi-fractal

Multi-fractal describes the different levels of features in the growth process of fractal geometry objects. First, using the scale of ξ, dividing the object of study into N is the small area. Then, the size of the module in the i – the small region is set as L_i, and the growth probability of the growth interface in the small region is set as P_i. As we can know, the growth probability of different small regions is different:

$$P_i(\xi) \propto \xi^{\alpha} \tag{1}$$

where α is singular exponent, which is used to reflect the degree of singularity within a small subset. If the element with a singular exponent of α is $N_\alpha(\xi)$, there is a power relation between- $N_\alpha(\xi)$ and α:

$$N_\alpha(\xi) \propto \xi^{-f(\alpha)} \tag{2}$$

where $f(\alpha)$ represents the fractal dimension of the fractal subset with a singularity exponent α. The infinite sequence of $f(\alpha)$ is called the multi-fractal spectrum and it can be used to measure the roughness, complexity, irregularity, and degree of unevenness of the research object.

In order to find out what α and $f(\alpha)$ are, the common method is the statistical moment method, which defines a partition function:

$$\chi q(\alpha) = \sum_{i=1}^{N_\alpha} P_i^q(\xi) \tag{3}$$

where q is any number, denoting the statistical order of probability. If $\chi_{q(\alpha)} = \xi^{\tau(q)}$ is true, the double logarithmic transformation of the partition function is carried out:

$$\ln \chi_{q(\alpha)} = \tau(q) \ln \xi \tag{4}$$

When ξ approaches 0 infinitely, there is a quality index $\tau(q)$:

$$\tau(q) = \lim_{\xi \to 0} \frac{\ln \chi_{q(\alpha)}}{\ln \xi} \tag{5}$$

According to the smooth function $\tau(q)$, α can be obtained:

$$\alpha(q) = \frac{\mathrm{d}}{\mathrm{d}q} \tau(q) \tag{6}$$

Then after the Legendre transformation of $\tau(q)$:

$$f(\alpha) = \alpha(q) q - \tau(q) \tag{7}$$

At the same time, we can get the generalized fractal dimension spectrum D_q [6, 7]:

$$D_q = \frac{\tau(q)}{q-1} (q \neq 1) \tag{8}$$

$$D_1 = \tau(1) \ln P_i(\xi)(q = 1) \tag{9}$$

It is easy to see that when $D_q = 1$, the equation can be replaced with $\tau(q) = q - 1$ being used to measure the multifractal degree. When the calculated fitting line between $\tau(q)$ and $q - 1$ is deviated infinitely, it indicates that the multifractal feature is strong and the fitted line is extremely stable. When q is assigned the value of 0, 1, 2. At that time, the corresponding dimensions are capacity dimension, information dimension and correlation dimension.

Fractal theory and soil thermal conductivity

Soil heat conduction is also an important part of soil research. It has great significance for ecological problems and energy consumption study soil thermal conductivity. The soil is a porous medium. Based on this property, it is uncontrollable. As a result, the thermal conductivity is usually measured by experiments. The introduction of fractal theory can effectively reduce the experimental cost and experimental error caused by human factors. According to the local fractal theory, the effective thermal parameters of porous medium with fractal structure, which has a local fractal scale l [8]:

$$E = f\left(\sum E_i, \varphi, D, l\right) \tag{10}$$

where E_i is thermal property of medium with every state and φ – the average void fraction by volume in porous medium. In order to simplify the calculation, Chen and Shi [8] deduced the relationship between soil effective thermal conductivity λ_e and gas-liquid mixture state λ_g, as well as the state of solid soil λ_s on this basis:

$$\lambda_e = \frac{\lambda_g \lambda_s (1 - a_1^{1/3} + a_1) + \lambda_g (a_1^{1/3} - a_1)}{\lambda_s (1 - a_1^{1/3}) + \lambda a_1^{1/3}} \tag{11}$$

where $a_1$is the concentration of solid.

