SOME KANNAN TYPE FIXED POINT RESULTS IN RECTANGULAR SOFT METRIC SPACE AND AN APPLICATION OF FIXED POINT FOR THERMAL SCIENCE PROBLEM

by

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The intention of current study to survey Kannan type mappings for rectangular soft metric space. Some Kannan type results are obtained by using rectangular soft metric and an application for thermal science problem is presented.

Key words: *soft metric, Kannan type mapping, fixed point*

Introduction

In the year 1999 Molodtsov [1] propounded the theory of soft sets as a contemporary mathematical subject for regarding with ambiguities. Recent works in theory of soft set and its applications in different scope such as economics, engineering, social and medical science have been developing quickly since Maji *et al.* [2] and many mathematicians [3-6].

Otherwise fixed point theory is an significant topic of mathematics which has a good deal of applications in different areas such as differential equations, thermal science, heat equations, *etc.* Bildik *et al.* [7] used the fixed point property for the approximatively solution of several sort of differential equations. Gulyaz and Inci [8] investigated the existence of integral equations via fixed point theorems and gave an application deal with the following heat equation of non-linear type in dimension one:

$$\begin{cases} u_t(x,t) = u_{xx}(x,t) + F(x,t,u,u_x), & -\infty < x < \infty, \quad 0 < t < T \\ u(x,0) = \varphi(x), & -\infty < x < \infty \end{cases}$$

Also thermal science problem which can be written as the following form:

$$\begin{cases} Lw(x,y) = \phi(x,x'), \ (x,x') \in \Omega \\ Bw(x,y) = \varphi(x,x'), \ (x,x') \in \partial\Omega \end{cases}$$

was perused by Qui [9] where L is a linear differential operator, B is a boundary operator, $\phi(x,x')$ and $\varphi(x,x')$ are two given functions, and $\Omega \in R^2$ is an open bounded domain together with boundary $\partial\Omega$. For other details for different types of differential equations [10, 11].

Beside soft metric spaces defined by Das and Samanta [12] in 2013 become an interesting area of fixed point theory. In 2017 Hosseinzadeh [13] improved the theory which mentioned by Das and Samanta [12] and gave a new aspect of the definition of soft metric.

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In current study we defined a renewed metric by using soft sets and rectangular metric which introduced by Branciari [14] and investigated in partially metric spaces by [15]. Then we prove some fixed point theorems by using rectangular soft metric and a special contractive mapping. Also we give the improved of our study [16, 17] by using Kannan type mappings [18, 19]. The new soft metric defined in current study will be a useful material for mathematicians especially working on fixed point theory. The findings obtained in this paper can be amplified to various areas and aspects of soft fixed point theory.

Preliminaries

Definition 1. Presume that $\mathbb{E}$ be a parameter set. A binary $(\mathcal{G},\mathbb{E})$ is said to be a soft set over the universe X, where $\mathcal{F}$ is a transformation given by $\mathcal{G}:\mathbb{E}\to\mathcal{P}(X)$ [1].

That is to say, a soft set over X, is a parameterized family of subsets of the universal set X. For any parameter $x\in\mathbb{E},\mathcal{G}(x)$ may be regarded as the set of $x-$approximative members of the soft set $(\mathcal{G},\mathbb{E})$.

Definition 2. Let $\mathbb{A}\subseteq\mathbb{E}$ be the parameter set. The ordered pairwise (α,t) is named to be a soft parametric scalar if $t\in\mathbb{R}$ and $\alpha\in\mathbb{A}$. The parametric scalar (α,t) said to be non-negative if $t\geq 0$. Assuming that (α,t) and (β,t') be two soft parametric scalars. (α,t) is said to be no less than (β,t') and it is written as $(\alpha,t)\succeq(\beta,t')$, if $t\geq t'$ [13].

Definition 3. Let $\mathbb{A}\subseteq\mathbb{E}$ be a set of parameters. Let (α,t) and (β,t') be two soft parametric scalars. Then the addition between soft parametric scalars and scalar multiplication on soft parametric scalars are described [13]:

$$(\alpha,t)\dot{+}(\beta,t')=(\{\alpha,\beta\},t+t'),\quad\text{and}\quad\lambda(\alpha,t)=(\alpha,\lambda t),\quad\text{for every}\quad\lambda\in\mathbb{R}.$$

Definition 4. Suppose that $(\mathcal{G},\mathbb{E})$ be a soft set over the universal X. The function f on $(\mathcal{G},\mathbb{E})$ is termed as parametric scalar valued, if there are maps $f_1:\mathbb{E}\to\mathbb{E}$ and $f_2:\mathcal{G}(\mathbb{E})\to\mathbb{R}$ such that $f(\mathcal{G},\mathbb{E})=(f_1,f_2)(\mathbb{E},\mathcal{G}(\mathbb{E}))$ [13].

In a similar manner, we can amplify the parametric scalar valued function previously defined as $f:(\mathcal{G},\mathbb{E})\times(\mathcal{G},\mathbb{E})\to(\mathbb{E},\mathbb{R})$ by $f(\mathbb{E}\times\mathbb{E},\mathcal{G}(\mathbb{E})\times\mathcal{G}(\mathbb{E}))=(f_1,f_2)(\mathbb{E}\times\mathbb{E},\mathcal{G}(\mathbb{E})\times\mathcal{G}(\mathbb{E}))$, where $f_1:\mathbb{E}\times\mathbb{E}\to\mathbb{E}$ and $f_2:\mathcal{G}(\mathbb{E})\times\mathcal{G}(\mathbb{E})\to\mathbb{R}$.

