BILINEARIZATION AND FRACTIONAL SOLITON DYNAMICS OF FRACTIONAL KADOMTSEV-PETVIASHVILI EQUATION

by

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Kadomtsev-Petviashvili equation is a mathematical model with many important applications in fluids. In this paper, a local fractional Kadomtsev-Petviashvili equation with Lax integrability is derived and solved by extending Hirota's bilinear method. More specifically, the local fractional Kadomtsev-Petviashvili equation is derived from a local fractional Lax equation. With the help of a suitable transformation, the local fractional Kadomtsev-Petviashvili equation is then bilinearized. Based on the bilinearized form, n-soliton solution with Mittag-Leffler functions is obtained. In order to gain more insights into the fractional n-soliton solution, the velocity of the fractional one-soliton solution is simulated. It is shown that the velocity of the fractional one-soliton changes with the fractional order.

Key words: *local fractional Kadomtsev-Petviashvili equation, n-soliton solution, fractional soliton dynamics, Hirtoa's bilinear method, Mittag-Leffler function*

Introduction

With the development of fractional calculus and its applications, dynamical processes and dynamical systems of fractional orders have attracted much attentions. Fujioka *et al.* [1] investigated fractional optical solitons by means of an extended non-linear Schrodinger equation with fractional dispersion term and fractional non-linearity term. Yang *et al.* [2] modeled fractal waves on shallow water surfaces by introducing a local fractional Korteweg-de Vries (KdV) equation.

Hirota's [3] bilinear method is a famous analytical method for constructing exact and explicit n-soliton solutions of non-linear PDE. Since put forward in 1970, Hirota's bilinear method has achieved considerable developments [4-14]. With the close attentions of fractional calculus and its applications [15-24], some of the natural questions are whether Hirota's bilinear method can be extended to non-linear PDE of fractional orders and what about the fractional soliton dynamics and integrability of fractional PDE. As far as we know there is no research reports on the bilinear method for non-linear PDE of fractional orders. This paper is motivated by the desire to extend the bilinear method to non-linear fractional PDE and then gain more insights into the fractional soliton dynamics of the obtained n-soliton solution. For such a purpose, we consider the following local fractional Kadomtsev-Petviashvili (LFKP) equation:

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$$\mathrm{D}_x^{(\alpha)}\left[\mathrm{D}_t^{(\alpha)}u+6u\mathrm{D}_x^{(\alpha)}u+\mathrm{D}_x^{(3\alpha)}u\right]+3\sigma^2\mathrm{D}_y^{(2\alpha)}u=0,\ 0<\alpha\le 1 \tag{1}$$

which is a generalization of the Kadomtsev-Petviashvili (KP) equation $(u_t + 6uu_x + u_{xxx})_x + 3\sigma^2u_{yy} = 0$ – the mathematical model with many important applications in fluids. In eq. (1), the local time-fractional derivative $\mathrm{D}_t^\alpha u$ at the point $t = t_0$ is defined [15]:

$$\mathrm{D}_t^{(\alpha)}u(x,t_0)=\left.\frac{\partial^\alpha u(x,t)}{\partial t^\alpha}\right|_{t=t_0}=\lim_{t\to t_0}\frac{\Delta^\alpha[u(x,t)-u(x,t_0)]}{(t-t_0)^\alpha} \tag{2}$$

where

$$\Delta^\alpha[u(x,t)-u(x,t_0)]\cong\Gamma(1+\alpha)[u(x,t)-u(x,t_0)]$$

some graceful properties [15] of the local fractional derivative have been used in this paper.

Derivation of the LFKP equation

For the LFKP eq. (1), we have the following *Theorem 1*.

Theorem 1. The LFKP eq. (1) has Lax integrability, which can be derived from the following Lax equation with local fractional derivatives:

$$\mathrm{D}_t^{(\alpha)}L+LA-AL=0 \tag{3}$$

where

$$L=\sigma\mathrm{D}_y^{(\alpha)}+\mathrm{D}_x^{(2\alpha)}+u(x,y,t),\ A=-4\mathrm{D}_x^{(3\alpha)}-6u\mathrm{D}_x^{(\alpha)}-3\mathrm{D}_x^{(\alpha)}u+3\sigma[{}_0I_x^{(\alpha)}\mathrm{D}_y^{(\alpha)}u],\ \sigma=\text{const} \tag{4}$$