In order to use fractal to simulate soil better, this paper uses the random Sierpinski carpet to construct this porous medium. As shown in the fig. 1, the black area represents the solid matrix in the soil, and the white area represents the pore structure in the soil. Unlike the Sierpinski carpet, each square is no longer a relationship of one third of the length of a fixed position, but a random relationship of one fifth, even one sixth, depending on the pore structure required for simulation.

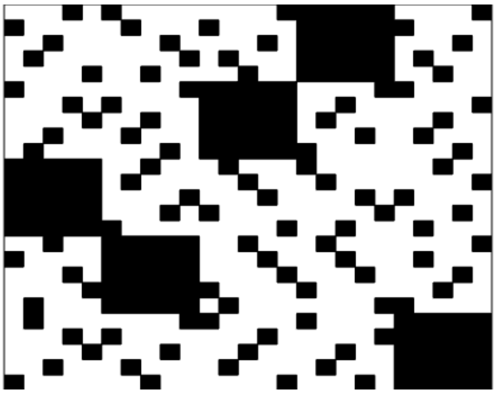

Figure 1. Simple soil structure by random Sierpinski carpet

So as to see this process more clearly, the fig. 1 is drawn by MATLAB as a random Sierpinski carpet with the length of one fifth and the two iterations. Also the study of soil thermal conductivity in subsequent cases is based on this.

Results

In this paper, we take the multifractal analysis of soil particle size and soil organic carbon content as the example using multifractal methods to study soil spatial variability and this case was conducted by Dai and Li [6] in different bushes in the Wulan Bush desert in the 2017.

The unique climate and geographical features of Ulan Bush desert determines the unstable soil structure locally. At the same time, frequent winds could easily lead to the soil loss of small particle size and accumulation of coarse-grained soil particles due to gravity. Because the bushes can block a part of the sand flow effectively, the soil particles can be deposited around the bushes without being blown away by the sand, which in turn affects the soil particle size and organic carbon content under the bushes. It is well known that the content of carbon and nitrogen determines some of the nutrients in the soil, so the higher the organic carbon content is in the soil spatial structure, the more favorable the vegetation becomes in the desert land. The relationship between the soil particle size and the soil organic carbon content can reveal the quality of the soil in the desert. In addition, the impact reflects the status of the local desert carbon pool, so as to help people master the mechanism of carbon sequestration.

They use five kinds of bushes suitable for the growth in Wulan Bush desert. They are Nitraria tangutorum, Ammopiptanthus, Hedysarum scoparium, Haloxylon and Oxytropis aciphylla. Also the soil particle of unvegetated flowing sand dune was studied in the control group and the predecessors compared the depth of soil particles from 0-100 m under various bushes. For ease of operation, we makes the labels of the six soil particle size *N*, *A*, *HS*, *H*, *O*, *U*. The original text groups soil depths by class interval. The first four groups have a class interval of 10 cm and the last three groups have a class interval of 20 cm. In this paper, we take the median of the corresponding group to analyze the fractal dimensions about the soil particle size distribution and organic carbon content under each bush. So in terms of previous data, the relevant data of each fractal parameter with depth change is shown in tab. 1 [6].

Seeing tab. 1 horizontally, we could find that the increase of capacity dimension and information dimension are not very big under the every brush in soil though the soil depth is increased, showing that the multifractal degree of the soil particle size distribution in the vertical space is not high. Therefore, to prove this point, we use the capacity dimension and the information dimension as the ordinate and the bushes depth as the abscissa drawing some scatter plots, as demonstrated in figs. 2 and 3.

In figs. 2 and 3 U_0 is the capacity dimensions of flowing sand dune, A_0 – the capacity dimensions of Ammopiptanthus, HS_0 – represents the capacity dimensions of Hedysarum scoparium, H_0 – the capacity dimensions of Haloxylon, O_0 – the capacity dimensions of Oxytropis aciphyll, N_0 – the capacity dimensions of Nitraria tangutorum. By that analogy, U_1, A_1, HS_1, H_1, O_1, and N_1 denote the information dimensions of the aforementioned bushes, respectively.