Definition 5. Let $(\mathcal{F},\mathbb{E})$ be a soft set over X and let $\breve{\varphi}:\mathbb{E}\times\mathbb{E}\to\mathbb{E}$ be parametric function. The parametric scalar valued function $\breve{\mathcal{D}}:(\mathcal{G},\mathbb{E})\times(\mathcal{G},\mathbb{E})\to(\mathbb{E},\mathbb{R}^+\cup\{0\})$ is called to be a soft meter on $(\mathcal{F},\mathbb{E})$ if $\breve{\mathcal{D}}$ provides following conditions [13]:

- $\breve{\mathcal{D}}((a,\mathcal{G}(a)),(a',\mathcal{G}(a')))\succeq(\breve{\varphi}(a,a'),0)$, and if $a=a'$ then the equality holds.
- $\breve{\mathcal{D}}((a,\mathcal{G}(a)),(a',G\mathcal{F}(a')))=\breve{\mathcal{D}}((a',\mathcal{G}(a')),(a,\mathcal{G}(a)))$ for all $a,a'\in E$.
- $\breve{\mathcal{D}}((a,\mathcal{G}(a)),(a',\mathcal{G}(a')))\preceq\breve{\mathcal{D}}(((a,\mathcal{G}(a)),(a',\mathcal{G}(a')))\dot{+}\breve{\mathcal{D}}((a',\mathcal{G}(a')),(a'',\mathcal{G}(a''))),$ for all $a,a',a''\in\mathbb{E}$

The pairwise $((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}})$ is said to be a soft metric space over X.

Definition 6. Assume A is a not null set and let $d_r:A\times A\to[0,\infty]$ carry out the below conditions for all $x,y\in A$ and all different $z,t\in A$ each of which is dissimilar from x and y [14]:

- (RM1) $d_r(x,y)=0\Leftrightarrow x=y$,
- (RM2) $d_r\left(x,y\right)=d_r\left(y,x\right)$,
- (RM3) $d_r(x,y)\leq d_r(x,z)+d_r(z,t)+d_r(t,y)$.

At that time the map d_r is named a rectangular metric and the pairwise (A, d_r) is named a rectangular metric space (RMS).

Definition 7. $A \neq \varnothing$ and an element $a \in A$ is a fixed point of $f : A \to A$ if $f(a) = a$ [20].

Definition 8. Let M be a mapping of a metric space X into X [20]. We assert that M is a contraction mapping if there subsist a number λ so that $0 < \lambda < 1$ and $\rho(Mx, My) \leq \lambda\rho(x, y)$ for all $x, y \in X$.

Theorem 9. Any contraction transformation of a complete non-empty metric space X into X has an only one fixed point in X [20].

Definition 10. Let (X, d) be a universalized metric space [18]. The self-map $M : X \to X$ is named a universalized Kannan contraction type map if:

$$d(Mx, My) \leq \alpha(d(x, y))[d(x, Mx) + d(y, My)] \quad \text{for any} \quad x, y \in X,$$

where $\alpha \to [0, \infty] \to [0, 1/2)$ is an increasing mapping.

Definition 11. Suppose that $\breve{\varphi} : \mathbb{E} \times \mathbb{E} \to \mathbb{E}$ be a scalar valued parametric function [16]. The parametric scalar valued function $\breve{\mathcal{D}}_R : (\mathcal{G}, \mathbb{E}) \times (\mathcal{G}, \mathbb{E}) \to (\mathbb{E}, \mathbb{R}^+ \cup \{0\})$ is pronounced to be a rectangular soft metric on $(\mathcal{G}, \mathbb{E})$ if $\breve{\mathcal{D}}_R$ satisfies the following properties:

– (RSM1) $\breve{\mathcal{D}}_R((x, \mathcal{G}(x)), (y, \mathcal{G}(y))) \geq (\breve{\varphi}(x, y), 0)$, if $x = y$, then the equality holds.

– (RSM2) $\breve{\mathcal{D}}_R((x, \mathcal{G}(x)), (y, \mathcal{G}(y))) = (\breve{\varphi}(x, y), 0) \Leftrightarrow$ for all $((x, \mathcal{G}(x)), (y, \mathcal{G}(y))) \in (\mathcal{G}, \mathbb{E})$, $(x, \mathcal{G}(x)) = (y, \mathcal{G}(y))$ [for all $x, y \in \mathbb{E}, x = y$]

– (RSM3) $\breve{\mathcal{D}}_R(x, \mathcal{G}(x)), (y, \mathcal{G}(y))) = \breve{\mathcal{D}}_R((y, \mathcal{G}(y)), (x, \mathcal{G}(x)))$, for all $x, y \in \mathbb{E}$.

– (RSM4)
$$\breve{\mathcal{D}}_R((x, \mathcal{G}(x)), (y, \mathcal{G}(y))) \leq \breve{\mathcal{D}}_R((x, \mathcal{G}(x)), (z, \mathcal{G}(z))) + \breve{\mathcal{D}}_R((z, \mathcal{G}(z)), (t, \mathcal{G}(t))) + \breve{\mathcal{D}}_R((t, \mathcal{G}(t)), (y, \mathcal{G}(y))), \text{ for all } x, y, z, t \in \mathbb{E}$$

Then we say the pair $((\mathcal{G}, \mathbb{E}), \breve{\mathcal{D}}_R)$ is a rectangular soft metric space over X.

Definition 12. Assume that $(\mathcal{G}, \mathbb{E})$ be a soft set on X [13]. A soft sequence in $(\mathcal{G}, \mathbb{E})$ is a map $f : \mathbb{N} \to (\mathcal{G}, \mathbb{E})$ equipped $f(n) = (\mathcal{G}_n, \mathbb{E})$ such that $(\mathcal{G}_n, \mathbb{E})$ is a soft subset of $(\mathcal{G}, \mathbb{E})$ for $n \in \mathbb{N}$, and it is symbolised by $\{(\mathcal{G}_n, \mathbb{E})\}_{n=1}^{\infty}$.

