COUPLING EFFECTS OF POROSITY AND PARTICLE SIZE ON SEEPAGE PROPERTIES OF BROKEN SANDSTONE BASED ON FRACTIONAL FLOW EQUATION

by

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Studying the seepage properties of broken rock is important for understanding the behavior of engineering projects and preventing seepage disasters from occurring. Therefore, a test system was developed to test the seepage properties of broken rock under different porosities and particle sizes. A non-linear seepage equation of broken rock was developed based on the Forchheimer equation and the theories of fraction calculus. The influence of the coupling mechanism of the porosity and particle size on the seepage properties of broken rock was analyzed. The results show that the non-linear seepage equation can describe the non-linear seepage properties of broken rock well. The relations between the permeability and the porosity and particles size can all be represented through an exponential function. It is thought that watercourses are developed in broken rock with high porosity and large particle size, which shows a stronger hydraulic conductivity capability. However, the inertial potential energy of a non-Darcy flow is relatively small.

Key words: *broken rock, fractional flow, seepage properties, non-Darcy*

Introduction

The broken rock is widely distributed in mining engineering projects, roadbed, and slopes, and has specific characteristics including a large porosity and high permeability. Its permeability is several orders of magnitude larger than that of intact rock, thereby causing seepage disasters to frequently occur in a broken rock mass [1]. The permeability of rocks can be traced by Darcy, a French hydrologist, [2]. Based on the Darcy's Law, Dupuit [3] added a quadratic term to describe the non-linear relationship between the hydraulic gradient and seepage velocity. Forchheimer [4] proposed the Forchheimer model of non-Darcy seepage for the problem of an excessive seepage velocity of the rock mass, which was widely used to describe the non-Darcy seepage properties of a fluid in a porous medium.

The Darcy's law can only describe the permeability of certain rocks [5], and the non-Darcy properties of broken rock is mostly based on the Forchheimer model, which had achieved fruitful results. Zoback *et al*. [6] compared the deformation behavior and permeability of highly broken calcite caused by an earthquake. Pradip *et al*. [7] studied the convergence

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speed of the non-Darcy flow on the broken coarse particle of marble, which was used to describe the flow behavior of broken rock by Forchheimer equation, Missbach equation, and Wilkins equation. Legrand [8] used the capillary model to obtain the relationship between structural parameters, Reynolds number, and the friction factor of each model during the seepage flow of broken rock. Ma *et al.* [9] used the Forchheimer equation, analyzed the non-linear properties of broken rock with the experimental data obtained under a steady seepage. Wang *et al*. [10] used the continuous gas-flow velocity method to study the permeability of broken salt rock.

Fractional calculus is an integral calculus used in the expansion of arbitrary order calculus, and is the study of the basic theory of arbitrary order of differentials and integrals, precisely describing the non-linear problem of an advantageous tool [11]. In recent years, Yang *et al*. [12-15] proposed a series of new operators for the applications of diffusion and rheology problems. He solved the fractional 3-D seepage motion equation using an iteration method [16]. Choudhary [17] used a numerical solution solve the flow problem of two incompatible liquids and applied it to oil production. Liu *et al*. [18] proposed two modified alternate directions method for solving the non-continuous seepage problems with fractional derivatives in 2-D homogeneous media. Wang *et al*. [19] used the Laplace transform of a fractional derivative sense Swartzendruber motion equation and an analytical expression of the experimental data fitting of an unsaturated seepage analysis of the fractional Swartzendruber precision. The classical Forchheimer formula plays an important role in the study of non-linear seepage properties, but it needs to be improved in terms of an accurate description of the non-linear seepage properties of the broken rock. This article first embarks from the Forchheimer formula, with the aid of the fractional differential method, to obtain the score expression form, and combined with the broken rock steady seepage test data, to give the analysis of the non-linear seepage properties of the broken rock.

Experiment set-up and testing procedure

Specimen preparation

In this test, the sandstone was obtained from a coal mine in Xuzhou, China. The main physical properties of the sandstone samples in this test are given in tab. 1. After the block sandstone was collected, it was divided into different groups through manual crushing and screening in a laboratory. First, the large sandstone was broken by a crusher [20]. Second, using the head of a hammer, the broken rock was broken into smaller pieces through secondary crushing. Finally, the rock particles were screened by a sieve to obtain broken sandstone pieces of different sizes, which were divided into groups of 10-15 mm (size 1), 15-20 mm (size 2), and 20-25 mm (size 3) in size. When a cylindrical specimen with a diameter of 127 mm was produced as recommended by the International Society for Rock Mechanics and Rock Engineering (ISRM), the maximum grain size of the particles used in this experiment was 25 mm [21, 22].