Table 1. Fractal dimension values of each bushes changing with soil depth

Sample plot	Fractal parameters	Soil depth [cm]							
		5	15	25	35	50	70	90	Mean
U	D_0	0.499	0.474	0.492	0.593	0.484	0.573	0.503	0.517
	D_1	1.399	1.355	1.342	1.360	1.350	1.365	1.355	1.352
A	D_0	1.846	1.183	1.865	1.040	2.065	1.859	1.851	1.673
	D_1	1.598	1.450	1.545	1.458	1.492	1.550	1.563	1.522
HS	D_0	1.757	1.796	1.809	1.857	1.914	1.090	1.149	1.624
	D_1	1.641	1.649	1.597	1.566	1.546	1.427	1.487	1.559
H	D_0	0.575	0.549	0.506	0.504	0.500	0.543	0.500	0.525
	D_1	1.386	1.380	1.364	1.365	1.368	1.370	1.369	1.372
O	D_0	1.883	1.860	1.811	1.079	1.886	1.050	1.057	1.518
	D_1	1.550	1.563	1.623	1.453	1.574	1.436	1.464	1.535
N	D_0	1.903	1.025	1.040	1.074	1.062	1.067	1.927	1.300
	D_1	1.568	1.456	1.448	1.427	1.444	1.446	1.527	1.474

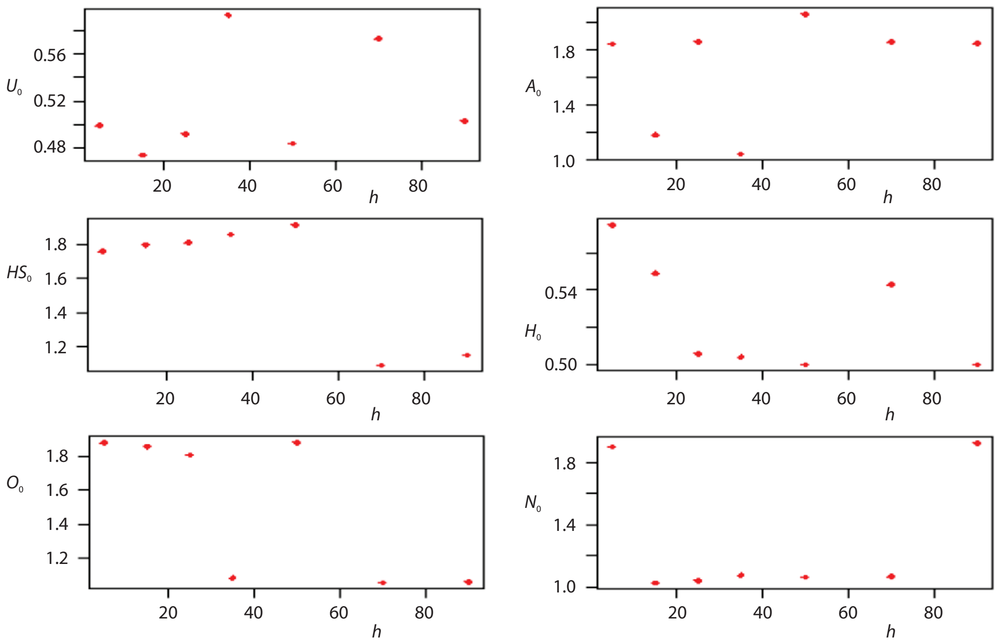

Figure 2. Scatter plots of capacity dimensions distribution under each bush

Discussion

The discussion on the case of multi-fractal

From fig. 2, it can be seen that the capacity dimensions of various bushes do not change in one direction in their respective scaleless regions, and each dimension is the fractal dimension of the soil particle within the box corresponding depth of the partition. So the variation of fractal dimension indicates the change of soil spatial structure. Contrasting the six

