STEPWISE LOADING-UNLOADING CREEP TESTING OF FLY ASH CONCRETE AND ITS CONSTITUTIVE MODEL

by

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Stepwise loading-unloading creep testing of concrete with fly ash content of 35%, and 50% was conducted. The time course curve of stepwise creep in fly ash concrete was obtained. Analyses have revealed that it had decelerated creep, constant velocity, and accelerated creep properties. Based on rheological theory, a non-linear viscoelastic-plastic rheological model (MSSB-NVPB) was constructed, and its constitutive relations and creep equations were obtained. Combined with experimental data, the model parameters were determined. The results showed that this model can characterize the creep properties of the fly ash concrete fairly well.

Key words: *creep, theMSSB–NVPB model, constitutive equation, concrete with fly ash*

Introduction

The rheological properties of concrete have always represented an important and difficult research topic in the studies of long-term concrete performance [1-3]. A number of factors influence concrete rheology, including water-cement ratio, admixtures, aggregate composition, temperature, moisture, loading age, dimensions, condition of maintenance, and so on [4, 5]. Many experimental results and rheological theory have been obtained for the research on the rheological properties of concrete [6-14].

Fly ash serves as an important admixture that improves the compactness of concrete materials with significant effectiveness. It has already been broadly applied to actual engineering projects, especially bridges, dams, reservoirs, and other water conservation facilities. Many researchers have conducted relevant testing studies and theoretical investigations of the creep properties of fly ash concrete. For example, Qin *et al.* [15] conducted a compression creep testing study on C50 fly ash concrete and proposed formulas for calculating the creep variability of different fly ash contents. Zhao *et al.* [16] studied the effects of single and compound factors such as slag grinding, fly ash, and water-cement ratio on the creep properties of concrete and conducted SEM analysis and investigation of its microscopic structure. Zhao *et al.* [17] studied the effect of maintenance systems on the creep properties of composite ultrafine fly ash concrete.

To study the mechanical properties of the fly ash concrete creep more extensively, we apply stepwise loading-unloading testing methods to analyze the short-term creep of C60 concrete with different fly ash content. Moreover, a non-linear viscoelastic-plastic rheological model of the fly ash concrete is constructed based on the analysis of the stepwise creep time

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course curve. The corresponding constitutive relations and creep equations were derived, and the theoretical models are validated.

Experimental

Materials

The cementitious materials used in the experiment were Portland cement and fly ash supplied by two local companies whose chemical components are shown in the tab. 1. Fine aggregate was river sand with the fineness modulus of 2.8 and the apparent density of 2769 kg/m^3. Coarse aggregate was crushed limestone with the size of 5-20 mm and the apparent density of 2719 kg/m^3. The slushing agent for concrete was carboxylic acid provided by Wulong, and the mix proportion of concrete is summarized in tab. 2.

Table 1. Chemical compositions of cement and fly ash [%]

Materials	SiO_2	Al_2O_3	Fe_2O_3	CaO	MgO	Na_2O	K_2O	SO_3
Cement	21.60	4.13	4.57	64.47	1.06	0.11	0.56	1.74
Fly ash	54.90	25.80	6.90	8.70	1.80	0.30	0.10	0.60

Table 2. Mix proportions of concretes

Fly ash Content [%]	Cement [kgm^{-3}]	Fly ash [kgm^{-3}]	Sand [kgm^{-3}]	Stone [kgm^{-3}]	Water-cement ratio [%]	Water [kgm^{-3}]
0.0	453.0	0.0	740.0	1112.0	32.0	145.0
20.0	362.0	91.0	740.0	1112.0	32.0	145.0
35.0	294.0	159.0	740.0	1112.0	32.0	145.0
50.0	226.5	226.5	740.0	1112.0	32.0	145.0

Test methods

The dimensions of the concrete sample were 100 mm × 100 mm × 300 mm, and the maintenance period was over 90 days. The Changchun Kexin YAS-5000 electrohydraulic servo-controlled compression testing machine was used. The increasing loading steps method was used for creep testing of fly ash concrete samples. First, the specimen was loaded at a rate of 3 kN/s to 30% of its strength value, held for 30 minutes, and then unloaded. Next, stepwise compression was performed using 10% of the strength value as the loading step. The load was applied for 30 minutes at each loading step, and then the specimen was unloaded. The process was repeated until the specimen failed. Strain and stress were measured using strain gauges and pressure sensors.

