A NON-LINEAR CREEP CONSTITUTIVE MODEL FOR SALT ROCK BASED ON FRACTIONAL DERIVATIVES

by

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The creep property of the salt rock is an important determinant of deep underground repository of energy storage for long-term operatio. Starting from fractional derivative, this paper proposes a new concept of super Abel dashpot based on Abel dashpot. According to the Nishihara model, the non-linear element of super Abel dashpot is introduced, and a new salt rock creep constitutive model is established. The theoretical analytic solution of this model is then deduced. Fitting analysis of the experimental data indicates that the model can well reflect the whole-process creep curve, especially the non-linear accelerated creep stage.

Key words*: creep, salt rock, fractional derivatives*

Introduction

Salt rock, with properties of good creep, low permeability, and damage self-recovery, is a useful medium for the storage of energy and high-level radioactive wastes [1-3]. However, the large deformation, loss of effective storage volume, or even ineffective storage function may occur in the underground cavities for the long term, which results in variation of the internal pressures, collapsing of roof cap, and changes in the creep property of salt rock. Therefore, the evaluation of the creep property of salt rock is very important [4].

In recent years, many studies have been performed on salt rock creep and various creep models have been proposed. In general, the creep constitutive model of salt rock consists of the empirical model, the component assembly model, the damage mechanism-based creep model, and the fractional derivative creep model. The empirical model demonstrates the functional relationship established based on salt rock creep experimental results. Considering the influence of confining pressure and axial pressure on time-dependent stress-strain behavior, Yang *et al*. [5] proposed an exponential equation describing initial creep and steady creep. With the high pertinence, the empirical model can accurately reflect the creep property of the specified rock material. However, the physical significance of the model parameters remains indistinct thus far. The component assembly model adopts the basic components of the model, including the Hooke viscoelastic body (H), Newton viscous body (N), and Saint-Venant's plastic body (St. V), which are assembled to simulate the creep mechanical behavior of the salt rock. It is widely applied in describing the creep property of the rock material because of its flexibility and

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convenience. Unfortunately, the empirical and component assembly models can hardly reflect the whole-process creep curve of the salt rock, especially the non-linear accelerated creep stage. With the increase in studies on the non-linear property of the soft rock creep in recent years, the evaluation of the creep models based on the damage mechanism has made significant progress, and many models have been proposed. By introducing the damage accelerating limit theory, Wang [6] proposed the creep damage model that describes the accelerated creep stage of the salt rock. The model holds that only after exceeding damage, accelerated limit can accelerate the creep stage, which greatly influences damage evolution and induces creep occurrence. Based on continuum medium damage mechanics, Chan *et al*. [7] studied the non-spring flow caused by damage and proposed the salt rock creep damage break multi-mechanism coupling model (MDCF). These various creep phenomenon models also show that the strain state at one point of the material is not only related to the stress state at that point at exactly the same instant but also the whole history of previous stress at that point before that instance.

Fractional time derivative is actually a differential-integral convolution operator, in which the integral term in the definition adequately reflects the historical dependence of the system functions development. It is a powerful mathematical tool for strong memory process modelling, thus making the fractional derivative an important tool for describing various complex mechanical and physical behaviors in recent years [8, 9]. Based on fractional calculus and typical Nishihara model, Zhou *et al.* [10] proposed a fractional derivative-based salt rock creep model. Considering crack evolution and damage accumulation in micro-scale during the accelerated creep stage, the authors introduced damage variables, and proposed the Abel dashpot of variable viscosity coefficient, thus enabling this model to better reflect the three-stages of salt rock creep, especially the accelerated creep stage. However, the derivative process is too complicated, and the Mittag-Leffler function convergence involved in parameter fitting is too slow and calculation very difficult.

Therefore, establishing a creep model that is not only simple but also able to reflect the stage characteristics of the non-linear accelerated creep stage has great value in engineering. Starting from the Riemann-Liouville fractional derivative, this paper proposes a new concept of the super Abel dashpot based on Abel dashpot. According to the Nishihara model, we then introduce the non-linear element of the super Abel dashpot and establish a new salt rock creep constitutive model. Fitting analysis of the experimental data is then applied to the model and results discussed.

Fractional calculus

Definition of Riemann-Liouville derivative

Suppose f is continuous by section between $[0, +\infty)$ and can be accumulated between bound subinterval of $[0, +\infty)$. When $t > 0$ and $\mathrm{Re}(v) > 0$, we have:

$$D^{-v} f(t) = \frac{1}{\Gamma(v)} \int_0^t (t-\xi)^{v-1} f(\xi)\mathrm{d}\xi \tag{1}$$

where $\Gamma(v)$ is gamma functi on.