and the local fractional integral of $u(\mu)$ of order $\alpha(0 < \alpha \le 1)$ is defined [15]:

$${}_aI_b^{(\alpha)}u(\mu)=\frac{1}{\Gamma(1+\alpha)}\int_a^b u(\mu)(\mathrm{d}\mu)^\alpha=\frac{1}{\Gamma(1+\alpha)}\lim_{\Delta\mu_k\to 0}\sum_{k=0}^{N-1}u(\mu_k)(\Delta\mu_k)^\alpha \tag{5}$$

where

$$\Delta\mu_k=\mu_{k+1}-\mu_k\ \text{ with }\ \mu_0=a<\mu_1<\cdots<\mu_{N-1}=b$$

Proof. A direct computation gives:

$$\mathrm{D}_t^{(\alpha)}L=\mathrm{D}_t^{(\alpha)}u \tag{6}$$

$$\begin{aligned}LA={}&-4\mathrm{D}_x^{(5\alpha)}-6u\mathrm{D}_x^{(3\alpha)}-3\mathrm{D}_x^{(\alpha)}u\mathrm{D}_x^{(2\alpha)}-u[4\mathrm{D}_x^{(3\alpha)}+6u\mathrm{D}_x^{(\alpha)}+3\mathrm{D}_x^{(\alpha)}u]-\\&-12\mathrm{D}_x^{(\alpha)}u\mathrm{D}_x^{(2\alpha)}-6\mathrm{D}_x^{(2\alpha)}\mathrm{D}_x^{(\alpha)}-6\mathrm{D}_x^{(2\alpha)}u-3\mathrm{D}_x^{(3\alpha)}u+\\&+\left\{-4\mathrm{D}_x^{(3\alpha)}-6u\mathrm{D}_x^{(\alpha)}-3\mathrm{D}_x^{(\alpha)}u+3\sigma\left[{}_0I_x^{(\alpha)}\mathrm{D}_y^{(\alpha)}u)\right]\right\}\sigma\mathrm{D}_y^{(\alpha)}+\\&+3\sigma\left[{}_0I_x^{(\alpha)}\mathrm{D}_y^{(\alpha)}u\right]\left[\mathrm{D}_x^{(2\alpha)}+u\right]+3\sigma^2\left[{}_0I_x^{(\alpha)}\mathrm{D}_y^{(2\alpha)}u\right]-3\sigma\mathrm{D}_x^{(\alpha)}\mathrm{D}_y^{(\alpha)}u-\\&-6\sigma\mathrm{D}_y^{(\alpha)}u\mathrm{D}_x^{(\alpha)}+6\sigma\mathrm{D}_y^{(\alpha)}u\mathrm{D}_x^{(\alpha)}+3\sigma\mathrm{D}_x^{(\alpha)}\mathrm{D}_y^{(\alpha)}\end{aligned} \tag{7}$$

$$\begin{aligned}AL={}&-4\mathrm{D}_x^{(5\alpha)}-6u\mathrm{D}_x^{(3\alpha)}-3\mathrm{D}_x^{(\alpha)}u\mathrm{D}_x^{(2\alpha)}-u\left[4\mathrm{D}_x^{(3\alpha)}+6u\mathrm{D}_x^{(\alpha)}+3\mathrm{D}_x^{(\alpha)}u\right]-\\&-12\mathrm{D}_x^{(\alpha)}u\mathrm{D}_x^{(2\alpha)}-12\mathrm{D}_x^{(2\alpha)}u\mathrm{D}_x^{(\alpha)}-4\mathrm{D}_x^{(3\alpha)}u-6u\mathrm{D}_x^{(\alpha)}u+\\&+\left\{-4\mathrm{D}_x^{(3\alpha)}-6u\mathrm{D}_x^{(\alpha)}-3\mathrm{D}_x^{(\alpha)}u+3\sigma\left[{}_0I_x^{(\alpha)}\mathrm{D}_y^{(\alpha)}u\right]\right\}\sigma\mathrm{D}_y^{(\alpha)}+\\&+3\sigma\left[{}_0I_x^{(\alpha)}\mathrm{D}_y^{(2\alpha)}u\right]\left[\mathrm{D}_x^{(2\alpha)}+u\right]\end{aligned} \tag{8}$$

Substituting eqs. (6)-(8) into eq. (3), we have:

$$\mathrm{D}_t^{(\alpha)}u+6u\mathrm{D}_x^{(\alpha)}u+\mathrm{D}_x^{(3\alpha)}u+3\sigma^2\left[{}_0I_x^{(\alpha)}\mathrm{D}_y^{(2\alpha)}u\right]=0 \tag{9}$$

Taking the local fractional derivative of eq. (9) with respect to x, we arrive at eq. (1). Thus, we finish the proof of *Theorem 1*. The process of proof shows that eq. (1) has Lax integrability.