Results and discussion

Figure 1 shows the stepwise creep time course curves for concrete with different fly ash contents. Analysis of these curves revealed that at low stress levels, amount of creep deformation in concrete remained very small as time increased, and the strain instantaneously and completely recovered after unloading without any elastic after-effects. With increasing load steps, the creep properties of concrete manifested as decelerated creep deformation. With increasing time, creep increased but creep velocity decreased. After unloading, strain mostly recovered instantaneously, and a small part recovered as time increased, finally reaching a stable value with elastic after-effects. When stress reached the yield strength, the creep curve showed full creep curve characteristics with decelerating, constant velocity, and accelerating creep stages.

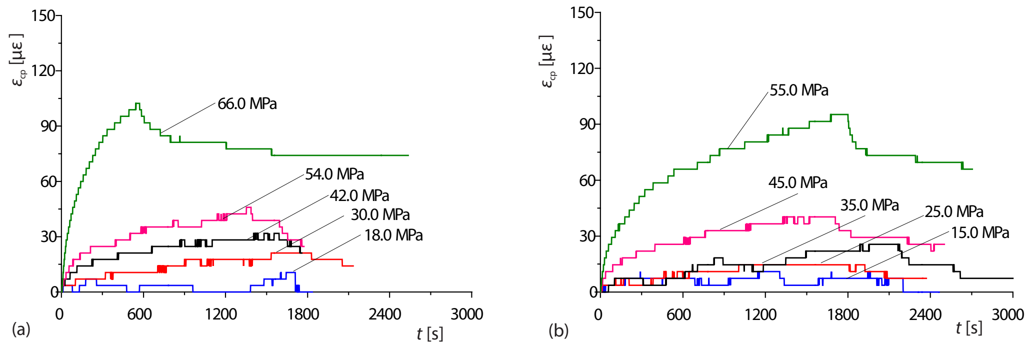

Figure 1. Creep-time curves of concrete mixed with fly ash without elastic deformation; (a) *m* = 35% , (b) *m* = 50%

The MSSB-NVPB rheological model

Model

The analysis showed that the modified Schofield-Scott-Blair (MSSB) elastic-viscoplastic-viscoelastic-plastic rheological model [18, 19] can be used to describe the deceleration and constant velocity creep properties of fly ash concrete. When the load reached the yield strength, the fly ash concrete exhibited accelerated unsteady creep properties. In this study, we introduced parameters of the non-linear viscoplastic body (NVPB) rheological model [20] with the MSSB model as a foundation in order to describe these properties. In summary, we used the MSSB–NVPB non-linear viscoelastic-plastic rheological model to characterize the creep properties of fly ash concrete, as shown in fig. 2.

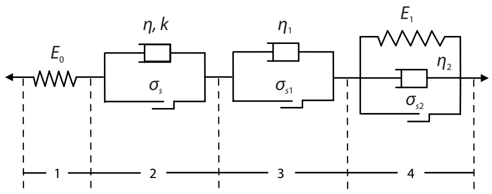

Figure 2. The MSSB-NVPB rheological model for concrete mixed with fly ash; E_0 and E_1 are the elastic moduli, η, η_1, and η_2 – the viscosity coefficients, σs_1 and σs_2 – the ultimate strengths of the corresponding models, and σ_s – the long-term strength

The second part of fig. 2 is the NVPB model, which is described in detail in the literature [20]. The creep equation of the model:

$$\varepsilon(t) = \frac{H(\sigma_0 - \sigma_s)}{\eta}\frac{t^k}{t_0^{k-1}} = \frac{H(\sigma_0 - \sigma_s)}{\eta}t^k \tag{1}$$

where k is the rheological index, which reflects the speed of the accelerated rheological rate of the rock, t_0 – the reference time which is set to 1, and σ_s – the yield stress or the long-term strength. The expression for $H(\sigma_0 - \sigma s)$:

$$H\left(\sigma_0 - \sigma_s\right) = \begin{cases} 0 & \sigma_0 \le \sigma_s \\ \sigma_0 - \sigma_s & \sigma_0 > \sigma_s \end{cases} \tag{2}$$

Clearly, when $\sigma < \sigma_s$, the MSSB-NVPB model reverts to the MSSB model. In that case, the model can be used to describe the elastic-viscoplastic-viscoelastic-plastic rheological properties of concrete. When $\sigma > \sigma_s$, the MSSB–NVPB model can be used to describe accelerating creep properties.

