INFLUENCE OF ELECTRICAL-CONDUCTIVITY OF WALLS ON MAGNETOHYDRODYNAMIC FLOW AND HEAT TRANSFER OF MICROPOLAR FLUID

by

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Original scientific paper
https://doi.org/10.2298/TSCI18S5591K

In this paper, flow and heat transfer in a horizontal channel with isothermal walls has been investigated. The upper and lower plate have been kept at the two constant different temperatures, micropolar fluid is electrically conducting, while the channel plates have arbitrary electrical-conductivity. Applied magnetic field is perpendicular to the flow and the full MHD model is investigated. The general equations that describe the discussed problem under the adopted assumptions are reduced to ODE and closed-form solutions are obtained. The profiles of velocity, microrotation, induced magnetic and temperature fields in function of electrical-conductivity and the coupling parameter and the spin-gradient viscosity parameter together with electrical-conductivity, are graphically shown and discussed

Key words: *electrical-conductivity, MHD, heat transfer, micropolar*

Introduction

Modern technology has stimulated the interest in fluid-flow studies, which involve the interaction of several phenomena. One of these phenomena is certainly viscous flow of electrically conducting micropolar fluid in the presence of a magnetic field. The theory of thermo-micropolar fluids has been developed by Eringen [1], taking into account the effect of micro elements of fluids on both the kinematics and conduction of heat. The concept of micropolar fluid is introduced in an attempt to explain the behavior of a certain fluid containing polymeric additives and naturally occurring fluids such as the phenomenon of the flow of colloidal fluids, real fluid with suspensions, liquid crystals and animal blood, *etc*...

The flow and heat transfer of a viscous incompressible electrically conducting fluid between two infinite parallel insulating plates has been studied by many researchers [2-4] due to its important applications in the further development of MHD technology. The MHD devices for liquid metals attracted the attention of metallurgist [5]. It was shown that the effect of magnetic field could be very helpful in the modernization of technological processes. The increasing interest in the study of MHD phenomena is also related to the development of fusion reactors where plasma is confined by a strong magnetic field [6]. Many exciting innovations were put forth in the areas of MHD propulsion [7], remote energy deposition for drag reduction [8], MHD control of flow and heat transfer in the boundary-layer [9-11].

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All the cited studies are limited to classical Newtonian fluids. There are many fluids which are important from the industrial point of view, and display non-Newtonian behavior. Due to the complexity of such fluids, several models have been proposed but the micropolar model is the most prominent one.

Eringen [1] initiated the concept of micropolar fluids to characterize the suspensions of neutrally buoyant rigid particles in a viscous fluid. The micropolar fluids exhibit microrotational and microintertial effects and support body couple and couple stresses. It may be noted that micropolar fluids take care of the microrotation of fluid particles by means of an independent kinematic vector called microrotation vector.

According to the theory of micropolar fluids proposed by Eringen [1] it is possible to recover the inadequacy of Navier-Stokes theory to describe the correct behavior of some types of fluids with micro-structure such as animal blood, muddy water, colloidal fluids, lubricants, and chemical suspensions. In the mathematical theory of micropolar fluids there is, in general, six degrees of freedom, three for translation and three for microrotation of micro elements. Extensive reviews of the theory and applications can be found in the review articles [12, 13] and in the recent books [14, 15].

The research interest in the MHD flows of micropolar fluids has increased substantially over the past decades due to the occurrence of these fluids in industrial and magneto-biological processes. These flows take into account the effect arising from the local structure and micro-motions of the fluid elements. A comprehensive review of the subject and applications of micropolar fluid mechanics was given by Chamkha *et al.* [16] and Bachok *et al.* [17].

The MHD heat transfer of micropolar fluid can be divided in two parts. One contains problems in which the heating is an incidental by product of electromagnetic fields as in MHD generators *etc.*, and the second consists of problems in which the primary use of electromagnetic fields is to control the heat transfer, Toshivo *et al.* [18]. Heat transfer in micropolar fluid-flow in the presence of magnetic field has gained considerable attention in recent years because of its various applications in contemporary technology. These applications include liquid crystals [19], blood flow in lungs or in arteries [20], flow and thermal control of polymeric processing [21], *etc.*

From the view of practical applications, conducting metal walls are typically present in cooling devices. One of the most prominent examples here are liquid-metal cooled blankets in future nuclear fusion devices. Petrykowski and Walker [22] considered the MHD flows of liquid metals in rectangular channels, where the lower and upper wall considered as a non-conductive, while the side walls are of the ideal conductivity.