Local fractional bilinearization and *n*-soliton solution

For the bilinearization of LFKP eq. (1), we have the following *Theorem 2*:

Theorem 2. By the transformation:

$$u=2\mathrm{D}_x^{(2\alpha)}\ln f(x,y,t) \tag{10}$$

the LFKP eq. (1) can be bilinearized:

$$\left(\mathbf{D}_t^{(\alpha)}\mathbf{D}_x^{(\alpha)}+\mathbf{D}_x^{(4\alpha)}+3\sigma^2\mathbf{D}_y^{(2\alpha)}\right)ff=0 \tag{11}$$

where $\mathbf{D}_x^{(\alpha)}$, $\mathbf{D}_y^{(\alpha)}$, and $\mathbf{D}_t^{(\alpha)}$ are local fractional Hirota's bilinear operator:

$$\begin{gathered}\mathbf{D}_x^{(m\alpha)}\mathbf{D}_y^{(n\alpha)}\mathbf{D}_t^{(q\alpha)}fg=\left[\mathrm{D}_x^{(\alpha)}-\mathrm{D}_{x'}^{(\alpha)}\right]^m\left[\mathrm{D}_y^{(\alpha)}-\mathrm{D}_{y'}^{(\alpha)}\right]^n\left[\mathrm{D}_t^{(\alpha)}-\mathrm{D}_{t'}^{(\alpha)}\right]^q\\ \cdot f(x,y,t)g(x',y',t')|_{x'=x,y'=y,t'=t}\end{gathered} \tag{12}$$

Proof. Directly substituting eq. (10) into eq. (1), and using the properties [15] of local fractional derivatives and local fractional integrals, we transform eq. (1):

$$\mathrm{D}_t^{(\alpha)}\left[\frac{\mathrm{D}_x^{(\alpha)}f}{f}\right]+6\left\{\mathrm{D}_x^{(\alpha)}\left[\frac{\mathrm{D}_x^{(\alpha)}f}{f}\right]\right\}^2+\mathrm{D}_x^{(3\alpha)}\left[\frac{\mathrm{D}_x^{(\alpha)}f}{f}\right]+3\sigma^2\mathrm{D}_y^{(\alpha)}\left[\frac{\mathrm{D}_y^{(\alpha)}f}{f}\right]=0 \tag{13}$$

It is easy to obtain:

$$\mathrm{D}_t^{(\alpha)}\left[\frac{\mathrm{D}_x^{(\alpha)}f}{f}\right]=\frac{\mathrm{D}_t^{(\alpha)}[\mathrm{D}_x^{(\alpha)}f]}{f}-\frac{[\mathrm{D}_x^{(\alpha)}f]\mathrm{D}_t^{(\alpha)}f}{f^2} \tag{14}$$

$$\mathrm{D}_x^{(\alpha)}\left[\frac{\mathrm{D}_x^{(\alpha)}f}{f}\right]=\frac{\mathrm{D}_x^{(2\alpha)}f}{f}-\frac{[\mathrm{D}_x^{(\alpha)}f]^2}{f^2} \tag{15}$$

$$\mathrm{D}_x^{(3\alpha)}\frac{\mathrm{D}_x^{(\alpha)}f}{f}=\frac{\mathrm{D}_x^{(4\alpha)}f}{f}-\frac{4[\mathrm{D}_x^{(3\alpha)}f][\mathrm{D}_x^{(\alpha)}f]}{f^2}-\frac{3[\mathrm{D}_x^{(2\alpha)}f]^2}{f^2}+\frac{12[\mathrm{D}_x^{(2\alpha)}f][\mathrm{D}_x^{(\alpha)}f]^2}{f^3}-\frac{6[\mathrm{D}_x^{(\alpha)}f]^3}{f^4} \tag{16}$$

$$\mathrm{D}_y^{(\alpha)}\left[\frac{\mathrm{D}_y^{(\alpha)}f}{f}\right]=\frac{\mathrm{D}_y^{(2\alpha)}f}{f}-\frac{[\mathrm{D}_y^{(\alpha)}f]^2}{f^2} \tag{17}$$

