THE MOTION OF A NON-NEWTONIAN NANOFLUID OVER A SEMI-INFINITE MOVING VERTICAL PLATE THROUGH POROUS MEDIUM WITH HEAT AND MASS TRANSFER

by

Nabil T. M. El-DABE[a], Mahmoud E. GABR[b],
Abd-Elhafez A. ELSHEKHIPY[c], and Sameh A. ZAHER[b]*

[a] Mathematics Department, Faculty of Education, Ain Shams University, Cairo, Egypt
[b] Mathematics Department, Faculty of Science, Zagazig University, Zagazig, Egypt
[c] Mathematics Department, Collage of Science, Imam Abdulrahman Bin Faisal University, Damma, Saudi Arabia

The motion of a non-Newtonian nanofluid over a semi-infinite moving vertical plate through porous medium stressed by an external uniform magnetic field with heat and mass transfer is investigated. The fluid under consideration obeys Eyring-Powell model. The effects of the physical parameters of the problem such as, permeability, chemical reaction as well as the fluid material parameters such as Hartmann number, Eckert number, and Reynolds number are discussed. The effects of external cooling (Gr > 0) of the plate by the free convection are considered. Graphical results are presented to highlight effects of various emerging parameters on velocity, temperature and concentration profiles.

Key words: *boundary-layer, moving surface, magnetic field, porous media, heat and mass transfer, nanofluids, Eyring-Powel model, chemical reaction, vertical plate*

Introduction

At the beginning of the 19th century, Maxwell introduced the basic concepts of dispersing solid particles in fluids to enhance its thermal conductivity. Maxwell was one of the first to analytically investigate conduction through suspended particles [1]. By the end of the 19th century nanotechnology is widely used in engineering industries. In petroleum engineering industries, for example, when oil and gas companies turn their attention new search areas. Nanotechnology can improve viscosity, density, specific gravity and any other physical properties of the drilling fluid to overcome any difficulties. Throughout any industrial facility, heat is added, removed, or moved from one process stream to another. These processes provide a source for energy recovery and fluid heating/cooling. Because of the rising development in modern technology, there is a need to develop new types of fluids that will be more effective in terms of heat exchange performance. Convectional heat transfer fluids, including oil, water, and ethylene glycol mixtures are poor heat transfer fluids, while the thermal conductivity of these fluids plays an important role in the heat transfer coefficient between the heat transfer medium and the heat transfer surface [2].The thermal conductivity of fluids can be enhanced by adding nanoparticles to the base fluids, [3-9].

---

* Corresponding author, e-mail: samehabdelzaher419@yahoo.com

1312

El-Dabe, N. T. M., *et al*.: The Motion of a Non-Newtonian Nanofluid over ...
THERMAL SCIENCE: Year 2020, Vol. 24, No. 2B pp. 1311-1321

In the last few decides many scientists and engineers studied the boundary-layer flow over semi-infinite porous plates [10-17] with concentration on the fluid chemical reactions with heat and mass transfer under the influence of uniform magnetic field. Rashidi *et al.* [18] investigated heat the transfer of a steady incompressible water based nanofluid-flow over a stretching sheet in the presence of transverse magnetic field with thermal radiation and buoyancy effects. Gupta *et al.* [19] studied the free convection of flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis [20] extended this problem to include mass transfer effects subjected to variable suction or injection. Raptis *et al.* [21] studied the free convection effects on flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of magnetic field. Soundalgekar *et al.* [22] analyzed the problem of free convection effects on Stokes problem for a vertical plate under the action of transversely applied magnetic field. The problem of steady laminar MHD mixed convection heat transfer about a vertical plate is solved numerically by Orhan Aydin and Ahmet Kaya [23] taking into account the effect of Ohmic heating and viscous dissipation. Tripathy *et al.* [24] considered chemical reaction impacts on free convection MHD fluid-flow past a moving vertical permeable plate. Turkyilmazoglu [25] studied Soret and Dufour effect on MHD flow with heat and mass transfer of an electrically conducting viscoelastic fluid past a vertical stretching surface in porous media.

Although different kinds of fluids are discussed by many researchers, other important fluids, such as thixotropic fluids, still needs more study. Thixotropic fluid is a fluid which takes a finite time to attain equilibrium viscosity when introduced to a steep change in shear rate. Eyring and Powell [26] introduced a mathematical model, known as Eyring-Powell model, explaining the behavior of these fluids. This mathematical model is used by many researchers. Hayat *et al.* [27] considered heat analysis of Eyring-Powell fluid-flow over a continuously moving surface with convective endpoint conditions. Reddy [28] studied the flow and heat transfer of Eyring Powell fluid over a continuously moving surface in the presence of a free stream velocity. Fluids satisfying Eyring-Powell model are studied by many researchers, [29-31]

In this paper we focus on studying the boundary-layer flow of a non-Newtonian nanofluid, byes Eyring-Powell model, over a semi-infinite moving vertical plate in the presence of external uniform magnetic. Using appropriate transformation the governing system of PDE of the problem is trans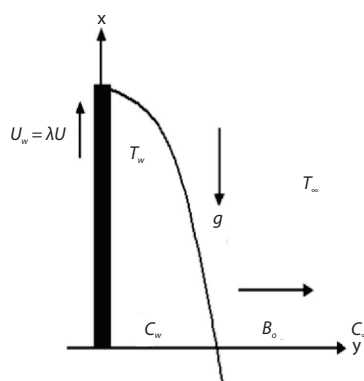formed to a system of ODE. The Mathematica Parametric NDSolve package is employed to obtain the results numerically. Moreover the effect of the variation of the permeability, chemical reaction, Hartmann number and Eckert number on velocity distribution, temperature distribution and concentration are obtained and presented in figures.