Keeping in view the wide area of practical importance of micropolar fluid-flow and heat transfer, as well as influence of electrical-conductivity of walls on flow and heat transfer, as previously mentioned, and continuing our previous work [23], the objective of the present study is to investigate the full MHD flow and heat transfer characteristics of incompressible micropolar fluid in a parallel plate channel with induced magnetic field effects and arbitrary electrical-conductivity of walls. The effects of the governing parameters on the flow and heat transfer aspects of the problem are discussed.

Mathematical and physical model

The problem of laminar MHD flow and heat transfer of an incompressible electrically conducting micropolar fluid between parallel plates is considered. The MHD channel flow analysis is usually performed assuming the fluid constant electrical-conductivity, while the upper and lower plate have arbitrary electrical-conductivity and treating the problem as a 1-D one: with these assumptions, the governing equations are considerably simplified and they can be solved analytically.

The physical model shown in fig. 1, consists of two infinite parallel plates extending in the x- and z-direction. Fully developed flow takes place between parallel plates that are at a distance, *h*, as shown in fig. 1. Electrically conductive fluid-flows through the channel due to the constant pressure gradient and applied magnetic field. A uniform magnetic field of the strength, *B*, is applied in the y-direction. Due the fluid motion magnetic field of the strength, B_x, is induced along the *x*-axis (Reynolds magnetic number takes values around one). The upper and lower plate have been kept at the two constant temperatures, T_1 and, T_2, respectively. The fluid velocity, $\vec{\mathrm{v}}$, and magnetic field $\vec{\mathrm{B}}$ are:

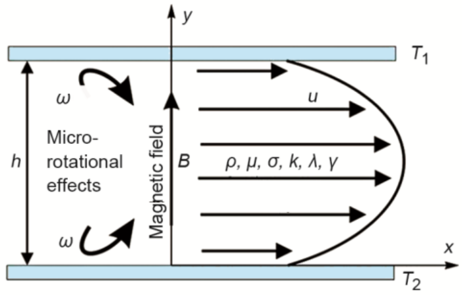

Figure 1. Physical model and co-ordinate system

$$\vec{\mathrm{v}} = u\vec{\mathrm{i}} \tag{1}$$

$$\vec{\mathrm{B}} = B_x\vec{\mathrm{i}} + B\vec{\mathrm{j}} \tag{2}$$

Described laminar MHD flow and heat transfer is mathematically presented with following equations in non-dimensional form:

$$(1+K)\frac{\mathrm{d}^2u}{\mathrm{d}y^2} + K\frac{\mathrm{d}\omega}{\mathrm{d}y} + \frac{\mathrm{Ha}^2}{\mathrm{Rm}}\frac{\mathrm{d}b}{\mathrm{d}y} + \mathrm{Re}\,P = 0 \tag{3}$$

$$\Gamma\frac{\mathrm{d}^2\omega}{\mathrm{d}y^2} - K\frac{\mathrm{d}u}{\mathrm{d}y} - 2K\omega = 0 \tag{4}$$

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}y^2} + (1+K)\,\mathrm{Pr}\,\mathrm{Ec}\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2 + \mathrm{Pr}\,\mathrm{Ec}\frac{\mathrm{Ha}^2}{\mathrm{Rm}^2}\left(\frac{\mathrm{d}b}{\mathrm{d}y}\right)^2 = 0 \tag{5}$$

$$\frac{1}{\mathrm{Rm}}\frac{\mathrm{d}^2b}{\mathrm{d}y^2} + \frac{\mathrm{d}u}{\mathrm{d}y} = 0 \tag{6}$$