Definition 13. Presume that $(\mathcal{G}, \mathbb{E})$ be a soft set on the universal set [16]. Suppose that $\breve{\mathcal{D}}_R$ be a rectangular soft metric on $(\mathcal{G}, \mathbb{E})$, $\{(\mathcal{G}_n, \mathbb{E})\}_{n=1}^{\infty}$ be a soft sequence in $(\mathcal{G}, \mathbb{E})$ and $(x, \mathcal{G}(x)) \in (\mathcal{G}, \mathbb{E})$. In this instance, we state that the $\{(\mathcal{G}_n, \mathbb{E})\}_{n=1}^{\infty}$ converges to $(x, \mathcal{G}(x))$, if for every positive number ϵ, there subsist a naturel number N so that for all $n \in \mathbb{N}$ with $n \geq N$, we have $\breve{\mathcal{D}}_R((a, \mathcal{G}_n(a)), (x, \mathcal{G}(x))) \leq (\breve{\varphi}(a, x), \epsilon)$.

Definition 14. Suspect that $(\mathcal{G}, \mathbb{E})$ be a soft set on X [16]. Let $\breve{\mathcal{D}}_R$ be a rectangular soft metric on $(\mathcal{G}, \mathbb{E})$ and $\{(\mathcal{G}, \mathbb{E})\}_{n=1}^{\infty}$ be a soft sequence in $(\mathcal{G}, \mathbb{E})$. Then we express that $\{(\mathcal{G}_n, \mathbb{E})\}_{n=1}^{\infty}$ is a Cauchy soft sequence, if there subsist a natural number N so that for every natural number n, m with $n, m \geq N$, we have $\breve{\mathcal{D}}_R((a, \mathcal{G}_n(a)), (a, \mathcal{G}_m(a))) \leq (\breve{\varphi}(a, a), \epsilon)$ for every positive number ϵ.

Proposition 15. Let $((\mathcal{G}, \mathbb{E}), \breve{\mathcal{D}}_R)$ be a rectangular soft metric space over X, [16], and let $\{(\mathcal{G}_n, \mathbb{E})\}_{n=1}^{\infty}$ be a convergent soft sequence in $(\mathcal{G}, \mathbb{E})$. Then $\{(\mathcal{G}_n, \mathbb{E})\}_{n=1}^{\infty}$ is a Cauchy soft sequence.

Theorem 16. Assume that $(\mathcal{G}, \mathbb{E})$ be a soft set on X [16]. Let $\breve{\mathcal{D}}_R$ be a metric on $(\mathcal{G}, \mathbb{E})$ and $\{((\mathcal{G}_n, \mathbb{E})\}_{n=1}^{\infty}$ be a soft sequence in $(\mathcal{G}, \mathbb{E})$. If $\{(\mathcal{G}, \mathbb{E})\}_{n=1}^{\infty}$ is convergent in $(\mathcal{G}, \mathbb{E})$, in this case it converges to only one element of $(\mathcal{G}, \mathbb{E})$.

Definition 17. Let $(\mathcal{G},\mathbb{E})$ be a soft set on X [16]. Let $\breve{\mathcal{D}}_R$ be a rectangular soft metric on $(\mathcal{G},\mathbb{E})$. $(\mathcal{G},\mathbb{E})$ is said to be a complete rectangular soft metric space if every Cauchy soft sequence converges in $(\mathcal{G},\mathbb{E})$.

Theorem 18. Let $((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)$ and $((\mathcal{G}',\mathbb{E}'),\breve{\mathcal{D}}'_R)$ be two rectangular soft metric spaces over X and Y in order [16]. Let $f=(f_1,f_2):((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)\to((\mathcal{G}',\mathbb{E}'),\breve{\mathcal{D}}'_R)$ be a soft transformation. In that case f is continuous in soft manner iff for all $(a,\mathcal{G}(a))\in(\mathcal{G},\mathbb{E})$ and every positive number ϵ, there exist a $\delta>0$ so that for every $(a',\mathcal{G}(a'))\in(\mathcal{G},\mathbb{E})$,

$$\breve{\mathcal{D}}'_R((f(a,\mathcal{G}(a))),f((a',\mathcal{G}(a'))))\le(\breve{\varphi}'(\breve{\varphi}(a,a')),\epsilon)$$

whenever $\breve{\mathcal{D}}_R((a,\mathcal{G}(a)),(a',\mathcal{G}(a')))\le(\breve{\varphi}(a,a')),\delta)$.

Definition 19. Let $((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)$ be a rectangular soft metric space over X and $f:((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)\to((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)$ be a soft mapping [16]. In this instance, f is called to be soft contractive if there is a nonnegative number λ with $0<\lambda<1$ such that:

$$\breve{\mathcal{D}}_R((\breve{M}(a,\mathcal{G}(a))),\breve{M}((a',\mathcal{G}(a'))))\le\lambda\breve{\mathcal{D}}_R((a,\mathcal{G}(a)),(a',\mathcal{G}(a'))),\ \text{for all}\ a,a'\in\mathbb{E}$$

Theorem 20. Soft contractive mapping is continuous in soft manner in rectangular soft metric space $((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)$ [16].

Definition 21. Let $((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)$ be a complete rectangular soft metric space over X and let:

$$f:((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)\to((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)$$

be a soft transformation. A fixed soft set for f is a soft subset of $(\mathcal{G},\mathbb{E})$ such as $(a,\mathcal{G}(a))$ so that $f((a,\mathcal{G}(a)))=(a,\mathcal{G}(a))$, [16].

Theorem 22. (*Banach contraction theorem for rectangular soft metric space*) [16]. Let $((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)$ be a complete rectangular soft metric space over X, and:

$$\breve{M}:((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)\to((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)$$

be rectangular soft contractive transformation. Then $\breve{M}$ has only one fixed soft set.

Some fixed point results for rectangular soft metric space by using Kannan type mappings

Lemma 1. Approve that $\mathbb{A}\subseteq\mathbb{E}$ is a set of parameters (α,t) and (β,t') be two soft parametric scalars that for every $\epsilon>0$ if $(\alpha,t)<(\beta,t'+\epsilon)$ then $(\alpha,t)\preceq(\beta,t')$.