Figure 1. Physical model and co-ordinate system

Formulation of the problem

Consider 2-D steady boundary-layer flow of an incompressible nanofluid over a moving semi-infinite vertical flat plate. Chose the Cartesian co-ordinate system such that the x-axis is taken along the continuous surface direction and the y-axis is normal to it, as shown in fig. 1. Let the fluid be influenced by uniform magnetic field $B = (0, B_o, 0)$ normal to the moving surface.

Let U being the velocity of the uniform free stream, $U_w = \lambda U$ being the flat plate velocity, where λ is the plate velocity parameter. Assume that T_w and C_w are the values of the moving surface temperature and concentration, respectively while in the ambient fluid these values are T_∞ and C_∞.

In Eyring-Powell model the stress tensor is written [26]:

$$\tau_{ij} = \mu \frac{\partial v_i}{\partial x_j} + \frac{1}{\beta_1} \sinh^{-1}\left(\frac{1}{c} \frac{\partial v_i}{\partial x_j} \right)$$

(1)

$$\sinh^{-1}\left(\frac{1}{c} \frac{\partial v_i}{\partial x_j} \right) \simeq \frac{1}{c} \frac{\partial v_i}{\partial x_j} - \frac{1}{6}\left(\frac{1}{c} \frac{\partial v_i}{\partial x_j} \right)^3, \qquad \left| \frac{1}{c} \frac{\partial v_i}{\partial x_j} \right| << 1$$

(2)

where μ is the dynamic viscosity, β_1 and c are the characteristics of the Eyring-Powell model.

Taking in consideration the steady-state flow of the boundary-layer, the governing equations of the considered problem:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(3)

$$u\frac{\partial u}{\partial x} + v\left(\frac{\partial u}{\partial y} \right) = \left(v + \frac{1}{\beta_1 c \rho_f} \right)\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho_f}\left(\sigma B_o^2 + \frac{\mu}{k} \right)u -$$

$$- \frac{1}{2\beta_1 c^3 \rho_f}\left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} + g\beta\left(T - T_\infty \right) + g\beta^*\left(C - C_\infty \right)$$

(4)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y} \right)^2 \right] +$$

$$+ \left(\frac{\mu}{(\rho c)_f} + \frac{1}{c\beta_1 (\rho c)_f} \right)\left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma}{(\rho c)_f}B_o^2 u^2 - \frac{1}{6c^3 \beta_1 (\rho c)_f}\left(\frac{\partial u}{\partial y} \right)^4$$

(5)

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2} \right) + k_\circ(C - C_\infty)$$

(6)

The boundary conditions of the problem:

$$v = 0, \quad u = U_w = \lambda U, \quad T = T_w, \quad C = C_w \text{ at } y = 0,$$
$$u \rightarrow U, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \text{ at } y = \infty$$

(7)

where u and v are the velocity components along the x- and y-axis, respectively, v – the kinematic viscosity coefficient, α – the fluid thermal diffusivity $\alpha = k_c/(\rho c)_f$ and $\tau = (\rho c)_p/(\rho c)_f$. Where k_c, k, k_o, and σ are the fluid thermal conductivity, permeability coefficient, the chemical reaction, and fluid electric conductivity, ρ_f, $(\rho c)_f$, and $(\rho c)_p$ are fluid density, fluid heat capacity and nanoparticles effective heat capacity, D_B and D_T are Brownian and thermophoretic diffusion coefficients.

Introducing the stream function ψ defined as $u = \partial\psi/\partial y$ and $v = \partial\psi/\partial x$, which identically satisfies the continuity eq. (3), we can write eqs. (4)-(6):

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y}-\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2}=\left(v+\frac{1}{c\beta_1 \rho_f}\right)\frac{\partial^3 \psi}{\partial y^3}-\frac{1}{\rho_f}\left(\sigma B_o^{2}+\frac{\mu}{k}\right)\frac{\partial \psi}{\partial y}-$$

$$-\frac{1}{2c^3\beta_1 \rho_f}\left(\frac{\partial^2 \psi}{\partial y^2}\right)^2\frac{\partial^3 \psi}{\partial y^3}+\mathrm{g}\beta\left(T-T_\infty\right)+\mathrm{g}\beta^*\left(C-C_\infty\right) \tag{8}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]+$$

$$+\left[\mu+\frac{1}{c\beta_1(\rho c)_f}\right]\left(\frac{\partial^2 \psi}{\partial y^2}\right)^2+\frac{\sigma}{(\rho c)_f}B_o^{2}\left(\frac{\partial \psi}{\partial y}\right)^2-\frac{1}{6c^3\beta_1(\rho c)_f}\left(\frac{\partial^2 \psi}{\partial y^2}\right)^4 \tag{9}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial \psi}{\partial x}\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right)+k_o(C-C_\infty) \tag{10}$$

By introducing the similarity variable $\eta = y(U/2vx)^{1/2}$ and the stream function $\psi = (2Uvx)^{1/2}f(\eta)$ together with $\theta(\eta) = (T(\eta) - T_\infty)/(T_w - T_\infty)$, and the system of eqs. (8)-(10) reduces to the following system of non-linear ODE:

$$(1+\varepsilon)f'''(\eta)+f(\eta)f''(\eta)-\left(\mathrm{M}+\frac{1}{k}\right)f'(\eta)-\varepsilon\delta f''^2(\eta)f'''(\eta)+2\mathrm{Re}_x(\mathrm{Gr}\theta+\mathrm{Gc}\varphi)=0 \tag{11}$$

$$\frac{1}{\mathrm{Pr}}\theta''(\eta)+f(\eta)\theta'(\eta)+Nb\theta'(\eta)\varphi'(\eta)+Nt\theta'^2(\eta)+\mathrm{Ec}(1+\varepsilon)f''^2(\eta)+$$

$$+\mathrm{M}\,\mathrm{Ec}f'^2(\eta)-\varepsilon\delta\mathrm{Ec}f''^4(\eta)=0 \tag{12}$$

$$\varphi''(\eta)+\frac{Nt}{Nb}\theta''(\eta)+\mathrm{Le}f\varphi'(\eta)+2\gamma\,\mathrm{Re}_x\,\mathrm{Le}\varphi(\eta)=0 \tag{13}$$