Table 1. Physical properties of sandstone

Nature density [gcm^{-3}]	Dry density [gcm^{-3}]	Saturated density [gcm^{-3}]	Porosity	Saturated water content [%]	Uniaxial compressive strength [MPa]	Elastic modulus [GPa]
2. 48	2. 42	2. 52	10. 90	2. 60	57. 5	7. 53

Experiment set-up

There are two methods of rock permeability test: transient permeability method and steady permeability method [23]. For an unsteady seepage of broken rock, because of its larger permeability, the transient testing method for the differential pressure at the ends of the specimen will disappear within a short period of time or tend to be a very small value, which is not convenient to observe; at this time, we should adopt a steady osmosis, which is measured by the pressure gradient and its varying rate of time series, as well as the extraction of the non-Darcy flow seepage characteristic parameters.

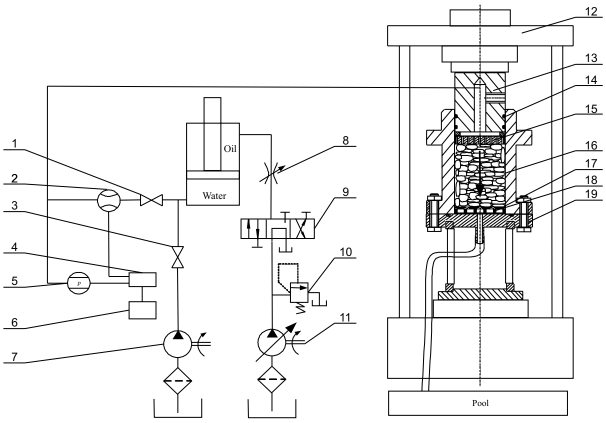

Figure 1. Experimental system; *1 – swich 1, 2 – flowmeter, 3 – swich 2, 4 – paperless recorder, 5 – pressure transmitter, 6 – computer, 7 – water pump, 8 – one-way restrictor valve , 9 – reversing valve, 10 – overflow valve, 11 – hydraulic pump station, 12 – the MTS816.03 system, 13 – inlet water piston, 14 – shape seal ring, 15 – upper permeable plate, 16 – sample, 17 – felt, 18 – lower permeable plate, 19 – pedestal*

The system, see fig. 1, used in this experiment is a self-developed seepage test system for broken rock, which can provide the axial pressure, water pressure, and penetration pressure required for the test. The axial pressure is provided by MTS816 rock mechanics test system, the dual function hydraulic cylinder, oil pump station, and water pump can provide an adjustable and stable penetration pressure, and can simulate the water pressure in the project site. The stable oil pressure is provided through the oil pump station control the double acting hydraulic cylinder and provide a stable penetration pressure. The pressure sensor, flow sensor, and data collector are used to collect the seepage pressure and flow during the test in real time.

Testing procedure

This experiment used a change in the displacement to control the change in porosity of the sample and obtain the sample permeability characteristics, which change with the porosity, fig. 2. The key is to obtain the seepage velocity and pore pressure gradient of two physical quantities.

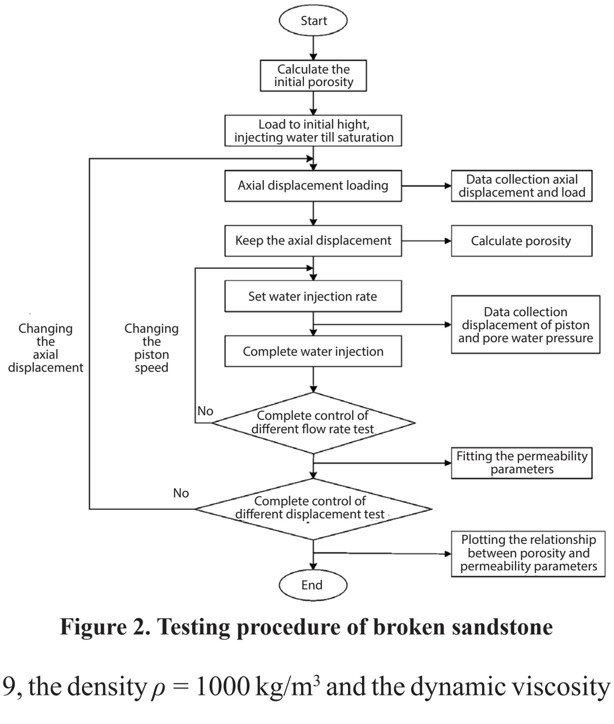

Figure 2. Testing procedure of broken sandstone

In this test, the particle size d = 10~25 mm, the porosity φ = 0.48~0.19, the density ρ = 1000 kg/m^3 and the dynamic viscosity μ = 1.01·10^3 Pa·s:

$$\mathrm{Re} = \frac{\rho \upsilon d}{\varphi \mu} = 0.645 \sim 9.773$$

Considering that the broken rock mass is the actual size in the mining engineering project, which is often 10 times, or even dozens of times, that of the test size, the Reynolds is far greater than 10, which is beyond the scope established by the Darcy's law, and belongs to the non-Darcy seepage.

Test parameter calculation

In this test, considering the broken sandstone sample with cross-sectional area, A, thickness, h, and radial rigid constraint, the pressures of the upper and lower ends of the sample are p_1 and p_2 ($p_1 > p_2$), respectively. Owing to the pressure difference ($p_1 > p_2$) at both ends of the rock sample, water flows from the top to the bottom in the sample. The results show that water flow in broken sandstone generally follows the Forchheimer relation, *i. e.*

$$G_p = \frac{\mu}{k}\upsilon + \rho\beta\upsilon^2 \tag{1}$$

where ρ is the mass density, μ – the dynamic viscosity of the water, k – the permeability, β – the non-Darcy flow, β is the factor of the broken sandstone, υ – the seepage velocity, and G_p – the absolute value of the pressure gradient with $G_p = (p_1 - p_2)/H$.

Taking the derivative of both sides of eq. (1) with respect to velocity υ, we get:

$$\frac{\mathrm{d}G_p}{\mathrm{d}\upsilon} = \frac{\mu}{k} + 2\rho\beta\upsilon \tag{2}$$

Equation (2) means that the derivative of pressure gradient with respect to the seepage velocity is linearly related to the seepage velocity.

The experimental results show that more satisfactory results can be obtained by replacing $\mathrm{d}Gp/\mathrm{d}\upsilon$ with the fractional derivative $\mathrm{d}^\alpha Gp/\mathrm{d}\upsilon^\alpha$ [11].

With the aid of the Caputo fractional derivative, we can write eq. (2) into the equation of form:

$${}_0^C\mathrm{D}_\upsilon^\alpha G_p = \frac{\mu}{k} + 2\beta\rho\upsilon \tag{3}$$

where ${}_0^C\mathrm{D}_\upsilon^\alpha$ is denoted as the Caputo fractional derivative [11], and the left subscript is the minimum value of the independent variable υ.

The Laplace transform of eq. (3) can be represented by the two-parameter Mittag-Leffler function [24]:

$$G_p = \frac{\mu}{k}\upsilon^\alpha E_{\alpha,\alpha+1}(2\beta\rho\upsilon^\alpha) \tag{4}$$

where the general special function is denoted [11]:

$$E_{\alpha,\alpha+1}(2\beta\rho\upsilon^\alpha) = \sum_{j=0}^{+\infty}\frac{(2\beta\rho\upsilon^\alpha)^j}{\Gamma(\alpha j + \alpha + 1)} \tag{5}$$

Experiment results

Non-linear flow properties of broken sandstone

Many research results, presented the water flow in porous media, which is characterized by the significant non-linearity between the pressure gradient and seepage velocity. In order to specify the non-linear seepage features of broken sandstone, we use the Monte-Carlo method to fit the relation between the pressure gradient and seepage velocity in the test process of three groups of particle size, the permeability and non-Darcy flow, and factor of broken sandstone, see tab. 2.

Table 2. Fitting parameters of fractional seepage model of broken sandstone

d [mm]	φ	k [10^{-15} ms^{-1}]	β [10^{12} kgm^{-4}]	α	R^2
10-15	0. 43	353. 00	1. 03	0. 83	0. 89
	0. 36	50. 38	4. 48	0. 69	0. 83
	0. 28	18. 30	20. 90	0. 93	0. 96
	0. 22	4. 62	27. 65	0. 76	0. 94
	0. 17	2. 27	120. 00	0. 97	0. 97
15-20	0. 43	437. 00	1. 19	1. 12	0. 91
	0. 36	180. 00	5. 42	0. 71	0. 96
	0. 32	27. 00	10. 06	0. 73	0. 98
	0. 28	10. 20	21. 45	1. 08	0. 93
	0. 23	8. 66	29. 30	0. 93	0. 98
	0. 17	2. 35	147. 00	0. 60	0. 99
20-25	0. 43	615. 00	2. 90	0. 43	0. 96
	0. 36	301. 00	6. 12	0. 52	0. 93
	0. 32	39. 90	14. 50	0. 55	0. 90
	0. 28	16. 59	23. 35	0. 75	0. 89
	0. 23	9. 52	29. 80	0. 67	0. 87
	0. 18	4. 31	157. 00	0. 99	0. 84