Theorem 2. Presume that $((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)$ is a complete rectangular soft metric space over X, and let $\breve{M}=(\breve{M}_1,\breve{M}_2):((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)\to((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)$ be a soft continuous mapping such that $\breve{M}(\varrho,\mathcal{G}(\varrho))=(\breve{M}_1(\varrho),\breve{M}_2(\mathcal{G}(\varrho)))=(\breve{M}_1(\varrho),\mathcal{G}(\breve{M}_1(\varrho)))$ for every $(\dot{e},\mathcal{G}(\dot{e}))\in(\mathcal{G},\mathbb{E})$ and it satisfies for some parametric scalar valued $\varphi:(\mathcal{G},\mathbb{E})\to(\mathbb{E},\mathbb{R}^+)$

$$\breve{\mathcal{D}}_R(\breve{M}((\varrho,\mathcal{G}(\varrho))),(\varrho,\mathcal{G}(\varrho)))<\varphi((\varrho,\mathcal{G}(\varrho)))-\varphi(\breve{M}(\varrho,\mathcal{G}(\varrho)))\tag{1}$$

Then $\{\breve{M}^n((\varrho,\mathcal{G}(\varrho)))\}$ converges to a fixed soft set, for every $(\varrho,\mathcal{G}(\varrho))\in(\mathcal{G},\mathbb{E})$.

Proof. Let $\breve{M}=(\breve{M}_1,\breve{M}_2)$ and $\varphi=(\varphi_1,\varphi_2)$. Now, if

$$\breve{M}((\varrho,\mathcal{G}(\varrho)))=(\breve{M}_1(\varrho),\breve{M}_2(\mathcal{G}(\varrho))),$$

we set $\breve{M}((\varrho,\mathcal{G}(\varrho)))=(\varrho_1,\mathcal{G}(\varrho_1))$, then $\varphi(\breve{M}((\varrho,\mathcal{G}(\varrho))))=(\varphi_1(\varrho_1),\varphi_2(\mathcal{G}(\varrho_1)))==(b_1,t_1)$. In a similar way we write $\breve{M}^n((\varrho,\mathcal{G}(\varrho)))=(\varrho_n,\mathcal{G}(\varrho_n))$ for $n=1,2,...$ Thus:

$$\varphi(\breve{M}^n((\varrho,\mathcal{G}(\varrho)))=(\varphi_1(\varrho_n),\varphi_2(\mathcal{G}(\varrho_n)))=(b_n,r_n)$$

It is required the following inequality:

$$\breve{\mathcal{D}}_R(\breve{M}((\varrho,\mathcal{G}(\varrho))),(\varrho,\mathcal{G}(\varrho)))+\varphi(\breve{M}(\varrho,\mathcal{G}(\varrho)))<\varphi((\varrho,\mathcal{G}(\varrho))) \tag{2}$$

By using *Lemma 1* we obtain that $\varphi(\breve{M}((\mathbb{E},\mathcal{G}(\varrho))))<\varphi((\varrho,\mathcal{G}(\varrho)))$.

This involves that $\varphi(\breve{M}^2((\varrho,\mathcal{G}(\varrho))))\le\varphi(\breve{M}((\varrho,\mathcal{G}(\varrho))))$. If we keep up this process, we obtain:

$$\varphi(\breve{M}^{n+1}((\varrho,\mathcal{G}(\varrho))))\le\varphi(\breve{M}^{n}((\varrho,\mathcal{G}(\varrho))))$$

This amounts to that $\{\varphi(\breve{M}^n((\varrho,\mathcal{G}(\varrho))))\}$ is a decreasing and thus the sequence $\{r_n\}$ of real numbers is decreasing. Therefore, there is a $t\in\mathbb{R}$ so that:

$$\lim_{n\to\infty}\varphi(\breve{M}^n((\varrho,\mathcal{G}(\varrho))))=(b,t) \quad (\text{for all } b\in\mathbb{E}) \tag{3}$$

Explicitly (b,t) is not negative. Then for all $n,m\in\mathbb{N}$ with $m\ge n,$ we have by using soft rectangular property:

$$\begin{aligned}
&\breve{\mathcal{D}}_R(\breve{M}^n((\varrho,\mathcal{G}(\varrho))),\breve{M}^m((\varrho,\mathcal{G}(\varrho))))\le\breve{\mathcal{D}}_R(\breve{M}^n((\varrho,\mathcal{G}(\varrho))),\breve{M}^{n+1}((\varrho,\mathcal{G}(\varrho))))\\
&+\breve{\mathcal{D}}_R(\breve{M}^{n+1}((\varrho,\mathcal{G}(\varrho))),\breve{M}^{n+2}((\varrho,\mathcal{G}(\varrho))))+\breve{\mathcal{D}}_R(\breve{M}^{n+2}((\varrho,\mathcal{G}(\varrho))),\breve{M}^{m}((\varrho,\mathcal{G}(\varrho))))\\
&\le\breve{\mathcal{D}}_R(\breve{M}^n((\varrho,\mathcal{G}(\varrho))),\breve{M}^{n+1}((\varrho,\mathcal{G}(\varrho))))+\breve{\mathcal{D}}_R(\breve{M}^{n+1}((\varrho,\mathcal{G}(\varrho))),\breve{M}^{n+2}((\varrho,\mathcal{G}(\varrho))))\\
&+\breve{\mathcal{D}}_R(\breve{M}^{n+2}((\varrho,\mathcal{G}(\varrho))),\breve{M}^{n+3}((\varrho,\mathcal{G}(\varrho))))+\breve{\mathcal{D}}_R(\breve{M}^{n+3}((\varrho,\mathcal{G}(\varrho))),\breve{M}^{n+4}((\varrho,\mathcal{G}(\varrho))))\\
&+\breve{\mathcal{D}}_R(\breve{M}^{n+4}((\varrho,\mathcal{G}(\varrho))),\breve{M}^{m}((\varrho,\mathcal{G}(\varrho))))\\
&\le\breve{\mathcal{D}}_R(\breve{M}^n((\varrho,\mathcal{G}(\varrho))),\breve{M}^{n+1}((\varrho,\mathcal{G}(\varrho))))+\breve{\mathcal{D}}_R(\breve{M}^{n+2}((\varrho,\mathcal{G}(\varrho))),\breve{M}^{n+3}((\varrho,\mathcal{G}(\varrho))))\\
&\le\breve{\mathcal{D}}_R(\breve{M}^n((\varrho,\mathcal{G}(\varrho))),\breve{M}^{n+1}((\varrho,\mathcal{G}(\varrho))))+\breve{\mathcal{D}}_R(\breve{M}^{n+2}((\varrho,\mathcal{G}(\varrho))),\breve{M}^{n+3}((\varrho,\mathcal{G}(\varrho))))\\
&+\breve{\mathcal{D}}_R(\breve{M}^{n+3}((\varrho,\mathcal{G}(\varrho))),\breve{M}^{n+4}((\varrho,\mathcal{G}(\varrho))))+\breve{\mathcal{D}}_R(\breve{M}^{n+4}((\varrho,\mathcal{G}(\varrho))),\breve{M}^{n+5}((\varrho,\mathcal{G}(\varrho))))\\
&+\breve{\mathcal{D}}_R(\breve{M}^{n+5}((\varrho,\mathcal{G}(\varrho))),\breve{M}^{n+6}((\varrho,\mathcal{G}(\varrho))))+\breve{\mathcal{D}}_R(\breve{M}^{n+6}((\varrho,\mathcal{G}(\varrho))),\breve{M}^{m}((\varrho,\mathcal{G}(\varrho))))\\
&\le\breve{\mathcal{D}}_R(\breve{M}^n((\varrho,\mathcal{G}(\varrho))),\breve{M}^{n+1}((\varrho,\mathcal{G}(\varrho))))+\breve{\mathcal{D}}_R(\breve{M}^{n+1}((\varrho,\mathcal{G}(\varrho))),\breve{M}^{n+2}((\varrho,\mathcal{G}(\varrho))))\\
&+\cdots+\breve{\mathcal{D}}_R(\breve{M}^{n+2k}((\varrho,\mathcal{G}(\varrho))),\breve{M}^{m}((\varrho,\mathcal{G}(\varrho))))\\
&\le\varphi(\breve{M}^n((\varrho,\mathcal{G}(\varrho)))-\varphi(\breve{M}^{n+1}((\varrho,\mathcal{G}(\varrho)))+\varphi(\breve{M}^{n+1}((\varrho,\mathcal{G}(\varrho)))-\varphi(\breve{M}^{n+2}((\varrho,\mathcal{G}(\varrho)))\\
&+\cdots+\varphi(\breve{M}^{n+2k}((\varrho,\mathcal{G}(\varrho)))-\varphi(\breve{M}^{m}((\varrho,\mathcal{G}(\varrho)))\\
&\le\varphi(\breve{M}^n((\varrho,\mathcal{G}(\varrho)))-\varphi(\breve{M}^{m}((\varrho,\mathcal{G}(\varrho)))\le\varphi(\breve{M}^n((\varrho,\mathcal{G}(\varrho)))-\varphi(\breve{M}^{m}((\varrho,\mathcal{G}(\varrho)))
\end{aligned}$$

Thus

$$\lim_{m,n\to\infty}\breve{\mathcal{D}}_R(\breve{M}^n((\varrho,\mathcal{G}(\varrho))),\breve{M}^m((b,\mathcal{G}(b))))=(b,0)$$

Hence $\{\breve{M}^n((\varrho,\mathcal{G}(\varrho)))\}$ is a soft Cauchy sequence. So there exist $(x,\mathcal{G}(x))\in(\mathcal{G},\mathbb{E})$ so that $\lim_{n\to\infty}\breve{M}^n((\varrho,\mathcal{G}(\varrho))=(x,\mathcal{G}(x))$.

Definition 3. Regard as $(\mathcal{G},\mathbb{E})$ is a soft set over X. We say that every member $(c,\mathcal{G}(c))\in(\mathcal{G},\mathbb{E})$ a soft point of $(\mathcal{G},\mathbb{E})$ where $c\in\mathbb{E}$. Usually, if for some $c\in\mathbb{E}, x\in\mathcal{G}(c)$ there is no need that (c,x) resides to $(\mathcal{G},\mathbb{E})$, but if it is occur we assume (c,x) as a soft point [13].

Definition 4. Suppose that $(\mathcal{G},\mathbb{E})$ is a soft set on X, let $\mathcal{D}_R$ be a soft rectangular meter on $(\mathcal{G},\mathbb{E})$ and t be a positive real number. The soft ball of radius (ϱ',t) about $(\varrho,\mathcal{G}(\varrho))$ is the set:

$$\left\{(c,\mathcal{G}(c))\in(\mathcal{G},\mathbb{E}):\breve{\mathcal{D}}_R((c,\mathcal{G}(c)),(\varrho,\mathcal{G}(\varrho)))\preceq(\hat{\alpha}(c,\varrho)=(\varrho',t))\right\}$$

We signify the soft ball of radius (ϱ',t) about $(\varrho,\mathcal{G}(\varrho))$ by $\mathcal{B}_{\breve{\mathcal{D}}_R}((\varrho,\mathcal{G}(\varrho)),(\varrho',t))$, and for more brief statement we just write:

$$\mathcal{B}_{\breve{\mathcal{D}}_R}((\varrho,\mathcal{G}(\varrho)),(\varrho',t)) \quad \text{or} \quad \mathcal{B}_{(\varrho',t)}(\varrho,\mathcal{G}(\varrho))$$

Theorem 5. Suppose that $((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)$ be a complete rectangular soft metric space on X, and let $\breve{M}:\mathcal{B}_{\breve{\mathcal{D}}_R}((\alpha,\mathcal{G}(\alpha)),(\alpha',t'))\to((\mathcal{G},\mathbb{E}),\breve{\mathcal{D}}_R)$ be a soft contraction transformation with:

$$\breve{\mathcal{D}}_R(\breve{M}((\alpha,\mathcal{G}(\alpha)),(\alpha,\mathcal{G}(\alpha))))\prec(1-2c)(\alpha',t') \tag{4}$$

where $0<c<1$. Then $\breve{M}$ has an only one fixed point set in $\mathcal{B}_{\breve{\mathcal{D}}_R}((\alpha,\mathcal{G}(\alpha)),(\alpha',t'))$.