Subject to the boundary conditions:

$$f(0)=0,\ \ f'(0)=\lambda,\ \ \theta(0)=1,\ \ \varphi(0)=1,\ \ \text{as}\ \ \eta\to 0$$
$$f'(\eta)\to 1,\ \ \theta(\eta)\to 0,\ \ \varphi(\eta)\to 0,\ \ \text{as}\ \ \eta\to\infty \tag{14}$$

where

$$\mathrm{M}=\frac{2x\sigma B_o^{2}}{U\rho_f},\ \ \ K=\frac{2x\mu}{U\rho_f k},\ \ \ \mathrm{Ec}=\frac{U^2}{(c_p)_f(T_w-T_\infty)},\ \ \ \mathrm{Le}=\frac{v}{D_B},\ \ \ \mathrm{Re}_x=\frac{Ux}{v}$$

$$\gamma=\frac{vk_\circ}{U^2},\ \ \ \mathrm{Pr}=\frac{v}{\alpha},\ \ \ Nb=\frac{(\rho c)_p D_B(C_w-C_\infty)}{(\rho c)_f v},\ \ \ Nt=\frac{(\rho c)_p D_T(T_w-T_\infty)}{v(\rho c)_f T_\infty} \tag{15}$$

$$\varepsilon=\frac{1}{\mu\beta c},\ \ \ \delta=\frac{U^3}{2vxc^2},\ \ \ \mathrm{Gr}=\frac{\mathrm{g}\beta v}{U^3}(T-T_\infty),\ \ \ \mathrm{Gc}=\frac{\mathrm{g}\beta^* v}{U^3}(C-C_\infty)$$

where M is the Hartmann number, K – the permeability parameter, Ec – the Eckert number, Le – the Lewis number, Re_x – the Reynolds number, γ – the chemical reaction parameter, Pr – the Prandtl number, Nb – the Brownian motion parameter, Gr – the Grashof number, Gc – the modified Grashof number, Nt – the thermophoresis parameter, and ε, δ are the fluid material parameters. Moreover the problem physical quantities (c_f is the skin-friction coefficient, Nu_x – the local Nussetl number, and Sh_x – the local Sherwood number) are defined:

El-Dabe, N. T. M., *et al.*: The Motion of a Non-Newtonian Nanofluid over ...
THERMAL SCIENCE: Year 2020, Vol. 24, No. 2B, pp. 1311-1321

1315

$$c_f = \frac{\tau_w}{\rho U^2}, \quad \mathrm{Nu}_x = \frac{xq_w}{k(T_w - T_\infty)}, \quad \mathrm{Sh}_x = \frac{xq_m}{D_B(C_w - C_\infty)}, \quad \tau_w = \left(\mu + \frac{1}{\beta c}\right)\frac{\partial u}{\partial y} - \frac{1}{6\beta}\left(\frac{1}{c}\frac{\partial u}{\partial y}\right)^3$$

$$q_w = -\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}, \quad q_m = -D_B\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{16}$$

Also, with the help of the similarity transformations we can write:

$$\sqrt{2\,\mathrm{Re}_x}\,c_f = (1+\varepsilon)f''(0) - \varepsilon\delta f''^3(0), \quad \theta'(0) = -\sqrt{\frac{2}{\mathrm{Re}_x}}\mathrm{Nu}_x, \quad \varphi'(0) = -\sqrt{\frac{2}{\mathrm{Re}_x}}\mathrm{Sh}_x \tag{17}$$

where c_f is the skin-friction coefficient, τ_w – the shear stress, q_w – theplate heat flux, q_m – the and plate mass flux.

Results and discussion

In this problem we studied the motion of non-Newtonian nanofluid with heat and mass transfer over a semi-infinite vertical flat plate stressed by a uniform external magnetic field. The problem is formulated mathematically by a system of non-linear PDE (3)-(7). Using suitable similarity variable the governing eqs. (3)-(7) are transformed into a system of non-linear ODE (11)-(14). The Mathematica ParametricNDSolve package is employed to solve the obtained system (11)-(14) numerically.

