A MODIFIED FOURIER-FICK ANALYSIS FOR MODELLING NON-NEWTONIAN MIXED CONVECTIVE FLOW CONSIDERING HEAT GENERATION

by

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Homotopic solutions for Jeffrey material in frames of buoyancy forces are constructed in this research. The improved Fourier-Fick laws are considered for formulation. In addition, variable liquid aspects (thermal conductivity, mass diffusivity) along with heat source are accounted. Prandtl's boundary-layer idea is utilized to model the problem. Involvement of similarity variables resulted into non-linear system of coupled equations. The well-known homotopic scheme is employed for non-linear analysis. Besides, a comprehensive discussion is reported for arising dimensionless variables vs. significant profiles. Our results indicate a rise in thermal and solutal fields when variable conductivity and mass diffusivity parameters are increased.

Key words: *improved Fourier-Fick laws, heat generation, Jeffrey material, mixed convective flow, variable liquid aspects*

Introduction

Fluids featuring non-Newtonian characteristics have vital contribution in distinct industrial utilizations due to their manifold attributes in nature. Numerous engineering structures comprise liquid crystals, polycrystalline materials, fibrous materials, course grain structures or colloidal suspensions elaborate features of non-Newtonian types. The classical Navier-Stokes concept is not able to predict their salient aspects. Therefore, distinct models elaborating non-Newtonian characteristics have been introduced. The considered model (Jeffrey liquid) is subcategory of rate type materials and has prospective to scrutinize the characteristics of relaxation/retardation aspects. Researchers considered Jeffrey model subjected to various configurations. For illustration, Chemically reacting Jeffrey material flow subjected to non-linear radiation is scrutinized by Raju *et al.* [1]. Hayat *et al.* [2, 3] formulated hydromagnetic characteristics for Jeffrey material stretched flow under heat sink/source aspects. Thermo diffusion impact in fractional hydromagnetic Jeffrey material in frames of radiation is explored by Imran *et al.* [4]. Waqas *et al.* [5] formulated thermally radiating stratified Jeffrey nanoliquid subjected to buoyancy forces.

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There are extremely effective utilizations of heat transportation mechanism for illustration chilling of nuclear reactor, medical utilizations like drug targeting and conduction of heat in tissues. Heat transport approach is firstly communicated through well-known Fourier law [6]. The thermal expression subjected to Fourier law has parabolic nature and so initial disruption is recognized continually throughout. To regulate such unrealistic characteristic in the thermal inertia factor, thermal expression is incorporated via stable heat-conduction. As a result, the innovative model revises the behavior of temporal solution and yields the heat-conduction expression into damped hyperbolic one [7]. Tibullo and Zampoli [8] established uniqueness results for incompressible nature problems subjected to modified Fourier law. Haddad [9] scrutinized thermal volatility in the permeable Brinkman medium employing modified Fourier law. A modified Fourier analysis considering magneto-Casson material under hydromagnetic characteristics is communicated by Malik *et al.* [10]. Waqas *et al.* [11] analytically addressed forced convective stratified Burgers material flow under modified Fourier law. Characteristics of modified Fourier law for computational analysis of non-Newtonian (Carreau and Jeffrey) material are communicated by Khan *et al.* [12, 13]. Waqas *et al.* [14] introduced heat source concept in mixed convective Burgers material subjected to improved Fourier-Fick expression. Variable liquid aspects in stratified Carreau nanoliquid subject to improved Fourier law are examined by Khan *et al.* [15].

Keeping aforementioned research attempts at mind, we found that improved Fourier law is less scrutinized in comparison traditional Fourier law. Thus our focus here is to formulate and examine the non-Newtonian (Jeffrey) fluid in frames of the improved Fourier Law. In addition, mixed convection, variable type conductivity/diffusivity and heat source characteristics are considered. Non-linear systems are tackled through homotopy algorithm [16-25]. The non-dimensional quantities are exhibited and deliberated.