The no slip conditions require that the fluid velocities are equal to the plate's velocities, boundary conditions on temperature are isothermal conditions and there is no microrotation near plates, while the electrical-conductivity of walls is arbitrary depending on the constants c $(c = \sigma_W T_W/\sigma h)$. The fluid and thermal boundary conditions for this problem are represented in non-dimensional form by equations:

$$\begin{aligned} &u = 0,\ \omega = 0,\ \theta = 0,\ -b = c\frac{\partial b}{\partial y} \quad \text{for} \quad y = 0, \\ &u = 0,\ \omega = 0,\ \theta = 1,\ b = c\frac{\partial b}{\partial y} \quad \text{for} \quad y = 1 \end{aligned} \tag{7}$$

In previous general equations and boundary conditions, used symbols are common for the theory of MHD flows and the following transformations have been used to transform previous equations to non-dimensional form:

$$u=\frac{u^*}{U_0},\quad y=\frac{y^*}{h},\quad P=-\frac{\partial p}{\partial x}=\text{const.},\quad \theta=\frac{T-T_2}{T_1-T_2},\quad b=\frac{B_x}{B_0},\quad \omega_0=\frac{U}{h},\quad K=\frac{\lambda}{\mu}$$

$$\Gamma=\frac{\gamma}{\mu h^2},\quad \text{Ha}=Bh\sqrt{\frac{\sigma}{\mu}},\quad \Pr=\frac{\mu c_p}{k},\quad \text{Rm}=\sigma\mu_0 Uh,\quad \text{Re}=\frac{U_0 h}{\nu},\quad \text{Ec}=\frac{U^2}{c_p(T_1-T_2)} \tag{8}$$

After basic mathematical transformations from eqs. (3) and (4), the equation for microrotation is:

$$\omega^{iv}-a\omega''+b\omega=0 \tag{9}$$

where:

$$a=\text{Rm}B^*-(A-2)D^*,\quad b=2\text{Rm}B^*D^*,\quad A=\frac{K}{1+K}$$

$$B^*=\frac{1}{1+K}\frac{\text{Ha}^2}{\text{Rm}},\quad C=\frac{\text{Re}\,P}{1+K},\quad D^*=\frac{K}{\Gamma} \tag{10}$$

The solution of eq. (9), are giving three possible cases:

$$\omega(y)=C_1\exp(\delta_1 y)+C_2\exp(\delta_2 y)+C_3\exp(\delta_3 y)+C_4\exp(\delta_4 y) \tag{11}$$

$$\omega(y)=(C_5+C_6 y)\exp(\xi_1 y)+(C_7+C_8 y)\exp(\xi_2 y) \tag{12}$$

$$\begin{aligned}\omega(y)=&\left[C_9\cos(\beta_1 y)+C_{10}\sin(\beta_1 y)\right]\exp(\alpha_1 y)+\\&+\left[C_{11}\cos(\beta_1 y)+C_{12}\sin(\beta_1 y)\right]\exp(-\alpha_1 y)\end{aligned} \tag{13}$$

Solutions for velocity, temperature, and magnetic field are, respectively:

$$u(y)=C_1\mathbf{D}_1\exp(\delta_1 y)+C_2\mathbf{D}_2\exp(\delta_2 y)+C_3\mathbf{D}_3\exp(\delta_3 y)+C_4\mathbf{D}_4\exp(\delta_4 y)+D_1$$

$$\begin{aligned}\theta(y)=&-(1+K)\Pr\text{Ec}\{\mathbf{T}_1\exp(2\delta_1 y)+\mathbf{T}_2\exp(2\delta_2 y)+\mathbf{T}_3\exp(2\delta_3 y)+\\&+\mathbf{T}_4\exp(2\delta_4 y)+\mathbf{T}_6\exp\left[(\delta_1+\delta_3)y\right]+\mathbf{T}_7\exp\left[(\delta_1+\delta_4)y\right]+\\&+\mathbf{T}_8\exp\left[(\delta_2+\delta_3)y\right]+\mathbf{T}_9\exp\left[(\delta_2+\delta_4)y\right]+\mathbf{T}_{11}\exp(\delta_1 y)+\mathbf{T}_{12}\exp(\delta_2 y)+\\&+\mathbf{T}_{13}\exp(\delta_3 y)+\mathbf{T}_{14}\exp(\delta_4 y)+\left(\mathbf{T}_5+\mathbf{T}_{10}+\mathbf{T}_{15}\right)y^2+D_7 y+D_8\}\end{aligned} \tag{14}$$