Constitutive equation

When $\sigma < \sigma_{s1}$, σ_{s2}, the first part of the MSSB–NVPB model describes the elastic properties with the constitutive equation, given:

$$\sigma = E_0\varepsilon \tag{3}$$

When $\sigma_{s1} \leq \sigma_{s2}$ and $\sigma_{s1} \leq \sigma \leq \sigma_{s2}$, the first and third parts of the MSSB-NVPB model gives the constitutive equation:

$$\dot{\varepsilon}=\frac{\dot{\sigma}}{E_0}+\frac{\sigma\text{-}\sigma_{\text{s1}}}{\eta_1} \tag{4}$$

When $\sigma_{s2} \leq \sigma_{s1}$ and $\sigma_{s2} \leq \sigma \leq \sigma_{s1}$, the first and fourth parts of the MSSB–NVPB model exists, and the constitutive equation can be written:

$$\dot{\varepsilon}+\frac{E_1}{\eta_2}\varepsilon=\frac{\dot{\sigma}}{E_0}+\frac{\sigma-\sigma_{\text{s2}}}{\eta_2}+\frac{E_1\sigma}{E_0\eta_2} \tag{5}$$

When $\sigma \geq \sigma_{s1}$, σ_{s2}, the first, third and fourth parts of the MSSB–NVPB model exists, and the constitutive equation is given:

$$\ddot{\varepsilon}+\frac{E_1}{\eta_2}\dot{\varepsilon}=\frac{\ddot{\sigma}}{E_0}+\left(\frac{E_1}{E_0\eta_2}+\frac{1}{\eta_1}+\frac{1}{\eta_2}\right)\dot{\sigma}+\frac{E_1\left(\sigma-\sigma_{s1}\right)}{\eta_1\eta_2} \tag{6}$$

When $\sigma \geq \sigma_{s1}$, σ_{s2}, σ_s, the all parts of the MSSB-NVPB model exists, and the constitutive equation is suggested:

$$\ddot{\varepsilon}+\frac{E_1\dot{\varepsilon}}{\eta_2}=\frac{\ddot{\sigma}}{E_0}+\left(\frac{kt^{k-1}}{\eta}+\frac{\eta_1+\eta_2}{\eta_1\eta_2}+\frac{E_1}{E_0\eta_2}\right)\dot{\sigma}+\frac{k\left(k-1\right)t^{k-2}\left(\sigma-\sigma_{\text{s}}\right)}{\eta}+$$
$$+\frac{E_1kt^{k-1}\left(\sigma-\sigma_{\text{s}}\right)}{\eta_1\eta_2}+\frac{E_1\left(\sigma-\sigma_{\text{s1}}\right)}{\eta\eta_2} \tag{7}$$

Creep equation

When $\sigma < \sigma_{s1}$, σ_{s2}, $\sigma = \sigma_0$, the model describes elastic properties of fly ash concrete for low level stress conditions without creep properties.

When $\sigma_{s1} \leq \sigma_{s2}$, $\sigma_{s1} \leq \sigma \leq \sigma_{s2}$, and $\sigma = \sigma_0$, the following creep equation which can characterize the constant creep property of concrete can be given [19]:

$$\varepsilon=\frac{\sigma_0}{E_0}+\frac{\sigma_0-\sigma_{\text{s1}}}{\eta_1}t \tag{8}$$

When $\sigma_{s2} \leq \sigma_{s1}$, $\sigma_{s2} \leq \sigma \leq \sigma_{s1}$, and $\sigma - \sigma_0$, the following creep equation can be written:

$$\varepsilon=\frac{\sigma_0}{E_0}+\frac{\sigma_0-\sigma_{\text{s1}}}{\eta_1}t+\frac{1}{E_1}\left(\sigma_0-\sigma_{\text{s2}}\right)\left(1-\text{e}^{-\frac{E_1}{\eta_2}t}\right) \tag{9}$$

Taking the first and second time derivatives on both sides of eq. (9) yields:

$$\dot{\varepsilon}=\frac{\left(\sigma_0-\sigma_{\text{s2}}\right)}{\eta_2}\text{e}^{-\frac{E_1}{\eta_2}t},\ \ \ddot{\varepsilon}=-\frac{\left(\sigma_0-\sigma_{\text{s2}}\right)E_1}{\eta_2^2}\text{e}^{-\frac{E_1}{\eta_2}t} \tag{10}$$

It can be obtained that $\dot{\varepsilon} > 0$, $\ddot{\varepsilon} < 0$, from eq. (10), and the model can characterize the decelerated creep property of the concrete.