Equation (1) is deemed as order ν Riemann-Liouville fractional integral of the function $f(t)$, where:

$$\Gamma(v) = \int_0^\infty t^{v-1} e^{-t} \mathrm{d}t$$

Definition 2: Suppose $f \in C$ and m is the minimal integral number larger than μ, then $v = \mathrm{m} - \mu > 0$:

$$D^{\mu} f(t) = D^{m}\left\{D^{-\nu}[f(t)]\right\},\ \mu > 0,\ t > 0 \tag{2}$$

Equation (2) is deemed as order μ Riemann-Liouville fractional integral of the function $f(t)$.

Abel dashpot

It is well-known that the ideal stress-strain relationship is that the elastic body fulfills the Hooke law $\sigma(t) \sim \varepsilon(t)$, and Newtonian fluid fulfills the Newtonian law $\sigma(t) \sim \mathrm{d}^1\varepsilon(t)/\mathrm{d}t^1$. If the Hooke law is revised to $\sigma(t) \sim \mathrm{d}^0\varepsilon(t)/\mathrm{d}t^0$, one can assume that the constitutive relationship of the intermediate material between purely elastic body and Newtonian fluid may obey $\sigma(t) \sim \mathrm{d}^\beta\varepsilon(t)/\mathrm{d}t^\beta$ $(0 \le \beta \le 1)$. The constitutive relationship of Abel dashpot is then given:

$$\sigma = \frac{\eta \mathrm{d}^{\beta}\varepsilon(t)}{\mathrm{d}t^{\beta}},\ (0 \le \beta \le 1) \tag{3}$$

where η is viscosity coefficient.

When the strain σ = const., the element will describe change of creep behavior, so the fractional integral is performed on both sides of eq. (3). According to the definition of fractional-order Riemann-Liouville integral, the following is obtained:

$$\varepsilon(t) = \frac{\sigma_0}{\mu}\frac{t^{\beta}}{\Gamma(1+\beta)},\ (0 \le \beta \le 1) \tag{4}$$

where σ_0 denotes normal stress.

The Abel dashpot is an integrated element including elastic element and damping element. Therefore, it can demonstrate the property of the entire viscoelastic body (N/H), which also provides evidence for the Abel dashpot replacing the entire viscoelastic body below.

Although the Abel dashpot has an integrated property of viscoelastic body, as its coefficient $0 \le \beta \le 1$, this element cannot reflect the non-linear accelerated creep stage.

A non-linear element: super Abel dashpot

Because the Abel dashpot cannot reflect the accelerated creep stage, to better reflect the whole-process creep curve of salt rock, especially the non-linear accelerated creep stage, it is necessary to propose a non-linear element. The basic reason as to why the Abel dashpot cannot reflect the non-linear accelerated creep stage is because its coefficient $0 \le \beta \le 1$. Therefore, if $\beta > 1$, this element can surely reflect the non-linear accelerated creep stage during the creep process.

According to our definition, when β is any real number larger than 1, the Abel dashpot will turn into super Abel dashpot, fig. 1. Therefore, the constitutive relationship of super Abel dashpot is:

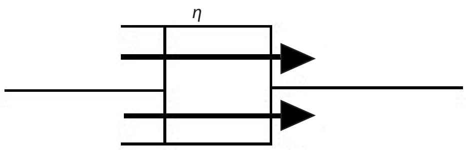

Figure 1. Super Abel dashpot

$$\sigma = \frac{\eta \mathrm{d}^{\beta}\varepsilon(t)}{\mathrm{d}t^{\beta}},\ (\beta > 1) \tag{5}$$

where η is viscosity coefficient.

Likewise, when strain σ = const., the element will describe the change of creep behavior. In this case, fractional integral is performed on both sides of eq. (5) based on the definition of fractional calculus Riemann-Liouville integral:

$$\varepsilon(t)=\frac{\sigma_0}{\mu}\frac{t^\beta}{\Gamma(1+\beta)},\ (\beta>1) \tag{6}$$

where σ_0 stands for normal stress.

Establishment and solution of the fractional derivative creep constitutive model

A typical creep process generally consists of stages: initial creep, steady creep, and accelerated creep. The assembly model has all along sought the precise description for these three-stages. In this section, the Abel dashpot replaces the entire viscoelastic body in the Nishihara model, and the super Abel dashpot replaces the Newton dashpot in the viscoplastic body, fig. 2. In the proposed creep constitutive model, suppose that the strains of Hooke body, viscoelastic body and that viscoplastic body are ε_e, ε_{ve}, and ε_{vp}, then the general strain ε can be described as