$$b(y)=-\frac{1}{B^*}\left[Cy+C_1\mathbf{A}_1\exp(\delta_1 y)+C_2\mathbf{A}_2\exp(\delta_2 y)+C_3\mathbf{A}_3\exp(\delta_3 y)+C_4\mathbf{A}_4\exp(\delta_4 y)+D_4\right]$$

$$
\begin{gathered}
u(y)=\left[\frac{1}{D^*}(C_6+\xi_1 C_5)-2\frac{C_5}{\xi_1}+2\frac{C_6}{\xi_1^2}\right]\exp(\xi_1 y)+\left[\frac{1}{D^*}(C_8+\xi_2 C_7)-2\frac{C_7}{\xi_2}+2\frac{C_8}{\xi_2^2}\right]\exp(\xi_2 y)+ \\
+\left(\frac{1}{D^*}\xi_1 C_6-2\frac{C_6}{\xi_1}\right)y\exp(\xi_1 y)+\left(\frac{1}{D^*}\xi_2 C_8-2\frac{C_8}{\xi_2}\right)y\exp(\xi_2 y)+D_2 \\
\theta(y)=-(1+K)\Pr\mathrm{Ec}[\mathbf{T}_{31}\exp(2\xi_1 y)+\mathbf{T}_{32}\exp(2\xi_2 y)+ \\
+\mathbf{T}_{34}\exp(\xi_1 y)+\mathbf{T}_{35}\exp(\xi_2 y)+\mathbf{T}_{36}y^2\exp(2\xi_1 y)+\mathbf{T}_{37}y^2\exp(2\xi_2 y)+ \\
+\mathbf{T}_{39}y\exp(2\xi_1 y)+\mathbf{T}_{40}y\exp(2\xi_2 y)+\mathbf{T}_{42}y\exp(\xi_1 y)+\mathbf{T}_{43}y\exp(\xi_2 y)+ \\
+\frac{1}{2}\mathbf{T}_{30}^*y^2+\frac{1}{6}\mathbf{T}_{27}y^3+\frac{1}{12}\mathbf{T}_{24}y^4+D_9 y+D_{10}] \\
b(y)=-\frac{1}{B^*}[Cy+(AC_5+\Im_3 C_5+\Im_5 C_6)\exp(\xi_1 y)+(AC_7+\Im_4 C_7+\Im_6 C_8)\exp(\xi_2 y)+ \\
+C_6(A+\Im_3)y\exp(\xi_1 y)+C_8(A+\Im_4)y\exp(\xi_2 y)+D_5]
\end{gathered}
\tag{15}
$$

$$
\begin{gathered}
u(y)=\left[(-\Im_1 C_9+\Im_2 C_{10})\sin(\beta_1 y)+(\Im_1 C_{10}+\Im_2 C_9)\cos(\beta_1 y)\right]\exp(\alpha_1 y)+ \\
+\left[-(\Im_1 C_{11}+\Im_2 C_{12})\sin(\beta_1 y)+(\Im_1 C_{12}-\Im_2 C_{11})\cos(\beta_1 y)\right]\exp(-\alpha_1 y)+D_3 \\
\theta(y)=-(1+K)\Pr\mathrm{Ec}\{\left[R_{28}\cos(2\beta_1 y)+R_{29}\sin(2\beta_1 y)+R_{36}\right]\exp(2\alpha_1 y)+ \\
+\left[R_{30}\cos(2\beta_1 y)-R_{31}\sin(2\beta_1 y)+R_{37}\right]\exp(-2\alpha_1 y)+ \\
+\left[R_{32}\sin(\beta_1 y)+R_{33}\cos(\beta_1 y)\right]\exp(\alpha_1 y)+\left[R_{34}\cos(\beta_1 y)-R_{35}\sin(\beta_1 y)\right]\exp(-\alpha_1 y)- \\
-R_{38}\cos(2\beta_1 y)-R_{39}\sin(2\beta_1 y)+\frac{1}{2}R_{19}y^2+D_{11}y+D_{12}\} \\
b(y)=-\frac{1}{B^*}\{Cy+\left[\mathbf{N}_9\cos(\beta_1 y)+\mathbf{N}_{10}\sin(\beta_1 y)\right]\exp(\alpha_1 y)+ \\
+\left[\mathbf{N}_{11}\cos(\beta_1 y)+\mathbf{N}_{12}\sin(\beta_1 y)\right]\exp(-\alpha_1 y)+D_6\}
\end{gathered}
\tag{16}
$$