When $\sigma \geq \sigma_{s1}$, σ_{s2}, and $\sigma = \sigma_0$, the following creep equation can be represented:

$$\varepsilon = \frac{\sigma_0}{E_0} + \frac{\sigma_0 - \sigma_{s1}}{\eta_1} t + \frac{1}{E_1}\left(\sigma_0 - \sigma_{s2}\right)\left(1 - e^{-\frac{E_1}{\eta_2}t}\right) \tag{11}$$

Taking the first and second time derivatives on both sides of eq. (11) yields:

$$\dot{\varepsilon} = \frac{\sigma_0 - \sigma_{s1}}{\eta_1} + \frac{\left(\sigma_0 - \sigma_{s2}\right)}{\eta_2} e^{-\frac{E_1}{\eta_2}t}, \quad \ddot{\varepsilon} = -\frac{\left(\sigma_0 - \sigma_{s2}\right)E_1}{\eta_2^2} e^{-\frac{E_1}{\eta_2}t} \tag{12}$$

It can be obtained that $\dot{\varepsilon} > 0$, $\ddot{\varepsilon} < 0$, from eq. (12). The model can characterize the decelerating creep and constant creep properties of concrete.

When $\sigma \geq \sigma_{s1}$, σ_{s2}, σ_s, and $\sigma = \sigma_0$, the following creep equation can be obtained from the eight-element non-linear viscoelastic-plastic MSSB–NVPB model according to the superposition principle:

$$\varepsilon = \frac{\sigma_0}{E_0} + \frac{\sigma_0 - \sigma_{s1}}{\eta_1} t + \frac{1}{E_1}\left(\sigma_0 - \sigma_{s2}\right)\left(1 - e^{-\frac{E_1}{\eta_2}t}\right) + \frac{\sigma_0 - \sigma_s}{\eta} t^k \tag{13}$$

Taking the first and second time derivatives on both sides of eq. (13) yields:

$$\dot{\varepsilon} = \frac{\sigma_0 - \sigma_{s1}}{\eta_1} + \frac{\left(\sigma_0 - \sigma_{s2}\right)}{\eta_2} e^{-\frac{E_1}{\eta_2}t} + \frac{k(\sigma_0 - \sigma_s)}{\eta} t^{k-1} \tag{14}$$

$$\ddot{\varepsilon} = -\frac{\left(\sigma_0 - \sigma_{s2}\right)E_1}{\eta_2^2} e^{-\frac{E_1}{\eta_2}t} + \frac{k\left(k-1\right)(\sigma_0 - \sigma_s)}{\eta} t^{k-2} \tag{15}$$

When $k \leq 1$, $\dot{\varepsilon} > 0$ and $\ddot{\varepsilon} < 0$, the model can characterize the instantaneous elastic strain, decelerated creep, and constant creep properties of concrete. When $k > 1$, if $\dot{\varepsilon} > 0$ but $\ddot{\varepsilon} < 0$, or if $\ddot{\varepsilon} = 0$, or if $\ddot{\varepsilon} > 0$, the model can characterize the decelerating creep, accelerating creep, and constant creep properties of concrete.

Parameter determination and verification of MSSB-NVPB model

Parameters E_0, E_1, η_2, and σ_{s2} are determined using the following eq. [21]:

$$E_0 = \frac{\sigma_0}{\varepsilon_0}, \quad E_1 = \frac{\sigma_0}{\varepsilon_{cp}(\infty) + \varepsilon_{cir}(\infty)}, \quad \sigma_{s2} = E_1 \varepsilon_{cir}(\infty), \quad \eta_2 = -\frac{E_1 t}{\ln\left[1 - \frac{\varepsilon_{cp}(t)}{\varepsilon_{cp}(\infty)}\right]} \tag{16}$$

where σ_0 is the initial stress, ε_0 – the initial strain created after applying the initial stress, $\varepsilon_{cp}(\infty)$ – the final creep at a given stress level after stabilization during the creep phase, $\varepsilon_{cir}(\infty)$ – the strain value at a given stress level that does not recover after unloading, $\varepsilon_{cp}(t)$ – the creep value at a given point on the creep curve, t – the creep time, ($\varepsilon_c(t)$, t) – a given point on the creep curve, and the model parameter η_2 is the mean value calculated from each data point during the creep phase.