As is seen in tab. 2, the permeability is distributed at a ratio $4.62\cdot10^{-16}$~$3.01\cdot10^{-12}$, and the non-Darcy flowfactor is distributed at a ratio of $1.85\cdot10^{11}$~$1.20\cdot10^{14}$. This indicates that the seepage in the broken sandstone presents obvious non-linear characteristics. To see these non-linear characteristics more directly, only a particle size of 20-25 mm was selected as an example.

Figure 3 shows the relation between the seepage velocity and pressure gradient in the broken rock under six types of initial porosity. It can be seen that the seepage velocity is positively correlated with the pressure gradient, and the relation shows obvious non-linear characteristics. The fractional flow equation can characterize the relation between the seepage velocity and pressure gradient in broken rock, and the corresponding characteristics of the non-Darcy flow can be reflected well in the seepage process of broken rock.

Effect of porosity on seepage parameters

Figure 4(a) shows the relationship between the permeability and porosity of a broken rock sample based on the fractional flow equation, in which the permeability is positively related to the porosity, which can be represented as the function:

$$k = 651.26\left(1 - \mathrm{e}^{-40.31\phi}\right)^{1.65}$$

This is because of the broken rock in the process of compression, in which the original pore structure is broken into tiny particles blocking the water channel, creating less porosity and