where in equations (11)-(16):

$$
\begin{gathered}
C_i,\ i=1,\dots 12;\quad \mathbf{D}_i,\ i=1,2,3,4;\quad \mathbf{T}_i,\ i=1,\dots 24;\quad \mathbf{A_i},\ i=1,2,3,4; \\
\mathrm{D}_i,\ i=1,\dots 12;\quad \Im_i,\ i=1,\dots 6;\quad R_i,\ i=28,\dots 39;\quad \mathbf{N}_i,\ i=9,\dots 12
\end{gathered}
\tag{17}
$$

represent constants, while:

$$
\delta_i,\ i=1,2,3,4;\quad \xi_i,\ i=1,2;\quad \alpha_1 \text{ and } \beta_1 \tag{18}
$$

are roots of characteristic eq. (9).

Results and discussion

In the previous section, it has been defined the mathematical model for the steady flow and heat transfer of an incompressible electrically conducting micropolar fluid between two infinite horizontal parallel plates under a constant pressure gradient and applied magnetic field. Obtained solutions for the velocity, microrotation, temperature and induced magnetic fields in function of electrical-conductivity, the coupling parameter and the spin-gradient viscosity parameter are generated graphically.

Figures 2 to 5 show the effect of electrical-conductivity of walls on the distribution of velocity, microrotation, induced magnetic and temperature fields.

Figure 2 shows the influence of electrical-conductivity of walls on velocity profiles of the flow. For the case when the walls are electrically insulated (c = 0), fluid velocity profile is more pronounced then in case of finite (c = 5) and approximately infinite (c = 1000) electrical conductivity of the walls. But, also it can be noted that increase of electrical conductivity of the walls is causing increase of fluid velocity, and decrease of influence of external magnetic field.

For the case when walls of the channel are electrically insulated (c = 0) microrotation of micropolar fluid has the maximum absolute value, while in the situation when electrical conductivity increase from some finite value (c = 5) to approximately infinite (c = 1000), this increase is causing the slight increase of microrotation of the fluid-flow, as shown in fig. 3.

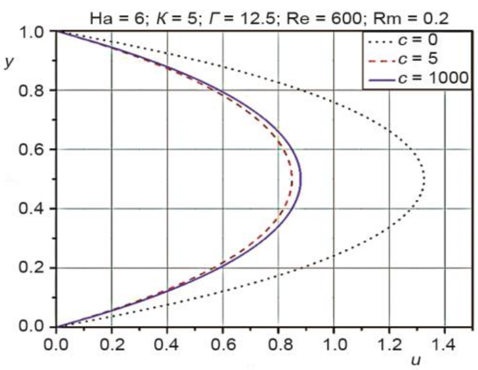

Figure 2. Velocity profiles for different values of electrical-conductivity of walls

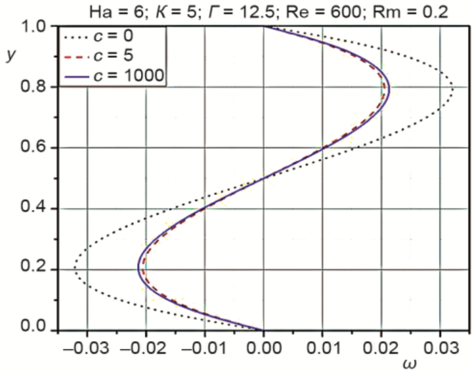

Figure 3. Microrotation in function of electrical-conductivity of walls

For the case when walls are electrically insulated there is no induced magnetic field on them, as it is expected. In the case when walls are of finite electrical conductivity, the induced magnetic field has the maximum values on walls comparing to the case of infinite electrical conductivity of walls. This is graphically presented on figure 4.