Parameters η, σ_s, and k are used for fitting and determining accelerating creep phase data for concrete, and they are found using non-linear regression methods. The parameters η_1 and σ_{s1} – the determined from the following formula [21]:

$$\eta_1 = \frac{\sigma_{02} - \sigma_{01}}{\dot{\varepsilon}_2 - \dot{\varepsilon}_1} \qquad \sigma_{s1} = \frac{\dot{\varepsilon}_2 \sigma_{01} - \dot{\varepsilon}_1 \sigma_{02}}{\dot{\varepsilon}_2 - \dot{\varepsilon}_1} \tag{17}$$

where σ_{01} and σ_{02} are the stress values corresponding to two stress levels, and ε_1, ε_2 – the creep velocities corresponding to the two stress levels during the constant velocity creep phase.

If constant creep data for the two stress levels are unavailable, the corresponding solutions for parameters E_0, E_1, η_2, σ_{s2}, η_1, and σ_{s1} are also found using non-linear regression analysis methods.

The parameters of the MSSB-NVPB model were determined using the methods described above. The results are shown in tab. 3. When the fly ash content m = 0%, η_1 = 1.15 GPa·s, σ_{s1} = 76.90 MPa, η = 1.03 GPa·s, σ_s = 9.18 MPa, and k = 1. Figure 3 shows the theoretical calculated values and experimental values for creep in concrete with different fly ash content. It can be seen that the model can describe creep in concrete rather well. The analysis showed that difference between the model parameter obtained through non-linear regression methods (σ_0 = 66.0 MPa) and the parameter calculated from theoretical value is rather large, which is worth noting in future studies.

Table 3. Parameters of MSSB-NVPB rheological model

m [%]	σ_0 [MPa]	E_0 [GPa]	E_1 [GPa]	η_2 [kGPa·s]	σ_2 [MPa]
35.00	30.00	50.61	849.88	324.33	12.00
	42.00	52.00	793.25	221.62	16.80
	54.00	50.77	764.92	217.49	18.90
	66.00	49.59	373.96	53.75	27.72
Mean	–	50.74	695.50	204.30	18.90
50.00	25.00	37.97	1137.76	379.64	8.33
	35.00	40.36	1061.86	221.62	7.78
	45.00	39.89	682.64	194.27	17.50
	55.0	39.69	341.32	116.41	22.50
Mean	–	39.50	805.90	228.00	14.03

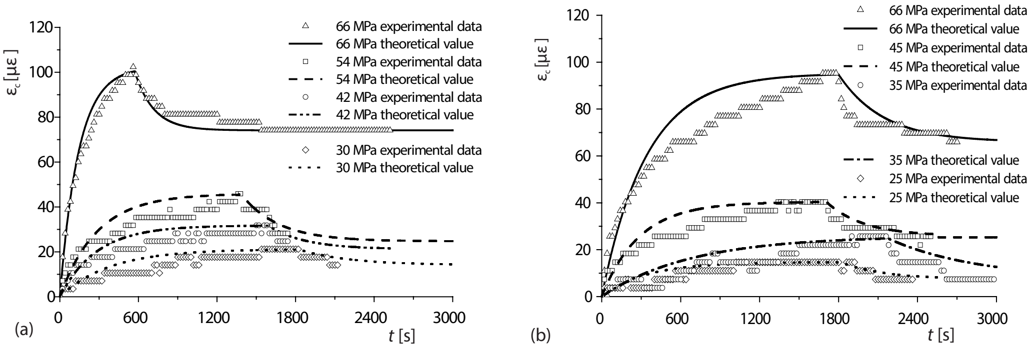

Figure 3. Comparison between theory and experimental results of concrete creep for different stress levels; (a) *m* = 35% (b) *m* = 50%

Conclusion

In this study, we explored the creep mechanical properties of concrete with 35%, and 50% fly ash content values. It is shown that the low stress levels fly ash concrete did not have elastic after-effects. With these parameters, the model yielded theoretical values for concrete with different fly ash contents at various load values. It was found that the model could characterize the creep properties of fly ash concrete at different stress levels fairly well.

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