We substitute eqs. (14)-(17) into eq. (13) and eliminate the denominator f^2, eq. (13) is reduced:

$$\begin{gathered}\mathrm{D}_t^{(\alpha)}[\mathrm{D}_x^{(\alpha)}f]-[\mathrm{D}_t^{(\alpha)}f]\mathrm{D}_x^{(\alpha)}f+f[\mathrm{D}_x^{(4\alpha)}f]-4[\mathrm{D}_x^{(\alpha)}f]\mathrm{D}_x^{(3\alpha)}f+3[\mathrm{D}_x^{(2\alpha)}f]^2+\\ +3\sigma^2\left\{[f[\mathrm{D}_y^{(2\alpha)}f]-[\mathrm{D}_y^{(\alpha)}f]^2\right\}=0\end{gathered} \tag{18}$$

With the help of the bilinearized form of the local fractional Hirota's bilinear operator (12), we have:

$$\mathrm{D}_t^{(\alpha)}[\mathrm{D}_x^{(\alpha)}f]-[\mathrm{D}_t^{(\alpha)}f]\mathrm{D}_x^{(\alpha)}f=\mathbf{D}_t^{(\alpha)}\mathbf{D}_x^{(\alpha)}ff \tag{19}$$

$$f[\mathrm{D}_x^{(4\alpha)}f]-4[\mathrm{D}_x^{(\alpha)}f]\mathrm{D}_x^{(3\alpha)}f+3[\mathrm{D}_x^{(2\alpha)}f]^2=\mathbf{D}_x^{(4\alpha)}f\,f \tag{20}$$

$$f[\mathrm{D}_y^{(2\alpha)}f]-[\mathrm{D}_y^{(\alpha)}f]^2=[\mathbf{D}_y^{(2\alpha)}]f\,f \tag{21}$$

By which we can rewrite eq. (18) as eq. (11). The proof of *Theorem 2* is then finished.

For the n-soliton solution of LFKP eq. (1), we have the following *Theorem 3*:

Theorem 3. The LFKP eq. (1) has the following fractional n-soliton solution:

$$u=2\mathrm{D}_x^{(2\alpha)}\ln\left[\sum_{\mu=0,1}E_\alpha\left(\sum_{j=1}^{n}\mu_j\xi_j+\sum_{1\le j<l}^{n}\mu_j\mu_l A_{jl}\right)\right] \tag{22}$$

where the summation $\sum_{\mu=0,1}$ refers to all possible combination of each $\mu_j = 0, 1$ for $j = 1, 2,...n$, $E_\alpha(\cdot)$ is the Mittag-Leffler function [15]:

$$\xi_j^\alpha=k_j(x+p_jy+\omega_jt)+\xi_j^0,\ \omega_j^\alpha=-k_j^{2\alpha}-3\sigma^2p_j^{2\alpha},\ \xi_j^0=\text{const.} \tag{23}$$

$$E_\alpha(A_{jl}^\alpha)=\frac{(k_j^\alpha-k_l^\alpha)^2-\sigma^2(p_j^\alpha-p_l^\alpha)^2}{(k_j^\alpha+k_l^\alpha)^2-\sigma^2(p_j^\alpha-p_l^\alpha)^2} \tag{24}$$

Proof. We suppose that:

$$f(x,y,t)=\sum_{i=0}^{\infty}\varepsilon^i f_i\,(f_0=1) \tag{25}$$

where ε is embedded parameter and f_i are undetermined functions of x, y, and t.