Modelling

An incompressible Jeffrey material mixed convective flow by stretchable surface is formulated. The improved Fourier-Fick laws are considered for formulation. In addition, variable liquid aspects (thermal conductivity, mass diffusivity) along with heat source are accounted. Viscous dissipation along with radiation is ignored. The governing 2-D expressions:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\begin{gathered}u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\nu}{1+\lambda_1}\left[\frac{\partial^2 u}{\partial y^2}+\lambda_2\left(\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+u\frac{\partial^3 u}{\partial x\partial y^2}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\right)\right]+\\ +\mathrm{g}\left[\Lambda_1(T-T_\infty)+\Lambda_2(C-C_\infty)\right]\end{gathered} \tag{2}$$

$$\begin{gathered}u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\lambda_T\cdot\\ \cdot\left[u\frac{\partial u}{\partial x}\frac{\partial T}{\partial x}+v\frac{\partial v}{\partial y}\frac{\partial T}{\partial y}+v\frac{\partial u}{\partial x}\frac{\partial T}{\partial x}+2uv\frac{\partial^2 T}{\partial x\partial y}+u^2\frac{\partial^2 T}{\partial x^2}+v^2\frac{\partial^2 T}{\partial y^2}-\frac{Q}{\rho c_p}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)\right]=\\ =\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left[K(T)\frac{\partial T}{\partial y}\right]+\frac{Q}{\rho c_p}(T-T_\infty)\end{gathered} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+\lambda_C\left(u\frac{\partial u}{\partial x}\frac{\partial C}{\partial x}+v\frac{\partial v}{\partial y}\frac{\partial C}{\partial y}+v\frac{\partial u}{\partial y}\frac{\partial C}{\partial x}+2uv\frac{\partial^2 C}{\partial x\partial y}+u^2\frac{\partial^2 C}{\partial x^2}+v^2\frac{\partial^2 C}{\partial y^2}\right)= =\frac{\partial}{\partial y}\left[D(C)\frac{\partial C}{\partial y}\right] \tag{4}$$

subject to conditions [14]:

$$\begin{gathered} u=U_w(x)=cx,\quad v=0,\quad T=T_w,\quad C=C_w \quad \text{at} \quad y=0 \\ u\to 0,\quad T\to T_\infty,\quad C\to C_\infty \quad \text{when} \quad y\to\infty \end{gathered} \tag{5}$$

where u and v elucidate velocities of fluid in horizontal and vertical directions, respectively, ρ – the liquid density, ν – the kinematic viscosity, λ_2 – the relaxation time, λ_1 – the relation between relaxation/retardation times, Λ_1 and Λ_2 – the thermal and solutal expansion coefficients, g – the gravitational acceleration, T and C – the fluid temperature and concentration, λ_T and λ_C – the heat and mass flux relaxation times, T_∞ and C_∞ – the ambient fluid temperature and concentration, Q – the heat source/sink coefficient, and c – the stretching rate. Mathematical forms of variable conductivity, $K(T)$, and variable mass diffusivity, $D(C)$, is [26]:

$$K(T)=K_\infty\left(1+\varepsilon_1\frac{T-T_\infty}{T_w-T_\infty}\right) \tag{6}$$

$$D(C)=D_\infty\left(1+\varepsilon_2\frac{C-C_\infty}{C_w-C_\infty}\right) \tag{7}$$

in which (K_∞, C_∞) illustrate ambient liquid (thermal conductivity, mass diffusivity) and (ε_1, ε_2) small parameters which elaborates the characteristics of temperature and concentration for thermal and solutal dependent conductivity and diffusivity.