Proof. It is clear that there is an $r_0\geq 0$ such that $0\preceq(\alpha',t_0)\preceq(\alpha',t)$ with:

$$\breve{\mathcal{D}}_R(\breve{M}((\alpha,\mathcal{G}(\alpha)),(\alpha,\mathcal{G}(\alpha))))\prec(1-2c)(\alpha',t_0)$$

By setting:

$$\breve{M}:\overline{\mathcal{B}_{\breve{\mathcal{D}}_R}((\alpha,\mathcal{G}(\alpha)),(\alpha',t_0))}\to\overline{\mathcal{B}_{\breve{\mathcal{D}}_R}((\alpha,\mathcal{G}(\alpha)),(\alpha',t_0))}$$

If

$$(\sigma,\mathcal{G}(\sigma)),(\phi,\mathcal{G}(\phi)),(\phi',\mathcal{G}(\phi'))\in\overline{\mathcal{B}_{\breve{\mathcal{D}}_R}((\alpha,\mathcal{G}(\alpha)),(\alpha',t_0))}$$

then:

$$\breve{\mathcal{D}}_R(\breve{M}((\sigma,\mathcal{G}(\sigma)),(\alpha,\mathcal{G}(\varepsilon))))\preceq\breve{\mathcal{D}}_R(\breve{M}((\sigma,\mathcal{G}(\sigma)),(\phi,\mathcal{G}(\phi))))$$

$$\begin{aligned}
\breve{\mathcal{D}}_R(\breve{M}((\sigma,\mathcal{G}(\sigma)),(\alpha,\mathcal{G}(\alpha)))) &\preceq \breve{\mathcal{D}}_R(\breve{M}((\sigma,\mathcal{G}(\sigma)),\breve{M}(\phi,\mathcal{G}(\phi))))\\
&\quad+\breve{\mathcal{D}}_R(\breve{M}((\phi,\mathcal{G}(\phi)),\breve{M}(\phi',\mathcal{G}(\phi'))))\\
&\quad+\breve{\mathcal{D}}_R(\breve{M}((\phi',\mathcal{G}(\phi')),(\alpha,\mathcal{G}(\alpha))))\\
&\preceq c\cdot\breve{\mathcal{D}}_R((((\sigma,\mathcal{G}(\sigma)),(\phi,\mathcal{G}(\phi))))\\
&\quad+c\cdot\breve{\mathcal{D}}_R((((\phi,\mathcal{G}(\phi)),(\phi',\mathcal{G}(\phi'))))+(1-2c)(\alpha',t_0)\\
&\preceq c\cdot(\alpha',t_0)+c\cdot(\alpha',t_0)+(1-2c)(\alpha',t_0)\\
&= (\alpha',t_0).
\end{aligned}$$

since

$$\overline{\mathcal{B}_{\breve{\mathcal{D}}_R}((\alpha,\mathcal{G}(\alpha)),(\alpha',t_0))}\,\breve{\subseteq}\,\overline{\mathcal{B}_{\breve{\mathcal{D}}_R}((\alpha,\mathcal{G}(\alpha)),(\alpha',t))}.$$

Now, by *Theorem 22* we obtained that $\breve{M}$ has a unique fixed point set in:

$$\overline{\mathcal{B}_{\breve{\mathcal{D}}_R}((\alpha,\mathcal{G}(\alpha)),(\alpha',t_0))}\,\breve{\subseteq}\,\overline{\mathcal{B}_{\breve{\mathcal{D}}_R}((\alpha,\mathcal{G}(\alpha)),(\alpha',t))}.$$

Theorem 6. Let $((\mathcal{G},\mathbb{E}), \breve{\mathcal{D}}_R)$ be a soft RMS. $\breve{M}:(\mathcal{G},\mathbb{E})\to(\mathcal{G},\mathbb{E})$ be a Kannan type mapping such that α satisfies for each r parametric scalar with $r\in[0,\infty)$ $\lim_{t\to r}\sup\alpha<1/2$ Suppose there exist on $(\eta_0,\mathcal{G}(\eta_0))\in(\mathcal{G},\mathbb{E})$ with the bounded orbit, namely the sequence $\{\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))\}$ is bounded. Additionally, assume that:

$$d((\eta,\mathcal{G}(\eta)),\breve{M}(\eta,\mathcal{G}(\eta)))<\infty \quad \text{for each} \quad (\eta,\mathcal{G}(\eta))\in(\mathcal{G},\mathbb{E}).$$

Then $\breve{T}$ has a fixed point $(\tilde{\rho},\mathcal{G}(\tilde{\rho}))\in(\mathcal{G},\mathbb{E})$ and $\lim_{n\to\infty}\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))=(\tilde{\eta},\mathcal{G}(\tilde{\eta}))$. Besides if $\breve{M}$ has a fixed point $(\tilde{y},\mathcal{G}(\tilde{y}))$, then either $\breve{\mathcal{D}}_R((\tilde{\eta},\mathcal{G}(\tilde{\eta})),(\tilde{y},\mathcal{G}(\tilde{y})))=\infty$ or $(\tilde{\eta},\mathcal{G}(\tilde{\eta}))=(\tilde{y},\mathcal{G}(\tilde{y}))$.