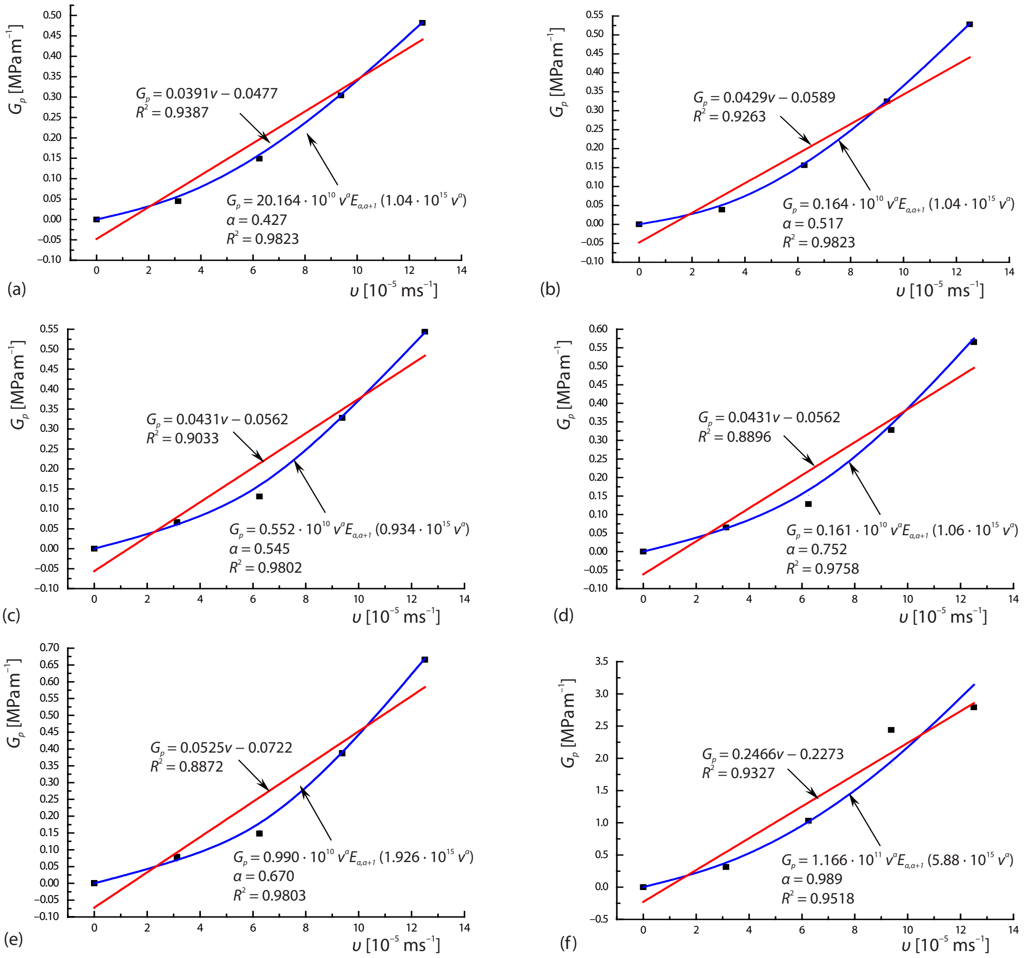

Figure 3. Relation between pressure gradient and seepage velocity under different porosity; (a) $\phi = 0.43$,(b) $\phi = 0.36$, (c) $\phi = 0.32$, (d) $\phi = 0.28$, and (f) $\phi = 0.18$

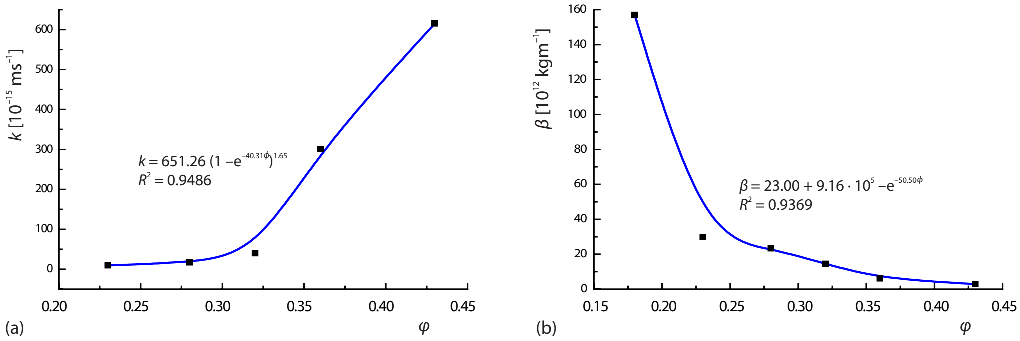

Figure 4. Relation between permeability and non-Darcy flowfactor with porosity

significantly weakening the water transmitting ability through the broken rock, particularly for a porosity, φ, of 0.1764, with a permeability k of the only $8.66 \cdot 10^{-16}$ m^2.

It can be seen from fig. 4(b), that the non-Darcy flowfactor has a negative correlation with the porosity, which can be well characterized:

$$\beta = 23.00 + 9.16 \cdot 10^5 \mathrm{e}^{-50.50\phi}$$

It can be seen that a broken rock sample with a high porosity contains more water diversion channels, and its non-Darcy flow inertia potential is relatively lower. In particular, when the porosity, φ, is 0.43, the non-Darcy flowfactor is only $2.09 \cdot 10^{-16}$ kgm^{-4}.

Effect of particle size on permeability parameters

Figures 5(a) and 5(b) show a broken sample based on the fractional flow equation of the non-Darcy flow of the infiltration parameters and particle size, where the relationship between the permeability and the particle size is positively related to $k = C_0 + C_1 e^{C_2\varphi}$ (where, C_0, C_1, and C_2 for fitting constants). This is because, in the compression process of a broken rock sample, a broken rock sample with large particle size has a relatively larger water channel, and thus its water conductivity is obviously stronger. However, the non-Darcy flow β factor is negatively correlated with the particle size, which can be represented well by $\beta = C_0 + C_1 e^{C_2}$ (where C_0, C_1, and C_2 are the fitting constants). It can be understood that the non-Darcy flow inertia potential of broken rock samples with larger water diversion channels is relatively lower.

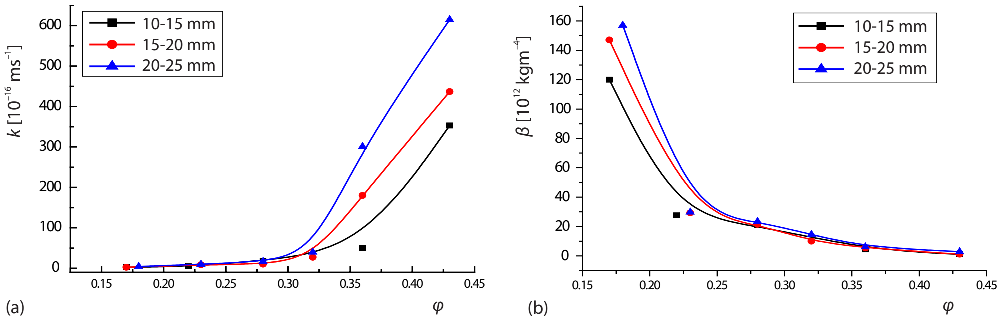

Figure 5. Relation between permeability and non-Darcy flow with factor porosity under different particle sizes