Using the default values of the physical parameters $\mathrm{Pr} = 1$, $\mathrm{Re}_x = 2$, $Nt = 0.1$, $Nb = 0.5$, $Le = 2$, $\varepsilon = 1$, $\delta = 0.5$, $\mathrm{Gr} = 0.5$, $\mathrm{Gc} = 0.5$, $\gamma = 0.5$, $\lambda = 0.2$, $M = 0.5$, $K = 0.5$, and $Ec = 2$, the velocity, temperature and concentration distributions as well as skin fraction and rate of the heat and mass transfer are discussed and illustrated graphically through set of figs. 2-7:
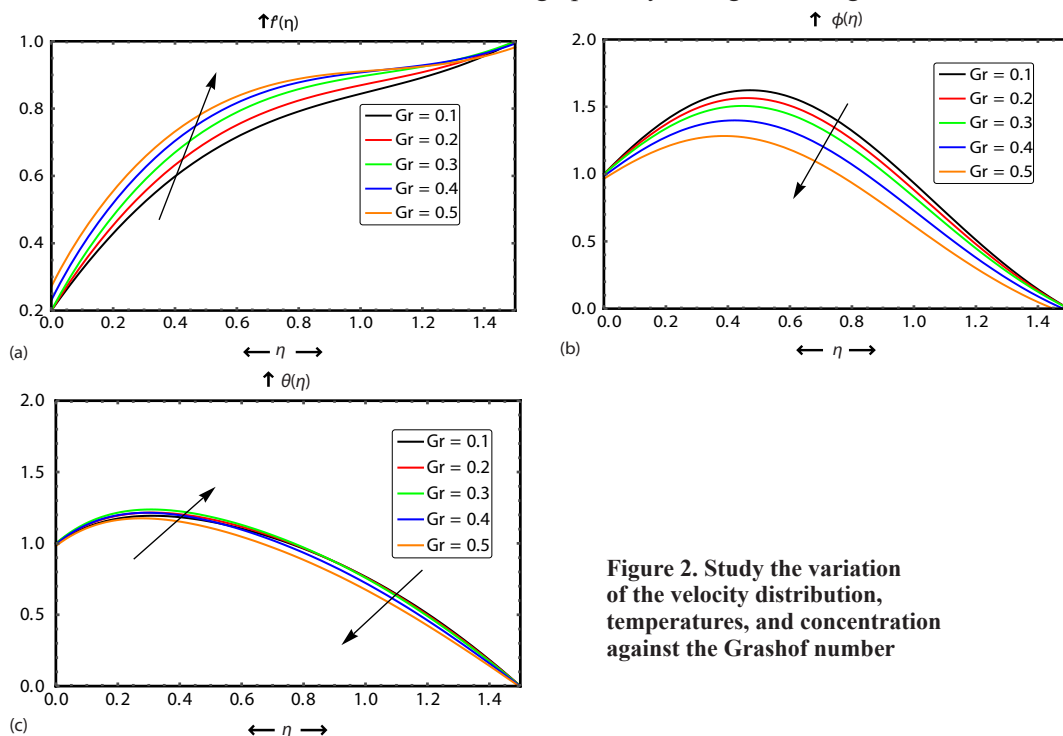

Figure 2. Study the variation of the velocity distribution, temperatures, and concentration against the Grashof number

1316

El-Dabe, N. T. M., *et al.*: The Motion of a Non-Newtonian Nanofluid over ...
THERMAL SCIENCE: Year 2020, Vol. 24, No. 2B pp. 1311-1321

Figure 2 shows the relation between the velocity $f'(\eta)$, the temperature $\theta(\eta)$, and the concentration $\varphi(\eta)$ with respect to the Grashof number. It is observed that the Grashof number signifies the relative effect of the buoyancy force to the hydrodynamic viscous force in the boundary-layer. The positive values of Grashof number correspond to cooling of the plate and the negative values of Grashof number correspond to heating of the plate by free convection. As expected, it is found that the increase in the Grashof number leads to increase the velocity, decrease the concentration and troubled increase in temperature due to enhancement in the buoyancy force.

Figure 3 illustrate the magnetic field effects on velocity, temperature, and concentration. The Lorentz force introduced by the magnetic field normal to the fluid motion decreases fluid velocity. This type of resisting force reduces fluid velocity, increase temperature and decrease concentration.

Figure 4 represents the influence of permeability parameter, K, on velocity, temperature, and concentration. Increasing the permeability parameter increases velocity and temperature while decreases concentration. The porous media creates huge resistance to fluid-flow and consequently minor changes occur in momentum boundary-layer.

Figure 5 illustrate the result of effects of Brownian motion parameter on velocity, temperature, and concentration. The study shows that the increase in the Brownian motion parameter increases velocity, temperature and decrease fluid concentration. This is due to the Brownian motion parameter is a random movement of microscopic particles suspended in liquids or gases resulting from the impact of molecules of the fluid surrounding the particles.
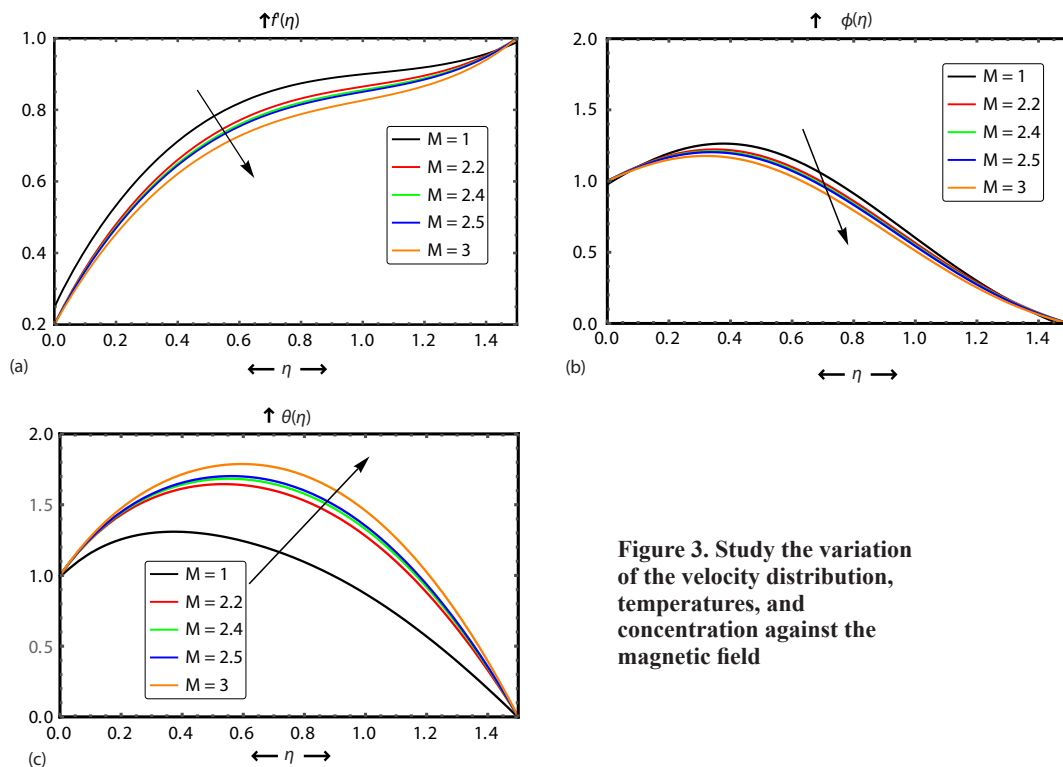