Proof. Take $(\eta_0,\mathcal{G}(\eta_0))\in(\mathcal{G},\mathbb{E})$ be arbitrary. Describe a sequence $\{(\mathcal{F}_n,\mathbb{E})\}_{n=1}^{\infty}$ as: $(\mathcal{G}_{n+1},\mathbb{E})=\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))$, $n=0,1,2,\ldots$ Since $\breve{M}$ is a Kannan type mapping, we get:

$$\begin{aligned}
\breve{\mathcal{D}}_R(\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0))) &= \breve{\mathcal{D}}_R(\breve{M}(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0))),\breve{M}(\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))))\\
&\le \alpha(\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))))\\
&\quad\cdot[\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)))\\
&\quad+\breve{\mathcal{D}}_R(\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0)))]\\
&\le \alpha(\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))))\\
&\quad\cdot(\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))))\\
&\quad+\alpha(\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))))\\
&\quad\cdot(\breve{\mathcal{D}}_R(\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0))))
\end{aligned}$$

$$\begin{aligned}
\breve{\mathcal{D}}_R(\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0)))-\alpha(\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))))&\\
\cdot(\breve{\mathcal{D}}_R(\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0))))&\\
\le\alpha(\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))))&\\
\cdot(\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))))&
\end{aligned}$$

$$\begin{aligned}
&\breve{\mathcal{D}}_R(\breve{M}^n(x_0,\mathcal{G}(x_0)),\breve{M}^{n+1}(x_0,\mathcal{G}(x_0)))(1-\alpha)(\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))))\\
&\qquad\le\alpha(\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))))\\
&\qquad\cdot(\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))))
\end{aligned}$$

$$\begin{aligned}
\breve{\mathcal{D}}_R(\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0)))\le&\frac{\alpha(\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))))}{(1-\alpha)(\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))))}\\
&(\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))))
\end{aligned}$$

Since $\alpha(t)\in[0,1/2)$, we get $\{\alpha(t)/[1-\alpha(t)]\}<1$. Then we get:

$$\breve{\mathcal{D}}_R(\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0)))<\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)))$$

It follows that $\{\breve{\mathcal{D}}_R(\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0)))\}_{n=1}^{\infty}$ is monotone decreasing. Similarly we can show the following statement.

$$\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)))\leq\frac{\alpha(\breve{\mathcal{D}}_R(\breve{M}^{n-2}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0))))}{(1-\alpha)(\breve{\mathcal{D}}_R(\breve{M}^{n-2}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0))))}(\breve{\mathcal{D}}_R(\breve{M}^{n-2}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0))))$$

Because $\alpha(t)$ is increasing $\{\alpha(t)/[1-\alpha(t)]\}$ also is increasing. Furthermore, from:

$$\{\breve{\mathcal{D}}_R(\breve{M}^{n-2}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)))\}_{n=1}^{\infty}$$

is monotone decreasing, then:

$$\breve{\mathcal{D}}_R(\breve{M}^{n-2}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)))<\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))$$

$$\frac{\alpha(\breve{\mathcal{D}}_R(\breve{M}^{n-2}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0))))}{(1-\alpha)(\breve{\mathcal{D}}_R(\breve{M}^{n-2}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0))))}<\frac{\alpha(\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))}{(1-\alpha)(\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))}$$

Then:

$$\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)))\leq\frac{\alpha(\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))}{(1-\alpha)(\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))}(\breve{\mathcal{D}}_R(\breve{M}^{n-2}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0))))$$

Repeating this relation, we get:

$$\breve{\mathcal{D}}_R(\breve{M}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^2(\eta_0,\mathcal{G}(\eta_0)))\leq\frac{\alpha(\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))}{(1-\alpha)(\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))}\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))$$

Now let:

$$h=\frac{\alpha(\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))}{(1-\alpha)(\breve{\mathcal{D}}_R((\alpha_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))}$$

then we can have:

$$\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)))\leq(h)^n(\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))\quad(5)$$

Let $m>n$; then from inequality (1), we have:

$$\begin{aligned}\breve{\mathcal{D}}_R(\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^m(\eta_0,\mathcal{G}(\eta_0)))&\leq\breve{\mathcal{D}}_R(\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0)))\\&+\breve{\mathcal{D}}_R(\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n+2}(\eta_0,\mathcal{G}(\eta_0)))\\&+\cdots+\breve{\mathcal{D}}_R(\breve{M}^{m-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^m(\eta_0,\mathcal{G}(\eta_0)))\\&\leq(h)^n\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))\\&+(h)^{n+1}\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))\\&+\cdots+(h)^{m-1}\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))\\&=[(h)^n+(h)^{n+1}+\cdots+(h)^{m-1}]\\&\cdot\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))\\&\leq\frac{(h)^n}{1-h}\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))\end{aligned}\quad(6)$$

Since

$$h=\frac{\alpha\,\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))}{(1-\alpha)\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))}\in[0,1/2)$$

it follows that $\{\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))\}_{n=1}^{\infty}$ is a Cauchy sequence in $(\mathcal{G},\mathbb{E})$ Since $(\mathcal{G},\mathbb{E})$ is complete RMS, there exists a point $(\tilde{\eta},\mathcal{G}(\tilde{\eta}))$ such that $\breve{M}^n(\tilde{\eta}_0,\mathcal{G}(\tilde{\eta}_0))\to(\tilde{\eta},\mathcal{G}(\tilde{\eta}))$. By rectangular property:

$$\begin{aligned}\breve{\mathcal{D}}_R(\breve{M}(\tilde{\eta},\mathcal{G}(\tilde{\eta})),(\tilde{\eta},\mathcal{G}(\tilde{\eta}))) &\le \breve{\mathcal{D}}_R(\breve{M}(\tilde{\eta},\mathcal{G}(\tilde{\eta})),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0))\\ &\quad+\breve{\mathcal{D}}_R(\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0)))\\ &\quad+\breve{\mathcal{D}}_R(\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0)),(\tilde{\eta},\mathcal{G}(\tilde{\eta})))\\ &\le \lambda[\breve{\mathcal{D}}_R(\breve{M}(\tilde{\eta},\mathcal{G}(\tilde{\eta})),(\tilde{\eta},\mathcal{G}(\tilde{\eta})))\\ &\quad+\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{F}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)))]\\ &\quad+h^n\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))\\ &\quad+\breve{\mathcal{D}}_R(\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0)),(\tilde{\eta},\mathcal{G}(\tilde{\eta})))\end{aligned}$$