Substituting eq. (25) into eq. (11) and then collecting all the coefficients with same order of ε, we derive a system of local fractional PDE (LFPDE):

$$\mathrm{D}_x^{(\alpha)}\mathrm{D}_t^{(\alpha)}f_1+\mathrm{D}_x^{(4\alpha)}f_1+3\sigma^2\mathrm{D}_y^{(2\alpha)}f_1=0 \tag{26}$$

$$2[\mathrm{D}_x^{(\alpha)}\mathrm{D}_t^{(\alpha)}f_2+\mathrm{D}_x^{(4\alpha)}f_2+3\sigma^2\mathrm{D}_y^{(2\alpha)}]f_2=-[\mathbf{D}_t^{(\alpha)}\mathbf{D}_x^{(\alpha)}+\mathbf{D}_x^{(4\alpha)}+3\sigma^2\mathbf{D}_{yy}^{(2\alpha)}]f_1f_1 \tag{27}$$

$$\mathrm{D}_x^{(\alpha)}\mathrm{D}_t^{(\alpha)}f_3+\mathrm{D}_x^{(4\alpha)}f_3+3\sigma^2\mathrm{D}_y^{(2\alpha)}f_3=-[\mathbf{D}_t^{(\alpha)}\mathbf{D}_x^{(\alpha)}+\mathbf{D}_x^{(4\alpha)}+3\sigma^2\mathbf{D}_{yy}^{(2\alpha)}]f_1f_2 \tag{28}$$

and so forth.

For the fractional one-soliton solution, we suppose:

$$f_1=E_\alpha(\xi_1^\alpha),\ \xi_1^\alpha=k_1(x+p_1y+\omega_1t)+\xi_1^0,\ \omega_1^\alpha=-k_1^{2\alpha}-3\sigma^2p_1^{2\alpha},\ \xi_1^0=\text{const.} \tag{29}$$

Substituting eq. (29) into above LFPDE shows that $f_2 = f_3 = ... = 0$. Setting $\varepsilon = 1$, we have:

$$f=1+E_\alpha(\xi_1^\alpha) \tag{30}$$

and obtain the fractional one-soliton solution of eq. (1):

$$u=2\mathrm{D}_x^{(2\alpha)}\ln[1+E_\alpha(\xi_1^\alpha)]=\frac{k_1^{2\alpha}}{2}\mathrm{sec}h_\alpha^2\frac{\xi_1^\alpha}{2} \tag{31}$$

where $\mathrm{sec}\,h_\alpha(\cdot)$ is the generalized hyperbolic secant function [15].

To construct the fractional two-soliton solution, we suppose:

$$f_1=E_\alpha(\xi_1^\alpha)+E_\alpha(\xi_2^\alpha),\ \xi_2^\alpha=k_2(x+p_2y+\omega_2t)+\xi_2^0,\ \omega_2^\alpha=-k_2^{2\alpha}-3\sigma^2p_2^{2\alpha},\ \xi_2^0=\text{const.} \tag{32}$$

and substitute eq. (32) into above LFPDE. Solving the LFPDE yields:

$$f_2 = E_\alpha(\xi_1^\alpha + \xi_2^\alpha + A_{12}^\alpha) \tag{33}$$

We therefore, obtain the fractional two-soliton solution of eq. (1):

$$u = 2\mathrm{D}_x^{(2\alpha)} \ln[1 + E_\alpha(\xi_1^\alpha) + E_\alpha(\xi_2^\alpha) + E_\alpha(\xi_1^\alpha + \xi_2^\alpha + A_{12}^\alpha)] \tag{34}$$

where

$$E_\alpha(A_{12}^\alpha) = \frac{(k_1^\alpha - k_2^\alpha)^2 - \sigma^2(p_1^\alpha - p_2^\alpha)^2}{(k_1^\alpha + k_2^\alpha)^2 - \sigma^2(p_1^\alpha - p_2^\alpha)^2} \tag{35}$$

Similarly, for the fractional three-soliton solution, we suppose:

$$f_1 = E_\alpha(\xi_1^\alpha) + E_\alpha(\xi_2^\alpha) + E_\alpha(\xi_3^\alpha),\ \xi_3^\alpha = \frac{k_3(x + p_3 y + \omega_3 t) + \xi_3^0}{\Gamma(1+\alpha)},$$
$$\omega_3^\alpha = -k_3^{2\alpha} - 3\sigma^2 p_3^{2\alpha},\ \xi_3^0 = \text{const.} \tag{36}$$

and have

$$f_2 = E_\alpha(\xi_1^\alpha + \xi_2^\alpha + A_{12}^\alpha) + E_\alpha(\xi_1^\alpha + \xi_3^\alpha + A_{13}^\alpha) + E_\alpha(\xi_2^\alpha + \xi_3^\alpha + A_{23}^\alpha) +$$
$$+ E_\alpha(\xi_1^\alpha + \xi_2^\alpha + \xi_3^\alpha + A_{12}^\alpha + A_{13}^\alpha + A_{23}^\alpha) \tag{37}$$