$$\varepsilon=\varepsilon_e+\varepsilon_{ve}+\varepsilon_{vp} \tag{7}$$

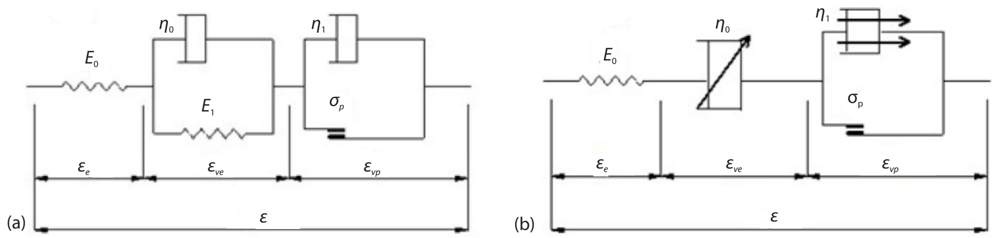

Figure 2. Schematic view of the creep constitutive model; (a) Nishihara model, (b) fractional derivative model

The stress-strain relationship of Hooke body (H) is:

$$\varepsilon_e=\frac{\sigma}{E_0} \tag{8}$$

where E_0 stands for the elastic modulus of the spring in the Hooke body.

The stress-strain relationship of the Abel dashpot (A):

$$\sigma=\frac{\eta_0 \mathrm{d}^\beta \varepsilon_{ve}(t)}{\mathrm{d}t^\beta},\ (0\le\beta\le1) \tag{9}$$

$$\varepsilon_{ve}=\frac{\sigma}{\eta_0}\frac{t^\beta}{\Gamma(1+\beta)},\ (0\le\beta\le1) \tag{10}$$

In the viscoplastic body (A/St. V), the strain of friction slip σ_p can be represented:

$$\sigma_p=\begin{cases}\sigma,\ \sigma<\sigma_s\\ \sigma_s,\ \sigma\ge\sigma_s\end{cases} \tag{11}$$

where σ_s stands for the yield stress.

According to the assembly model theory, the following equation is obtained:

$$\sigma=\sigma_d+\sigma_p \tag{12}$$

where σ stands for total stress of viscoplastic body and σ_d is the stress in the Abel dashpot.

When $\sigma < \sigma_s$, combining eqs. (1) and (2), we obtain $\sigma_d = 0$, that is:

$$\varepsilon_{vp} = 0 \tag{13}$$

When $\sigma \geq \sigma_s$ combining with eq. (12), the following constitutive relation is obtained:

$$\eta_1 D^\gamma(\varepsilon_{vp}) + \sigma_s = \sigma, \ (\gamma > 1) \tag{14}$$

$$D^\gamma(\varepsilon_{vp}) = \frac{\sigma - \sigma_s}{\eta_1}, \ (\gamma > 1) \tag{15}$$

Combining with initial condition $t = 0$, $\varepsilon_{vp} = 0$, eq. (15) can be solved:

$$\varepsilon_{vp} = \frac{\sigma - \sigma_s}{\eta_1} \frac{t^\gamma}{\Gamma(1+\gamma)}, \ (\gamma > 1) \tag{16}$$

i. e.,

$$\varepsilon_{vp} = \begin{cases} 0, \ \sigma \leq \sigma_s \\ \dfrac{\sigma - \sigma_s}{\eta_1} \dfrac{t^\gamma}{\Gamma(1+\gamma)}, \ \sigma \geq \sigma_s, \ \gamma > 1 \end{cases} \tag{17}$$

By comprehensively considering all three parts of strain, the constitutive equation of the fractional derivative creep model can be represented:

$$\varepsilon(t) = \begin{cases} \dfrac{\sigma}{E_0} + \dfrac{\sigma}{\eta_0} \dfrac{t^\beta}{\Gamma(1+\beta)}, \ \sigma < \sigma_s, \ (0 < \beta < 1) \\ \dfrac{\sigma}{E_0} + \dfrac{\sigma}{\eta_0} \dfrac{t^\beta}{\Gamma(1+\beta)} + \dfrac{\sigma - \sigma_s}{\eta_1} \dfrac{t^\gamma}{\Gamma(1+\gamma)}, \ \sigma \geq \sigma_s, \ (0 < \beta < 1, \gamma > 1) \end{cases} \tag{18}$$

Creep experiment

Experimental set-up

In this experiment, a program-controlled creep experiment instrument of Sichuan University, fig. 3, is used. The sample is pure salt obtained by drilling at a depth of 1982 m on the ground of Hubei Jianghan Oilfield Wangchu Mine 1. The dimensions of the standard sample in the first process is 75 mm (diameter) × 150 mm (height).

Parameters of experiment instrument: confining pressure: 0-30 MPa, axial load 0-600 kN, and precision 1%. The environmental temperature and humidity indoors are controlled by an air conditioner.

Figure 3. Experimental set-up of creep test of salt rock

Experimental method

The experiment adopts the method of monomer step loading to add load in a stepwise manner, with size variation from small size to big size. The first stage loading stress is 4 MPa. For each successive stage, 2 MPa is added. The loading period for each stage is 14 days. Figure 3 shows an eight-stage loading general creep curve lasting for 4 and half months.