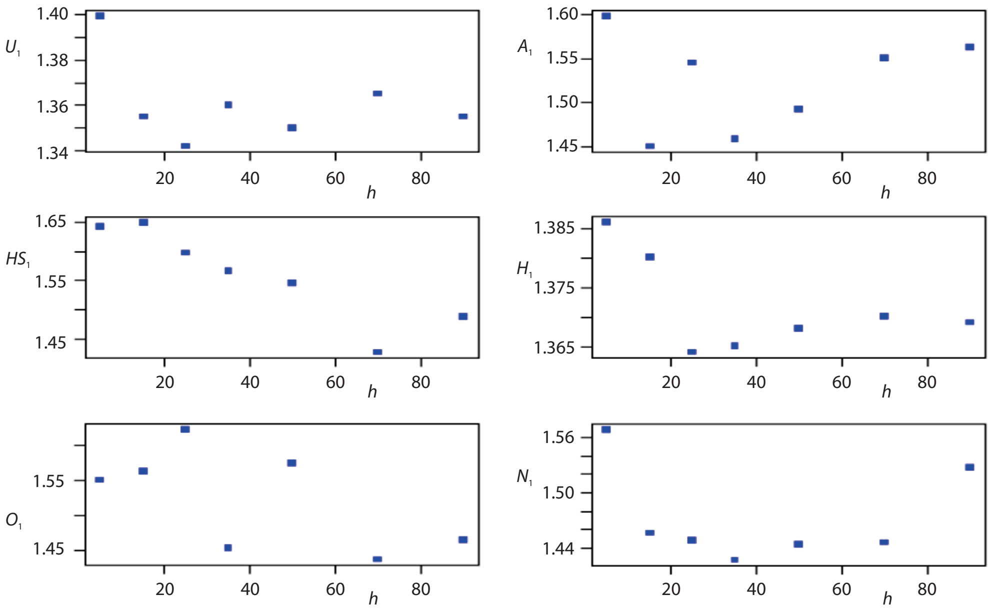

Figure 3. Scatter plots of information dimensions under each bush

pictures in fig. 2, each capacity dimension of soil particle is almost unchanged within a certain range of depth, that is to say, in their corresponding depth range the space structure of every bush approximates single fractal. For each soil particle, the capacity dimension of the flowing sand dune is the smallest at each depth and has the smallest change, indicating that the distribution range of the soil particle size in the flowing sand dune is the smallest and the degree of homogeneity is the highest, which is consistent with the objective situation. In addition, the mean value of capacity dimension about Ammopiptanthus is the largest. Because in the depth of the first half meter, the multifractal condition is obvious and in the second half of the depth, the soil particle size distribution is large, also indicating that the soil spatial variability of Ammopiptanthus in the deep soil is the largest.

Figure 3 shows that the distribution of information dimension is different from the capacity dimension, and the upper limit of the flowing sand dune is smaller than the lower limit value of all other soil particles about the information dimension, demonstrating that the concentration of soil particle in flowing sand dune is poorest, and the uniformity is the best, which is identified with the performance of the capacity dimension. Besides, the soil information dimension of each bush exhibits a certain tendency except for Oxytropis aciphylla, that is to say, with the increase of depth, the concentration trend of soil particle size distribution shows a certain degree of monotony, and the spatial variation of each soil in the bushes increases in different degrees. When the measurement data is enough, if the multi-fractal dimension can fit a regression curve, then for soil spatial variability we can take targeted steps to make soil resources better for people to utilize.

Next, they analyze the relationship between soil particle size distribution, soil organic carbon content, and fractal parameters. On the basis of research, the levels of soil particle size from small to large are clay *C*, silt *S*, fine sand *FS*, coarse sand *CS,* and gravels *G*. The results are shown in tab. 2 [6].