Figure 3. Study the variation of the velocity distribution, temperatures, and concentration against the magnetic field
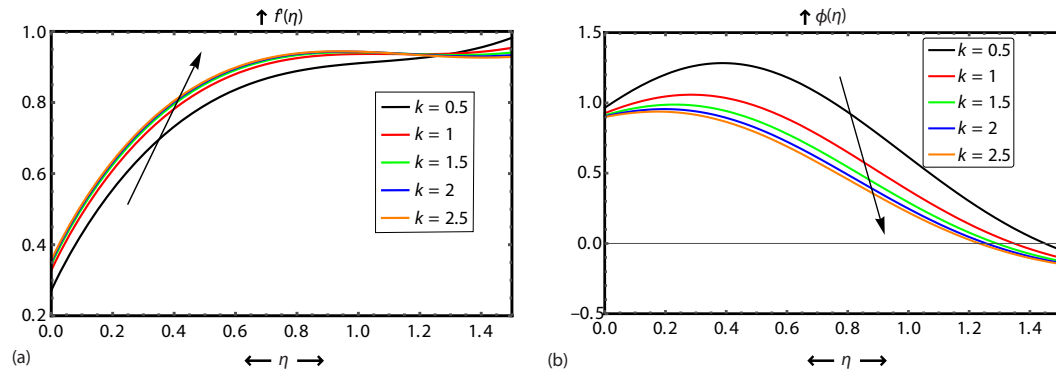
El-Dabe, N. T. M., *et al.*: The Motion of a Non-Newtonian Nanofluid over ...
THERMAL SCIENCE: Year 2020, Vol. 24, No. 2B, pp. 1311-1321

1317

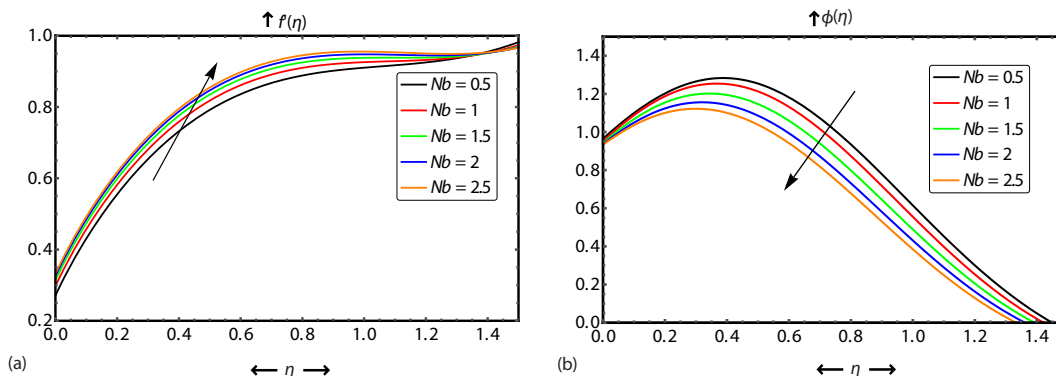**Figure 4. Study the variation of the velocity distribution, temperatures, and concentration against the permeability parameter**

Figure 5. Study the variation of the velocity distribution, temperatures, and concentration against the Brownian motion parameter *Nb*

1318

El-Dabe, N. T. M., *et al*.: The Motion of a Non-Newtonian Nanofluid over ...
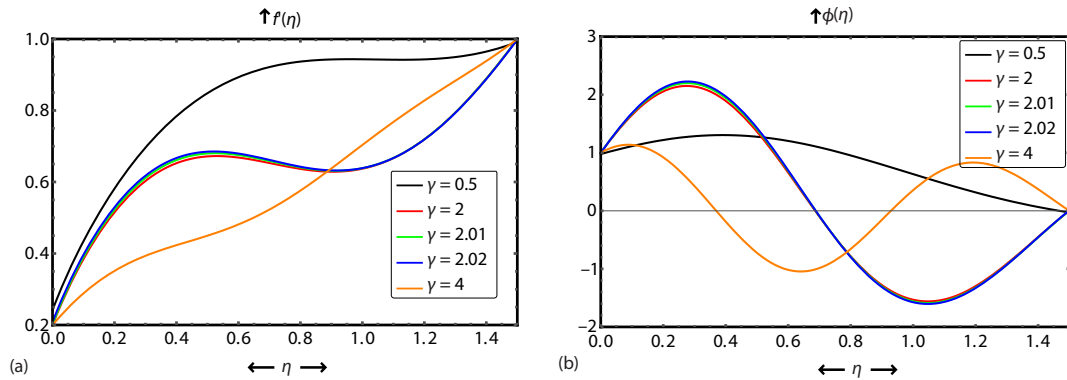THERMAL SCIENCE: Year 2020, Vol. 24, No. 2B pp. 1311-1321