$$\begin{aligned}\breve{\mathcal{D}}_R(\breve{M}(\tilde{\eta},\mathcal{G}(\tilde{\eta})),(\tilde{\eta},\mathcal{G}(\tilde{\eta}))) &\le \lambda\breve{\mathcal{D}}_R(\breve{M}(\tilde{\eta},\mathcal{G}(\tilde{\eta})),(\tilde{\eta},\mathcal{G}(\tilde{\eta})))\\ &\quad+\lambda\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)))\\ &\quad+h^n\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))\\ &\quad+\breve{\mathcal{D}}_R(\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0)),(\tilde{\eta},\mathcal{G}(\tilde{\eta})))\end{aligned}$$

$$\begin{aligned}(1-\lambda)\breve{\mathcal{D}}_R(\breve{M}(\tilde{\eta},\mathcal{G}(\tilde{\eta})),(\tilde{\eta},\mathcal{G}(\tilde{\eta}))) &\le \lambda\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)))\\ &\quad+h^n\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))\\ &\quad+\breve{\mathcal{D}}_R(\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0)),(\tilde{\eta},\mathcal{G}(\tilde{\eta})))\end{aligned}$$

$$\breve{\mathcal{D}}_R(\breve{M}(\tilde{\eta},\mathcal{G}(\tilde{\eta})),(\tilde{\eta},\mathcal{G}(\tilde{\eta})))\le\frac{\lambda}{(1-\lambda)}\breve{\mathcal{D}}_R(\breve{M}^{n-1}(\eta_0,\mathcal{G}(\eta_0)),\breve{M}^n(\eta_0,\mathcal{G}(\eta_0)))$$

$$+\frac{h^n}{(1-\lambda)}\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))\le h^n\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))$$

$$+\frac{h^n}{(1-\lambda)}\breve{\mathcal{D}}_R((\eta_0,\mathcal{G}(\eta_0)),\breve{M}(\eta_0,\mathcal{G}(\eta_0)))+\frac{1}{(1-\lambda)}\breve{\mathcal{D}}_R(\breve{M}^{n+1}(\eta_0,\mathcal{G}(\eta_0)),(\tilde{\eta},\mathcal{G}(\tilde{\eta})))$$

Letting $n\to\infty$ and using the fact that, $\breve{\mathcal{D}}_R((\mathcal{G}'_n,\mathbb{E}),(y,\mathcal{G}(y)))\to\breve{\mathcal{D}}_R((a,\mathcal{G}(a)),(y,\mathcal{G}(y)))$ and whenever $(\mathcal{G}'_n,\mathbb{E})$ is a soft sequence in $(\mathcal{G},\mathbb{E})$ with $(\mathcal{G}'_n,\mathbb{E})\to(a,\mathcal{G}(a))\in(\mathcal{G},\mathbb{E})$ we have $(\tilde{x},\mathcal{G}(\tilde{x}))=\breve{M}(\tilde{x},\mathcal{G}(\tilde{x}))$. Now we show that M has a unique fixed point. For this, assume that there exists another point $(\tilde{y},\mathcal{G}(\tilde{y}))\in(\mathcal{G},\mathbb{E})$ such that $(\tilde{y},\mathcal{G}(\tilde{y}))=\breve{M}(\tilde{y},\mathcal{G}(\tilde{y}))$.

Now:

$$\begin{aligned}\breve{\mathcal{D}}_R((\tilde{y},\mathcal{G}(\tilde{y})),(\tilde{\eta},\mathcal{G}(\tilde{\eta}))) &= \breve{\mathcal{D}}_R(\breve{M}(\tilde{y},\mathcal{G}(\tilde{y})),\breve{M}(\tilde{\eta},\mathcal{G}(\tilde{\eta})))\\ &\le \lambda\breve{\mathcal{D}}_R((\tilde{y},\mathcal{G}(\tilde{y})),\breve{M}(\tilde{y},\mathcal{G}(\tilde{y})))\\ &\quad+\breve{\mathcal{D}}_R((\tilde{\eta},\mathcal{G}(\tilde{\eta})),\breve{M}(\tilde{\eta},\mathcal{G}(\tilde{\eta})))\end{aligned}$$

$$\begin{aligned} &\leq \lambda \breve{\mathcal{D}}_R((\tilde{y},\mathcal{G}(\tilde{y})),(\tilde{y},\mathcal{G}(\tilde{y})))+ \\ &\quad +\breve{\mathcal{D}}_R((\tilde{\eta},\mathcal{G}(\tilde{\eta})),(\tilde{\eta},\mathcal{G}(\tilde{\eta})))= \\ &= 0 \end{aligned}$$

Hence $(\tilde{\eta},\mathcal{F}(\tilde{\eta}))=(\tilde{y},\mathcal{F}(\tilde{y}))$.

An application of fixed point for thermal science problem

A thermal science problem which can be expressed:

$$\begin{cases} Lw(x,y)=\phi(x,x'),\ (x,x')\in\Omega \\ Bw(x,y)=\varphi(x,x'),\ (x,x')\in\partial\Omega \end{cases}$$

is one of the boundary value problems. In this thermal equation L denotes a linear differential operator, B is a boundary operator, $\phi(x,x')$ and $\varphi(x,x')$ are two given functions, Ω is an open bounded region and $\partial\Omega$ represents the boundary of Ω. By using fixed point property and techniques investigated in [7, 8], we can guarantee the existence and uniqueness of the solution.

Conclusion

In this paper, Kannan type fixed point results are obtained by using rectangular soft metric. Also fixed point property associated with thermal science problem which is a kind of boundary value problem.

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