Dimensionless temperature in function of electrical-conductivity of walls is shown in fig. 5. When the electrical-conductivity of walls is finite or infinite the viscous dissipation is causing the increase of temperature in the middle of the channel.

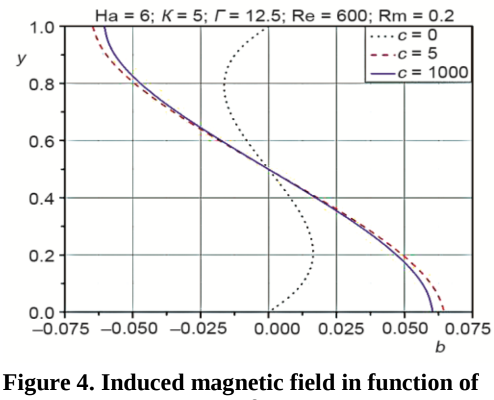

Figure 4. Induced magnetic field in function of electrical-conductivity of walls

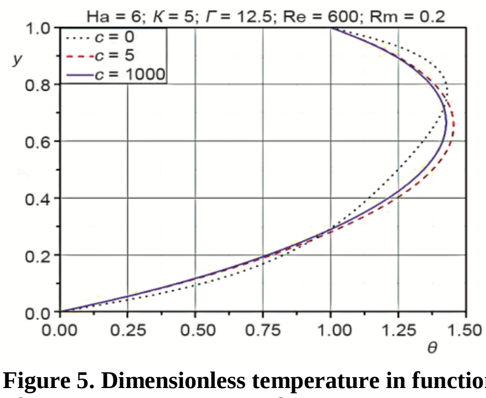

Figure 5. Dimensionless temperature in function of electrical-conductivity of walls

From fig. 6 it can be noted that increase of the coupling parameter is causing decrease of velocity in entire width of the cannel and in the case of infinite electrical-conductivity of walls this leads to intensive reduction of fluid-flow.

In fig. 7 is shown profile of microrotation depending on electrical-conductivity of walls and coupling parameter, *K*.

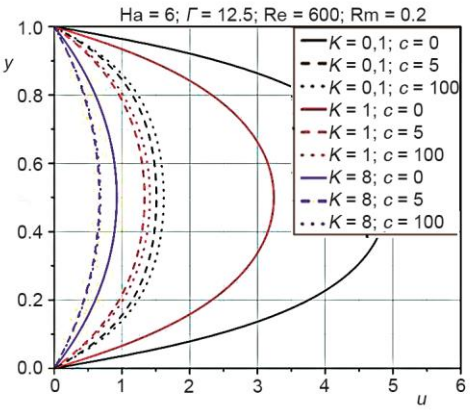

Figure 6. Velocity profiles for different values of electrical-conductivity of walls and the coupling parameter

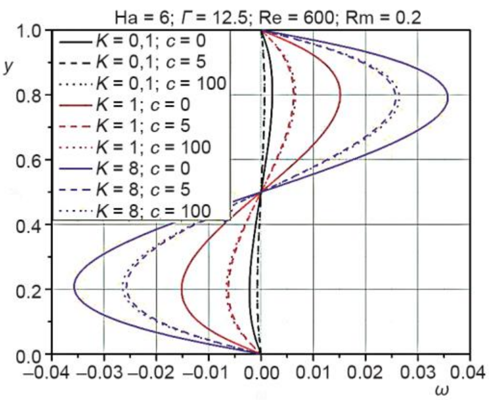

Figure 7. Microrotation profiles for different values of electrical-conductivity of walls and the coupling parameter

As the increase of the coupling parameter, *K*, cause the increase of microrotation, in the case when the walls are electrical insulated the microrotation will have the maximum absolute value.

Figure 8 shows, as it is expected, that increasing of coupling parameter cause decrease in absolute values of induced magnetic field. Furthermore, when the walls are electrical insulated there is no induced magnetic field on them.