Figure 6. Study the variation of the velocity distribution, temperatures, and concentration against the Chemical reaction parameter
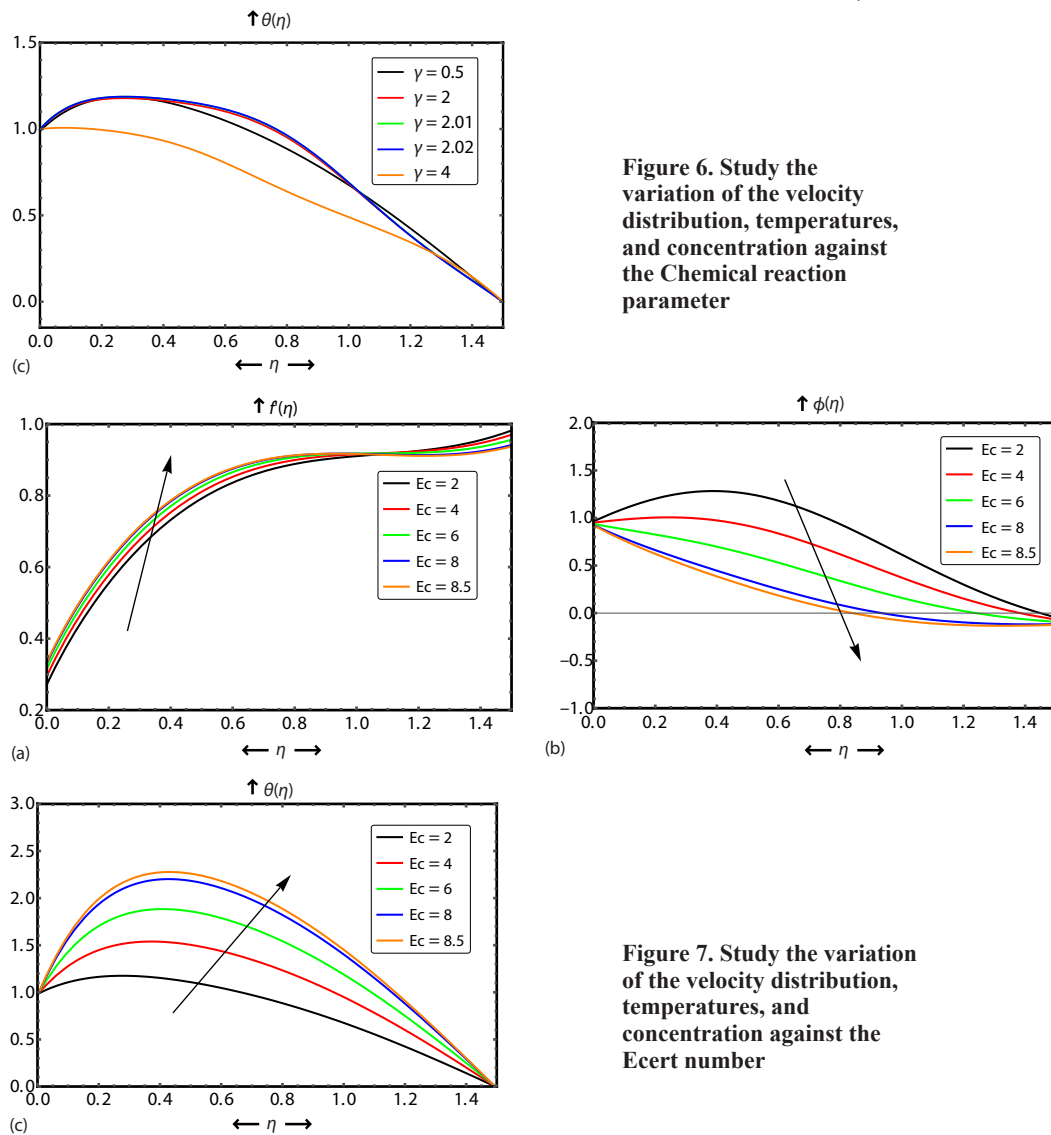

Figure 7. Study the variation of the velocity distribution, temperatures, and concentration against the Ecert number

Figure 6 study the relation between velocity, temperature, and concentration for different values of the chemical reaction parameter. Based in this study the increase in chemical reaction γ increases the velocity, temperature and concentration decrees as a result of the converted chemical force into energy.

Figure 7 shows the effect of Eckert number on velocity, temperature and concentration. The positive values of Eckert number indicates plate cooling, that is, loss of heat from the surface to the fluid so that the greater values of viscous dissipative heat cause enhancement of temperature and velocity profile as well. In addition, the magnitude of the thermal boundary-layer became large because of viscous dissipation effect. This leads to the increase in velocity and temperature but decreasing concentration.

Conclusions

Based on the aforementioned study we conclude the following results.
- The effects of some physical quantities (Eckert, Prandtl, Lewis, and Reynolds numbers, chemical reaction, Brownian motion parameter, thermophoretic parameter, and the fluid material parameter it has significant effects on both the velocity, temperature and concentration.
- The Grashof number signifies the relative effect of the buoyancy force to the hydrodynamic viscous force in the boundary-layer.
- Any increase in the Grashof number increases the velocity but decrease concentration and make trouble increase in the temperature due to the enhancement in the buoyancy force.
- Increasing the permeability parameter increases the velocity profile throughout the boundary-layer.
- The presence of porous media creates huge resistance to fluid-flow due to which trifling changes occurs in momentum boundary-layer and hence inciting of fluid temperature and decreasing in concentration appeared.
- Increase Eckert number increase fluid temperature and velocity.