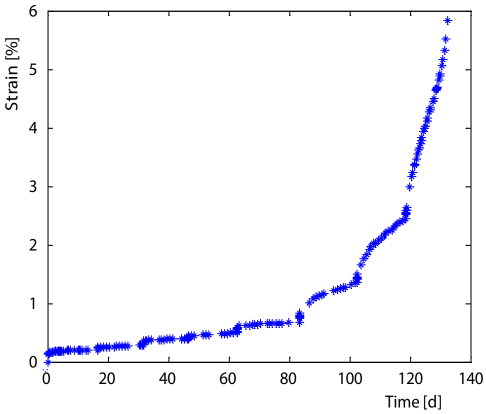

Figure 4. The whole-process creep curve

Experiment creep curve

Figure 4 presents an eight-stage general creep curve, from which we can see that the non-linear accelerated creep stage appears in the eighth creep stage.

Fitting analysis of parameters of fractional derivative creep constitutive model

When $\sigma \geq \sigma_s$, the eight-stage experimental data were separately fitted with the Nishihara and fractional derivative models (initial parameters were taken as $\sigma = 18$, $\sigma_s = 12$, $E_0 = 1.8$, $\eta_0 = 18$, $E_1 = 1.8$, $\eta_1 = 60$, $\beta = 0.3$, and $\gamma = 1.1$. The creep constitutive relation of the Nishihara model:

$$\varepsilon(t) = \frac{\sigma}{E_0} + \frac{\sigma}{E_1}\left[1 - \exp\left(\frac{-E_1}{\eta_0}t\right)\right] + \frac{\sigma - \sigma_s}{\eta_1}t \tag{19}$$

In comparison with eqs. (18) and (19), fitting analysis is performed on the eighth stage experimental data using the non-linear least squares method, see tab. 1 for results.

Table 1. Parameters determined by fitting analysis based on creep tests of salt rock

Model	Parameters					
	E_0/GPa	E_1/GPa	η_0/(GPa h^β)	η_1/(GPa h^γ)	β	γ
Fractional derivative model	0.7113		44.075	$1.8275 \cdot 10^{13}$	0.6988	7.224
Nishihara model	0.718	6.147	15.431	73.794		

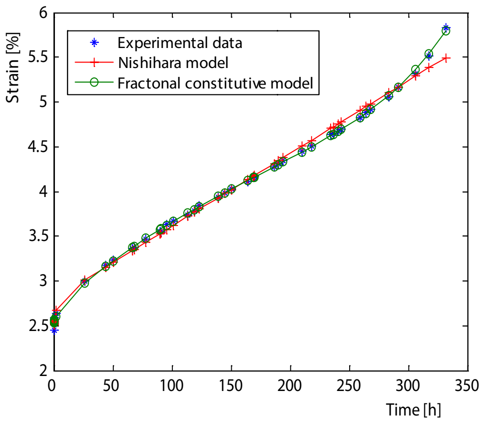

Figure 5. Experimental data and the fitting curves obtained by the Nishihara model and the fractional derivative model

Figure 5 shows the experimental data and model fitting curve. From tab. 1, it can be seen that the least squares deviation of the fractional derivative model is less than that of the Nishihara model. From fig. 5 we can see that the fractional derivative model matches well with the experimental data, and it can better reflect the whole-process creep curve, especially the non-linear accelerated creep stage.

Conclusions

This paper first proposed the new concept of the non-linear element of the Abel dashpot. We then replace the entire viscoelastic body in the Nishihara model with the Abel dashpot, and the Newton dashpot in the viscoplastic body with the super Abel dashpot to construct a new creep constitutive model, thus realizing simulation of all three-stages in the salt rock creep process, especially the non-linear accelerated creep stage. Based on experimental results pertaining to salt rock creep, we determined the parameters in the model using the fitting analysis method. Based on results, the following conclusions are obtained.

Starting from fractional derivative, this work proposed a new concept of super Abel dashpot based on the fractional order Abel dashpot. The super Abel dashpot can well reflect the non-linear accelerated creep stage. We then introduced the non-linear element of the fractional order super Abel dashpot, established a new salt rock creep constitutive model, and deduced a theoretical analytic solution of the model. Using fitting analysis of the experimental data, this model was found to well reflect the whole-process creep curve, especially the non-linear accelerated creep stage.

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Nomenclature

E_0 – elastic modulus of spring in Hooke body, [MPa]
E_1 – elastic modulus of spring in viscoelastic body, [MPa]

Greek symbols

η_0 – viscosity coefficient of Abel dashpot, [GPa h^{β}]
η_1 – viscosity coefficient of super Abel dashpot, [GPa h^{γ}]

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