Employing [14]:

$$\begin{gathered} \eta=y\sqrt{\frac{c}{\nu}},\quad u=cxf'(\eta),\quad v=-\sqrt{c\nu}f(\eta) \\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \end{gathered} \tag{8}$$

Equation (1) is validated automatically while eqs. (2)-(5) are:

$$f'''+(1+\lambda_1)(ff''-f'^2)+\beta(f''^2-ff^{iv})+\lambda(\theta+N\phi)=0 \tag{9}$$

$$(1+\varepsilon_1\theta)\theta''+\varepsilon_1\theta'^2+\Pr f\theta'+\Pr\delta\theta-\Pr\delta\gamma_1 f\theta'-\Pr\gamma_1(ff'\theta'+f^2\theta'')=0 \tag{10}$$

$$(1+\varepsilon_2\phi)\phi''+\varepsilon_2\phi'^2+\mathrm{Sc}f\phi'-\mathrm{Sc}\gamma_2(ff'\phi'+f^2\phi'')=0 \tag{11}$$

$$f(0)=0,\quad f'(0)=1,\quad f'(\infty)\to 0 \tag{12}$$

$$\theta(0)=1,\quad \theta(\infty)\to 0 \tag{13}$$

$$\phi(0)=1,\quad \phi(\infty)\to 0 \tag{14}$$

where

$$\beta = \lambda_2 c, \quad \lambda = \frac{\text{Gr}_x}{\text{Re}_x^2}, \quad N = \frac{\text{Gr}_x^*}{\text{Gr}_x}, \quad \text{Gr}_x = \frac{g\Lambda_1(T_w - T_\infty)x^3}{\nu^2}, \quad \text{Gr}_x^* = \frac{g\Lambda_2(C_w - C_\infty)x^3}{\nu^2}$$

$$\text{Re}_x = \frac{xU_w(x)}{\nu}, \quad \text{Pr} = \frac{\mu c_p}{K_\infty}, \quad S = \frac{Q}{c\rho c_p}, \quad \gamma_1 = \lambda_T c, \quad \gamma_2 = \lambda_C c, \quad \text{and} \quad \text{Sc} = \frac{\nu}{D_\infty}$$

where β is the Deborah number, λ – the thermal buoyancy factor, N – the ratio of solutal to thermal buoyancy, Gr_x – the thermal Grashof number, Gr_x^* – thesolutal Grashof number, Re_x – the Reynolds number, Pr – the Prandtl number, S – the heat generation factor, γ_1 – the thermal relaxation factor, γ_2 – the solutal relaxation factor, and Sc – the Schmidt number, respectively.

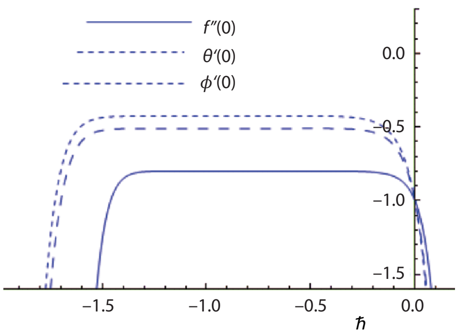

Figure 1. The $\hbar$-curves for f, θ, and ϕ

Solution scheme and convergence

We employed homotopy algorithm [16-25] for non-linear analysis of eqs. (9)-(11) subject to eqs. (12)-(14). Undoubtedly $\hbar$-curves are vital to certify convergence of eqs. (9)-(11). Thus we revealed $\hbar$-curves in fig. 1 for such purpose. Flat portions in fig. 1 help to attain allowable values of $\hbar_f$, $\hbar_\theta$, and $\hbar_\phi$. We noticed $-1.35 \leq \hbar_f \leq -0.30$, $-1.50 \leq \hbar_\theta \leq -0.40$, and $-1.40 \leq \hbar_\phi \leq -0.40$, with $\beta = 0.4$, $\lambda_1 = 0.5$, $\lambda = 0.3$, $N = \gamma_1 = 0.2$, $S = \gamma_2 = \varepsilon_1 = \varepsilon_2 = 0.1$, Sc = 0.8, and Pr = 1.0. Additionally, convergence is confirmed numerically via tab. 1. Clearly eq. (8) converges at 20th order approximation while eqs. (9) and (10) converge at 25th order approximation, respectively.