The coupling effect of porosity and particle size on permeability parameters

From the analysis in upper sections, the non-linear properties of the broken sandstone seepage rate parameter, k, and non-Darcy flow, β, factor are both influenced by the porosity and

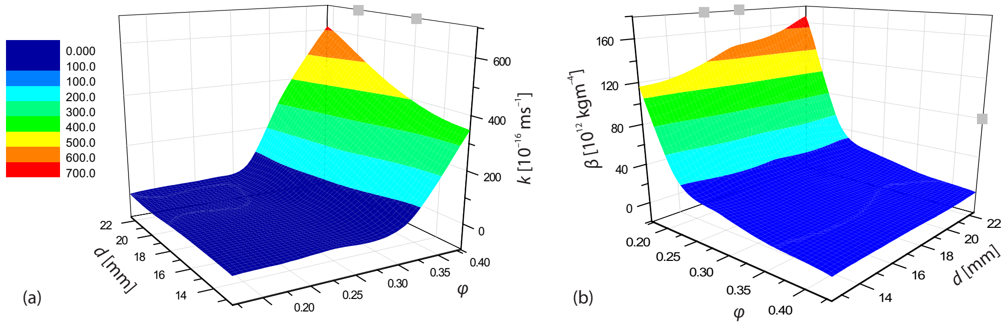

Figure 6. The coupling effect of porosity and particle size on permeability and non-Darcy flowfactor

particle size, and the following is from the angle of the coupling analysis showing how both non-linear parameters are influenced.

The permeability of broken sandstone with different grain sizes is positively correlated with the porosity, that is, the higher the porosity is, the stronger the permeability. With the same porosity, the greater the particle size is, the stronger the permeability, which is due to the geometric shape of the broken rock, whereas the greater the particle size of rock is, namely, its irregularity, the greater the probability of a possible channel occurring during the process of compaction, and the greater the effect of the permeability properties of the broken rock and the developed channel.

With the increase in porosity, the non-Darcy flow, β, factor shows a rapid decrease at first, and after a slowly decreasing trend, the inflection point reduction reaches near a porosity of 0.225, which is due to the broken sandstone in the compaction process; in addition, with crushed, broken, and mobile stones blocking part of the seepage channels, a decrease in the entire rock porosity occurs, voiding the change in spatial distribution, and gradually deviating the permeability from the Darcy characteristics, which are significant characteristics of a non-Darcy flow.

Conclusion

Making use of the control of the displacement of broken sandstone seepage properties, a self-developed and broken rock seepage test system under the steady-state osmosis was studied under the different pressure gradients, non-linear seepage properties of the broken sandstone, and the porosity and particle size effect on the permeability parameters. Based on the Forchheimer equation and fractional calculus theory, the non-linear seepage equation of broken rock is established. The test results show that the equation can describe the non-linear seepage properties of broken rock well. The permeability of the sandstone was positively related to the relationship with the porosity, which is due to the broken rock during the compression process, the original pore structure broken tiny particles filling blocking water channel, smaller broken rock porosity water less obviously. The non-Darcy flow, β, factor has a negative correlation with the porosity, which is due to the fact that the broken rock sample with a large porosity contains more water conduction channels, and its non-Darcy flow inertia potential is relatively low. The non-linear properties of broken sandstone, namely, permeability and non-Darcy flow factor, are positively and negatively correlated with the particle size, which can be characterized through the exponential function. This is due to the broken rock during the compression process, in which larger particle size and broken rock sample incur a relatively larger water channel, and significantly stronger water capacity; however, the non-Darcy flow inertia potential is relatively lower.

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Nomenclature

b – broken rock particle size, [mm]
d – particle size
G_p – pressure gradient, [MPam^{-1}]
k – permeability, [ms^{-1}]

p – water pressure, [MPa]
R^2 – corellation coefficient

Greek symbols

β – non-Darcy flow factor, [kgm^{-4}]
μ – dynamic viscosity, [Pa·s]
ρ – broken rock density, [kgm^{-3}]
υ – seepage velocity, [ms^{-1}]
φ – broken rock porosity, [–]

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