Next fig. 9 is showing the influence of coupling parameter on dimensionless temperature in three possible cases, when the walls are electrical insulated ($c = 0$) or they have fi-

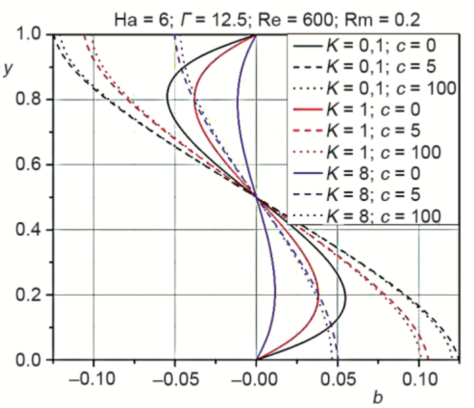

Figure 8. Induced magnetic field for different values of electrical-conductivity of walls and the coupling parameter

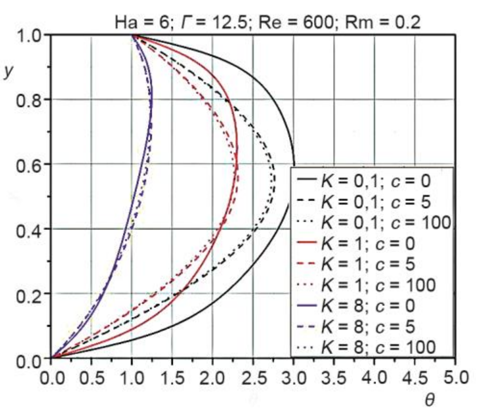

Figure 9. Dimensionless temperature for different values of electrical-conductivity of walls and the coupling parameter

nite (c = 5) or approximately infinite (c = 1000) electrical conductivity. Increasing of the coupling parameter is causing decrease of dimensionless temperature in entire width of channel. In the case when $K \to 0$ the dominant heat transfer mechanism will be conduction.

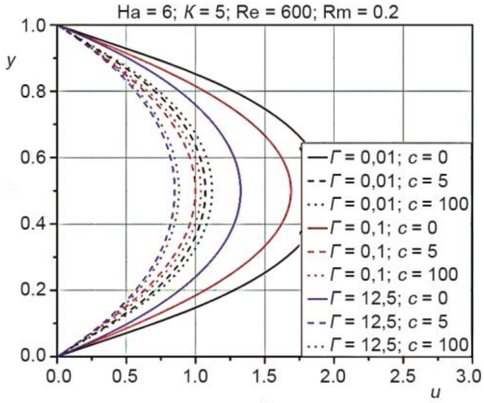

Figure 10. Velocity profiles for different values of electrical-conductivity of walls and the spin-gradient viscosity parameter

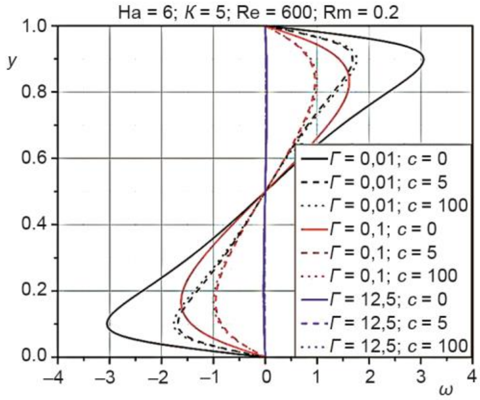

Figure 11. Microrotataion for different values of electrical-conductivity of walls and the spin-gradient viscosity parameter

Figure 10 shows the effect of the spin-gradient viscosity parameter, Γ, on velocity, which predicts that the velocity decrease as the spin-gradient viscosity parameter increase. When walls are electrical insulated and the spin-gradient viscosity parameter $\Gamma \to 0$ the profile of fluid velocity is same like in the case of laminar-flow of viscous fluid.

Microrotation in function of the spin-gradient viscosity parameter is shown in the fig. 11. Increasing of the spin-gradient viscosity parameter cause decrease in absolute values of microrotation. Furthermore, from fig. 11, as well as from figs. 7 and 3, it can be concluded that increase of electrical-conductivity of the walls is causing decrease of microrotation which lead to the fact that increase of electrical-conductivity of the walls reduces micropolar characteristics of the fluid-flow.