Table 2. Correlation among soil particle size, organic carbon content and fractal parameters

Project	Organic carbon content, w	$\varphi(C)$	$\varphi(S)$	$\varphi(FC)$	$\varphi(CS)$	$\varphi(G)$	D_1	D_2
Organic carbon content, w	1	0.545**	0.555**	0.260	–0.528**	–0.093	0.653**	0.652**
$\varphi(C)$		1	0.824**	0.273	–0.632**	–0.230	0.907**	0.804**
$\varphi(C)$			1	0.048	–0.501**	–0.033	0.926**	0.798**
$\varphi(C)$				1	–0.826**	-0.793**	0.146	0.084
$\varphi(C)$					1	0.462**	–0.592**	–0.496**
$\varphi(C)$						1	–0.057	0.011
D_1							1	0.939**
D_0								1

Note: ** indicates a significant difference at the significance level of 0. 01

From the data in tab. 2, it could be seen that the soil organic carbon content is significantly positively correlated with the content of clay and silt which belong to the small-sized soil particle, and has a significant negative correlation with the coarse sand particle content. In addition, there is also a significant positive correlation between the soil organic carbon content and the information dimension, as well as capacity dimension. By referring to the correlation coefficient of clay, silt and coarse sand particles with organic carbon, it can be considered that the three indexes of clay, silt, and coarse sand can reflect the change of soil organic carbon a certain extent. The index can reflect the change of soil organic carbon a certain extent. Also the information dimension and capacity dimension show a high positive correlation with the contents of clay and silt, and indicate moderate negative correlation with the content of coarse. At the same time,it still turns out that these three indicators can be used to reflect the spatial variability of other soil characteristics.

The discussion on soil thermal conductivity

Because the three indexes of clay, silt and coarse sand can be used to reflect the spatial variability in soil, these three kinds of particles are utilized to study the thermal effect. In this paper, an ideal experiment is carried out. That is, hypothesis is made that all the solid components in the soil are one of the three, then under the same conditions, the thermal conductivity of the three components is used to study the effect of porosity on the effective thermal conductivity of the soil.

While the water content is 15%, referring to the relevant literature, the thermal conductivity of clay, grain and coarse sand is as shown in the tab. 3.

Table 3. Three particle thermal conductivity

Soil type	Thermal conductivity
Clay	1.7
Silt	1.3
Coarse	3.3

The porosity is set to (0.2710, 0.2977, 0.3301, 0.3703, 0.42123, 0.4880, and 0.5781). The result is shown in fig. 4.

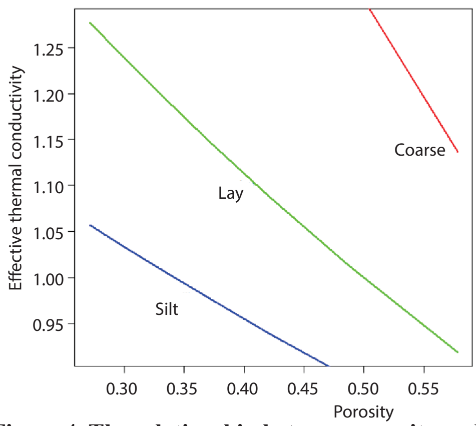

Figure 4. The relationship between porosity and effective thermal conductivity

It can be seen from the fig. 4 that when the effective thermal conductivity of the three kinds of particles has a linear relationship with the porosity. All they decrease with the increase of the porosity, and the coarse with the largest particle size decreases fastest, and the clay with the smallest particle has a moderate rate of decline. When the porosity is definite, the effective thermal conductivity of the coarse sand is the largest, the effective thermal conductivity of the clay with the highest thermal conductivity is second, and the effective thermal conductivity of the silt is the smallest.

Conclusion

Multi-fractal uses the form of multi-fractal spectrum to make up for the deficiency of single fractal dimension [9] in spatial transformation. By planting different shrubs, the non-uniformity of soil particle size can be increased from different degrees. Moreover, the content of clay, grain, and coarse can be used to reflect the spatial variability of soil to some extent.

It is of great significance to study the thermal conductivity of soil by using random Sierpinski carpet. While coarse, clay, and silt are used to the whole solid matrix, the effective thermal conductivity of the three kinds of particles has a linear relationship with the porosity. And they decrease with the increase of the porosity simultaneously.

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