Nomenclature

$\vec{B}$ – external uniform magnetic field, [kgs^{-2}A^{-1}]
C – nanoparticle volume fraction, [–]
C_w – nanoparticle volume fraction at the plate, [–]
c_f – skin-friction coefficient, [–]
c_p – specific heat at constant pressure, [JKg^{-1}K^{-1}]
D_B – Brownian diffusion coefficient, [–]
D_T – thermophoretic diffusion coefficient, [–]
f – dimensionless stream function, [–]
Gr – Grashof number, [–]
Gc – modified Grashof number, [–]
k – thermal conductivity, [Wm^{-1}K^{-1}]
Le – Lewis number, [–]
Nb – Brownian motion parameter, [–]
Nt – thermophoresis parameter, [–]
Nu$_x$ – local Nusselt number, [–]
Pr – Prandtl number [–]
q_w – plate heat flux, [–]
q_m – plate mass flux, [–]
Re$_x$ – local Reynolds number, [–]

Sh$_x$ – local Sherwood number, [–]
T – temperature, [K]
T_w – temperature at the plate, [K]
U – uniform velocity of the free stream low, [–]
U_w – uniform velocity of the moving plate, [–]
u, v – velocity components along x- and y-axes, [ms^{-1}]
x, y – Cartesian co-ordinates measured along the wall and normal to it, respectively, [–]

Greek symbols

α – thermal diffusivity, [–]
β^* – coefficient of expansion with concentration, [K^{-1}]
β – coefficient of thermal expansion [K$^{-1}c_p$]
η – similarity variable, [–]
θ – dimensionless temperature, [–]
λ – plate velocity ratio, [–]
μ – dynamic viscosity, [kgm^{-1}s^{-1}]
ν – kinematic viscosity, [kgm^{-1}s^{-1}]

ρ_f – fluid density, [kgm^{-3}]

ρ_p – nanoparticle mass density, [kgm^{-3}]

$(\rho c)_f$ – heat capacity of the fluid, [kgm^{-3}]

$(\rho c)_p$ – effective heat capacity of the nanoparticle
material, [kgm^{-3}]

σ – electrical conductivity, [sm^{-1}]

τ – heat capacity ratio, [–]

φ – dimensionless nanoparticle volume
fraction, [–]

ψ – stream function, [–]

Applications

The most interesting applications of the considered problem are in petroleum industries, human blood flow, heat and mass transfer and rheology, as follows.

−   The demand of a better drilling fluid increases when oil and gas companies turn their attention new search areas such as the unstable shale areas and high pressure. Nanotechnology can improve viscosity, density, specific gravity and any other physical properties of the drilling fluid to overcome any difficulties.

−   Human blood can be treated as a kind of a considered fluid due to the presence of several substances like, protein, fibrinogen and globulin in aqueous base plasma.

−   The importance of rheology science comes from its industrial applications where synthetic polymers and their solutions in different solvents is necessary for polymers and industrial application.

−   A wide variety of industrial processes involve the transfer of heat energy. Throughout any industrial facility, heat must be added, removed, or moved from one process stream to another and it has become a major task for industrial necessity. These processes provide a source for energy recovery and process fluid heating/cooling.

Acknowledgment

References

[1]   Maxwell, J. C., *Electricity and Magnetism*, Clarendon Press, Oxford, UK, 1873

[2]   Kakac, S., Pramuanjaroenkij, A., *Int. J. Heat Mass Transf*, *52* (2009), 13-14, pp. 3187-3196

[3]   Muhtamilselvan, M., Heat Transfer Enhancement of Copper-Water Nanofluids in a Lid-Driven Enclosure, *Commun. Non-linear Sci. Numer. Simul*, *15* (2010), 6, pp. 1501-1510

[4]   Choi, S. U. S., Enhancing Thermal Conductivity of Fluids with Nanoparticles, *Proceedings*, ASME International Mechanical Engineering Congress and Exposition, San Francisco, Cal., USA, 1995, Vol. 66, pp. 99-105

[5]   Eastman, J. A., *et al.*, Thompson, Anomalously Increased Effective Thermal Conductivities of Ethylene Glycol-Based Nanofluids Containing Copper Nanoparticles, *Appl. Phys. Lett.*, *78* (2001), 6, pp.718-720

[6]   Eldabe, N. T. M., *et al.*, Peristaltic Transport of Magneto-Hydrodynamic Carreau Nanofluid with Heat and Mass Transfer Inside Asymmetric Channel, *American J. Comput. Math.*, *7* (2017), 1, pp. 1-20

[7]   Eldabe, N. T. M., *et al.*, Magnetohydrodynamic Peristaltic Flow of Jeffry Nanofluid with Heat Transfer through a Porous Medium in a Vertical Tube, *Appl. Math. Inf. Sci.*, *11* (2017), 4, pp. 1097-1103

[8]   Eldabe, N. T. M., *et al.*, Magnetohydrodynamic Peristaltic Flow with Heat and Mass Transfer of Micropolar Biviscosity Fluid through a Porous Medium between Two co-Axial Tubes, *Arab. J. Sci. Eng.*, *39* (2014), Apr., pp. 5045-5062

[9]   Eldabe, N. T. M., *et al.*, The 2-D Boundary-Layer Flow with Heat and Mass Transfer of Magneto HydroDynamic Non-Newtonian Nanofluid through Porous Medium over a Semi-Infinite Moving Plate, *Microsystem Technologies*, *24* (2018), Jan., pp. 2919-2928

[10]   Venkateswarlu, M., *et al.*, Effects of Chemical Reaction Andheat Generation on MHD Boundary Layer Flow of a Moving Vertical Plate Withsuction and Dissipation, *Eng. Int.*, *1* (2013), 1 pp. 27-40