Increasing of the spin-gradient viscosity parameter cause decrease in absolute values of induced magnetic field, which is shown in fig. 12. Other fact which can be concluded from

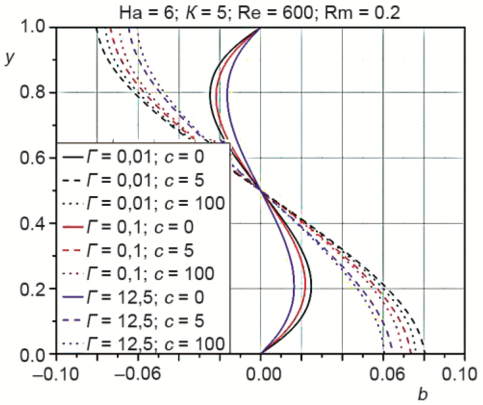

Figure 12. Induced magnetic field for different values of electrical-conductivity of walls and the spin-gradient viscosity parameter

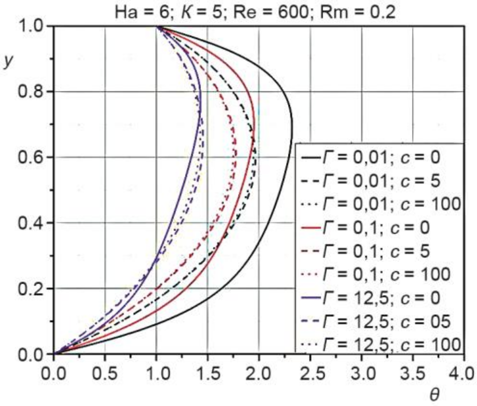

Figure 13. Dimensionless temperature for different values of electrical-conductivity of walls and the spin-gradient viscosity parameter

fig. 12 is that walls of finite electrical-conductivity has a little bit bigger absolute value of induced magnetic field comparing to the case of infinite electrical-conductive walls.

In fig. 13 is shown that increasing of the spin-gradient viscosity parameter cause decrease of dimensionless temperature over the entire width of the channel. Increase of the spin gradient viscosity reduce the amount of energy transformed in the fluid. As gyro-viscosity increases the dominant heat transfer mechanism is conduction.

Conclusion

Flow and heat transfer in a horizontal channel with isothermal walls has been investigated. The upper and lower plate have been kept at the two constant different temperatures, micropolar fluid is electrically conducting, while the channel plates have arbitrary electrical-conductivity. Applied magnetic field is perpendicular to the flow and the full MHD model is investigated. The general equations that describe the discussed problem under the adopted assumptions are reduced to ODE and closed-form solutions are obtained. The influences of each of the governing parameters on dimensionless velocity, dimensionless temperature, microrotation and induced magnetic field are discussed with the aid of graphs.

Acknowledgment

This paper is supported by the Serbian Ministry of Sciences and Technological development (Project No. TR 35016; Research of MHD flow in the channels, around the bodies and application in the development of the MHD pump).

Nomenclature

$\vec{B}$ – magnetic field vector, [T]
c_p – specific heat capacity, [$Jkg^{-1}K^{-1}$]
Ec – Eckert number
Ha – Hartmann number
h – height of channel, [m]
k – thermal conductivity of fluid, [$WK^{-1}m^{-1}$]
Pr – Prandtl number
p – pressure, [Pa]
Rm – Reynolds magnetic number
T – temperature, [K]
u – fluid velocity, [ms^{-1}]
x – longitudinal co-ordinate, [m]
y – transversal co-ordinate, [m]

Greek symbols

γ – spin gradient viscosity, [$kgms^{-1}$]
θ – dimensionless temperature
λ – vortex viscosity, [$kgm^{-1}s^{-1}$]
μ – dynamic viscosity, [$kgm^{-1}s^{-1}$]
ν – kinematic viscosity, [m^2s^{-1}]
ρ – density of fluid, [kgm^{-3}]
σ – electrical conductivity, [Sm^{-1}]
ω – microrotation vector, [s^{-1}]

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Paper submitted: March 23, 2018
Paper revised: July 6, 2018
Paper accepted: July 10, 2018