Table 1. Convergence analysis for distinct order approximations when $\beta = 0.4$, $\lambda_1 = 0.5$, $\lambda = 0.3$, $N = \gamma_1 = 0.2$, $S = \gamma_2 = \varepsilon_1 = \varepsilon_2 = 0.1$, Sc = 0.8, and Pr = 1.0

Order of approximations	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1	0.8680	0.7080	0.7400
5	0.8046	0.4529	0.5266
10	0.7999	0.4275	0.5106
15	0.8003	0.4270	0.5110
20	0.8003	0.4276	0.5114
25	0.8003	0.4276	0.5114
30	0.8003	0.4276	0.5114

Analysis

This section emphasizes the noteworthy characteristics of arising variables against thermal field, $\theta(\eta)$, and solutal field, $\phi(\eta)$. Thus, figs. 2-8 are sketched and explained in detail. Figure 2 reports S characteristics on $\theta(\eta)$. Clearly $\theta(\eta)$ increases when S is augmented. Heat transports promptly via higher S estimations. Outcome of Prandtl number *vs.* $\theta(\eta)$ is evaluated via fig. 3. Larger Prandtl number values yields less diffusivity which accordingly reduces $\theta(\eta)$.

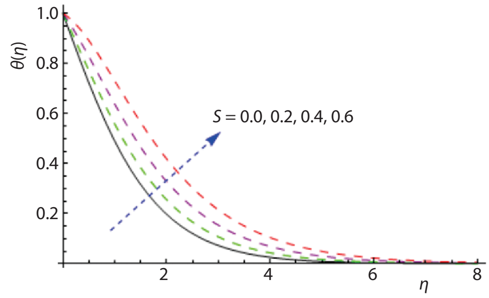

Figure 2. The S *vs.* $\theta(\eta)$

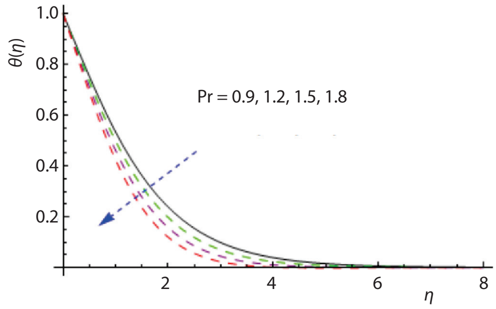

Figure 3. Prandtl number *vs.* $\theta(\eta)$

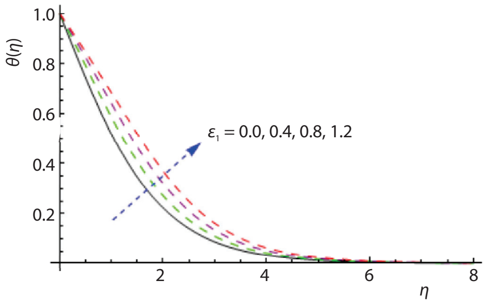

Figure 4. The ε_1 *vs.* $\theta(\eta)$

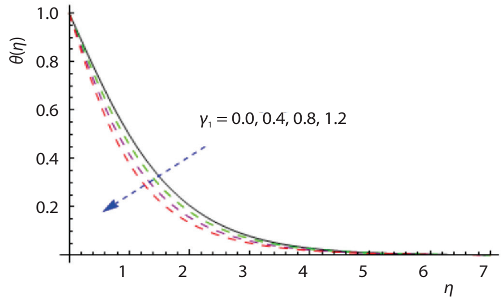

Figure 5. The γ_1 *vs.* $\theta(\eta)$

Figure 4 interprets variation in $\theta(\eta)$ for ε_1. As anticipated, larger ε_1 corresponds to $\theta(\eta)$ enhancement. In fact material's conductivity upsurges when ε_1 is incremented. Consequently extra heat quantity is switched from surface towards material and as a result $\theta(\eta)$ is boosted. Effects of γ_1 and γ_2 *vs.