Thus we obtain the fractional three-soliton solution of eq. (1):

$$u = 2D_x^{(2\alpha)} \ln[1 + E_\alpha(\xi_1^\alpha) + E_\alpha(\xi_2^\alpha) + E_\alpha(\xi_3^\alpha) + E_\alpha(\xi_1^\alpha + \xi_2^\alpha + A_{12}^\alpha) + E_\alpha(\xi_1^\alpha + \xi_3^\alpha + A_{13}^\alpha) +$$
$$+ E_\alpha(\xi_2^\alpha + \xi_3^\alpha + A_{23}^\alpha) + E_\alpha(\xi_1^\alpha + \xi_2^\alpha + \xi_3^\alpha + A_{12}^\alpha + A_{13}^\alpha + A_{23}^\alpha)] \tag{38}$$

where

$$E_\alpha(A_{13}^\alpha) = \frac{(k_1^\alpha - k_3^\alpha)^2 - \sigma^2(p_1^\alpha - p_3^\alpha)^2}{(k_1^\alpha + k_3^\alpha)^2 - \sigma^2(p_1^\alpha - p_3^\alpha)^2},\ E_\alpha(A_{23}^\alpha) = \frac{(k_2^\alpha - k_3^\alpha)^2 - \sigma^2(p_2^\alpha - p_3^\alpha)^2}{(k_2^\alpha + k_3^\alpha)^2 - \sigma^2(p_2^\alpha - p_3^\alpha)^2} \tag{39}$$

By induction, we can finally reach the fractional n-soliton solution determined by eqs. (22)-(24) of eq. (1). Thus the proof of *Theorem 3* is end.

Fractional soliton dynamics

In order to gain more insights into the soliton dynamics of the obtained fractional n-soliton solution (22), we consider the cases of $n = 1$.

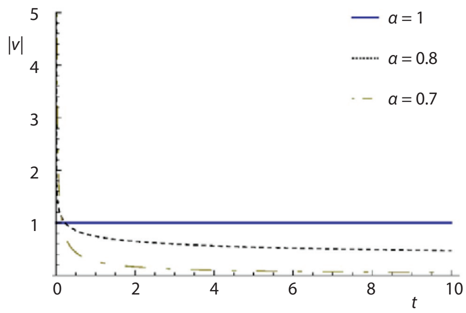

Figure 1. Velocity image of fractional one-soliton determined by solution (31)

In fig. 1, we simulate the velocity of the fractional one-soliton solution (31) with different values of α, propagating along the positive direction of x-axis. Here the parameters are selected as $\xi_1^0 = 0$, $k_1 = 1$, $p_1 = 1$, $y = 0$, and $\sigma^2 = -1$. With the help of the velocity image and the velocity expression $v = -(2)^{1/\alpha}$, we can see that the one-soliton has different velocities depending on the values of α. More specifically, the smaller the value of α is selected, the faster the soliton propagate.

Conclusion

In summary, we have derived and solved the Lax integrable LFKP eq. (1). This is due to Hirota's bilinear method extended to non-linear PDE with local fractional derivatives. To the best of our knowledge, combined with the Mittag-Leffler functions the obtained fractional n-soliton solution (22) and its special cases, the fractional one-soliton solution (31), two-soliton solution (34) and three-soliton solution (38), are all new, they have not been reported in literature. It is graphically shown that the fractional order of the LFKP eq. (1) influences the velocity of the fractional one-soliton solution (31) with Mittag-Leffler function in the process of propagations. More importantly, the fractional scheme of the bilinear method presented in this paper for constructing n-soliton solution of the LFKP eq. (1) can be extended to some other integrable local fractional PDE. In soliton theory, there are many other analytical methods such as those in [25-29] for solving non-linear PDE with integer order derivatives. How to extend these analytical methods to non-linear local fractional PDE is worthy of study.

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Nomenclature

k_j, p_j – constant, [–]
m, n, q, j, l – positive integer, [–]
t – time co-ordinate, [s]
x, y – space co-ordinates, [m]
$\partial^\alpha/\partial t^\alpha$ – local fractional partial derivative, [–]

Greek symbol

α – fractional order, [–]

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