[11]   Reddy, G. V. R., e *et al.*, Similarity Transformations Heat and Mass Transfer Effects on Steady MHD Free Convection Dissipative Fluid-Flow Past an Inclined Porous Surface with Chemical Reaction, *Journal Naval Archit. Mar. Eng.*, *11* (2014), Dec., pp. 157-166

El-Dabe, N. T. M., *et al.*: The Motion of a Non-Newtonian Nanofluid over ...
THERMAL SCIENCE: Year 2020, Vol. 24, No. 2B, pp. 1311-1321

1321

[12] Malik, M. Y., Rehman, K. U., Effects of Second Order Chemical Reaction on MHD Free Convection Dissipative Fluid-Flow Past an Inclined Porous Surface by Way of Heat Generation: A Lie Group Analysis, *Inf. Sci. Lett.*, (2016), 5, pp. 35-45

[13] Nayak, M. K., *et al.*, Heat and Mass Transfer Effects on MHD Viscoelastic Fluid over a Stretching Sheet through Porous Medium in Presence of Chemical Reaction, *Propul. Power Res.*, 5 (2016), 2, pp. 70-80

[14] Eid, M. R., Chemical Reaction Effect on MHD Boundary-Layer Flow of Two-Phasenanofluid Model over an Exponentially Stretching Sheet with a Heat Generation, *Journal Molecular Li.*, 220 (2016), 7, pp. 18-25

[15] Seth, G. S., *et al.*, Natural-Convection Heat and Mass Transfer Flow with Hall Current, Rotation, Radiation and Heat Absorption Past an Accelerated Moving Vertical Plate with Ramped Temperature, *Journal of Applied Fluid Mechanics*, 8 (2015), 1, pp. 7-20

[16] Seth, G. S., *et al.*, The MHD Natural-Convection Heat and Mass Transfer Flow Past a Time Dependent Moving Vertical Plate with Ramped Temperature in a Rotating Medium with Hall Effects, Radiation and Chemical Reaction, *Journal of Mechanics*, 31 (2015), 1, pp. 91-104

[17] Seth, G. S., *et al.*, Effects of Hall Current and Rotation on Unsteady MHD Natural Convection Flow with Heat and Mass Transfer Past an Impulsively Moving Vertical Plate in the Presence of Radiation and Chemical Reaction, *Bulgarian Chemical Communications*, 46 (2014), 4, pp. 704-718

[18] Rashidi, M. M., *et al.*, Buoyancy Effect on MHD Flow of Nanofluid over a Stretching Sheet in the Presence of Thermal Radiation, *Journal Moleculare Liq.*, 198 (2014), Oct., pp. 234-238

[19] Gupta, A. S., *et al.*, Free Convection Effects on the Flow Past an Accelerated Vertical Plate in an Incompressible Dissipative Fluid, *Rev. Roum. Sci. Techn. Mec. Apl.*, 24 (1979), Apr., pp. 561-568

[20] Kafousias, N. G., Rapits, A. A., Mass Transfer and Free Convection Effects on the Flow Past an Accelerated Vertical Infinite Plate with Variable Suction or Injection, *Rev. Roum. Sci. Techn. Mec. Apl.*, 26 (1981), Jan.-Feb., pp. 11-22

[21] Rapits, A. A., Hydromagnetic Free Convection Flow Past an Accelerated Vertical Infinite Plate Withvariable Suction and Heat Flux, *Letters in Heat and Mass Transfer*, 8 (1981), 2, pp. 137-143

[22] Soundalgekar, V. M., *et al.*, Free Convection Effects on the MHD Stokes Problem for a Vertical Plate, *Nucl. Eng. Des.*, 51 (1979), 3, pp. 403-407

[23] Aydin, O., Kaya, A. The MHD Mixed Convection of a Viscous Dissipating Fluid about a Permeable Vertical Flat Plate, *Appl. Math. Model.*, 33 (2009), 11, pp. 4086-4096

[24] Tripathy, R. S., *et al.*, Chemical Reaction Rffect on MHD Free Convective Surface over a Moving Vertical Plate through Porous Medium, *Alexandria Engineering Journal*, 54 (2015), 3, pp. 673-679

[25] Turkyilmazoglu, M., The Analytical Solution of Mixed Convection Heat Transfer and Fluid-Flow of a MH*D* Viscoelastic Fluid over a Permeable Stretching Surface, *International Journal of Mechanical Sci*ences, 77 (2013), Dec., pp. 263 -268

[26] Powell, R. E, *et al*, Mechanism for the Relaxation Theory of Viscosity, *Nature*, 154 (1944), pp. 427-428

[27] Hayat, T., *et al.*, Steady Flow of an Eyring Powell Fluid over a Moving Surface with Convective Boundary Conditions, *International Journal of Heat and Mass Transfer*, 55 (2012), 7-8, pp. 1817-1822

[28] Reddy, M. G, Scaling Transformation for Heat and Mass Transfer Effects on Steady MHD Free Convection Dissipative Flow Past an Inclined Porous Surface, *International Journal Applied Mathematics Mechanics*, 9 (2013), 10, pp. 1-18

[29] Singh, G., *et a*l, Effects of Ohmic Heating and Viscous Dissipation on Steady MHD Flow Near a Stagnation Point on an Isothermal Stretching Sheet, *Thermal Science*, 13 (2009), 1, pp. 5-12

[30] Ghani, F., *et al.*, Unsteady Magnetohydrodynamics Thin Film Flow of a Third Grade Fluid over an Oscillating in-Clined Belt Embedded in a Porous Medium, *Thermal Science*, 21 (2017), 2, pp. 875-887

[31] Hayat, T., *et al.*, Magnetohydrodynamic Flow of Powell-Eyring Fluid by a Stretching Cylinder with Newtonian Heating, *Thermal Science*, 22 (2018), 1B, pp. 371-382