* $\theta(\eta)$ and $\phi(\eta)$ is explained in figs. 5 and 6. Larger γ_1 and γ_2 signify non-conducting trend due to which $\theta(\eta)$ and $\phi(\eta)$ dwindles. Further, $\theta(\eta)$ and $\phi(\eta)$ are large for $\gamma_1 = 0 = \gamma_2$ when compared with $\gamma_1 > 0$ and $\gamma_2 > 0$. Figure 7 addresses Schmidt number influence on $\phi(\eta)$. Here $\phi(\eta)$ reduces for higher Schmidt number estimation. Physically, the Schmidt number expression involves Brownian diffusivity which dwindles subject to higher Schmidt number values. Characteristics of ε_2 *vs.* $\phi(\eta)$ are expressed via fig. 8. Clearly $\phi(\eta)$ enhances subject to higher ε_2. Undoubtedly liquids having higher mass diffusivity corresponds to higher concentration. In fact liquid mass diffusivity is increased via larger ε_2.

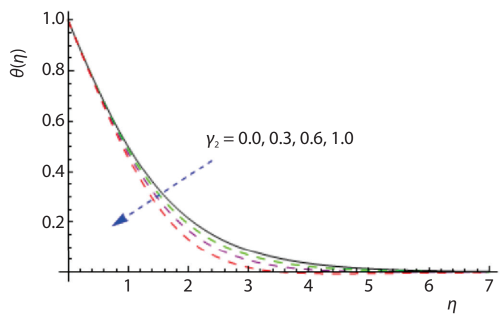

Figure 6. The γ_2 *vs.* $\phi(\eta)$

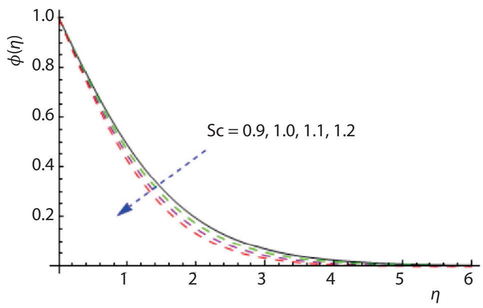

Figure 7. The Schmidt number *vs.* $\phi(\eta)$

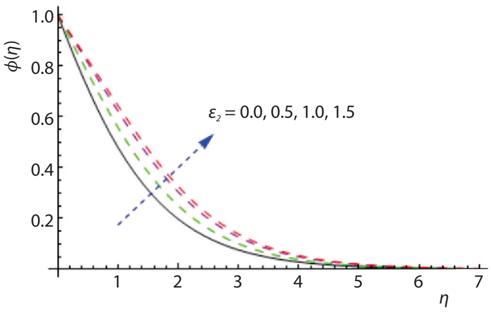

Figure 8. The ε_2 *vs.* $\phi(\eta)$

Conclusions

This communication reports heat source, variable (thermal conductivity, mass diffusivity) and buoyancy forces aspects in non-Newtonian (Jeffrey) material flow by moving vertical surface. Improved Fourier-Fick laws are utilized for formulation of energy and concentration equations. We acquired following noteworthy points through aforementioned investigation.

- Thermal field $\theta(\eta)$ upsurges when variable conductivity, ε_1, heat source, S, factors are augmented.
- Larger thermal relaxation, γ_1, and Prandtl number correspond to $\theta(\eta)$ decline.
- A rise in solutal relaxation, γ_2, and Schmidt number yield lower solutal field $\phi(\eta)$.
- The well-known Fourier-Fick laws can be recovered by letting $\gamma_1 = 0 = \gamma_2$ in eqs. (10) and (11).
- Jeffrey liquid model yields viscous liquid outcomes when $\lambda_1 = 0 = \beta$.

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