A COMPREHENSIVE NOTE ON THERMALLY STRATIFIED FLOW AND NON-FOURIER HEAT FLUX THEORY

by

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Here an analysis is presented to investigate the characteristics of non-Fourier flux concept in flow induced by stretching cylinder. Unlike the convectional approach, the heat flux by Cattaneo-Christov theory is considered. Stagnation point flow in the presence of thermal stratification and temperature dependent thermal conductivity is addressed. Rheological properties are examined for hyperbolic tangent material. Theory of the boundary-layer is implemented for the formulation purpose. The relevant transformations yield the strong non-linear differential systems which are numerically computed. Plots are presented for the solution expressions of velocity and temperature. Surface drag force is calculated and discussed. Here velocity and temperature are enhanced for the larger curvature parameter. Larger values of thermal relaxation factor give rise to be decrease in temperature.

Key words: *non-Fourier heat flux theory, thermally stratified medium, variable properties, hyperbolic tangent liquid*

Introduction

The transport phenomenon of non-Newtonian materials emerges in several branches of chemical, materials, and mechanical engineering. These materials illustrate shear stress/strain association which deviate considerably from the conventional Newtonian model *i. e Navier-Stokes model*. Different models for non-Newtonian materials comprise few adjustments in momentum conservation expressions [1-10]. Hyperbolic tangent four constants fluid is one of such models which can describe the shear thinning properties. This model is interpreted through the kinetic concept of liquids instead of empirical relation. Apparent viscosity of hyperbolic tangent fluid varies between zero to infinity. Moreover, this model has several advantages over other models including the ease of computation, simplicity and physical robustness. Some recent flow investigations involving hyperbolic tangent liquid and other fluids models are given through [11-20] and many studies therein.

Theory of heat conduction established by Fourier [20] communicates heat flux in a direct manner with temperature gradient utilizing coefficient of thermal conductivity. Under cer-

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tain circumstances the Fourier relation is authentic enough for distinct engineering problems apart from the problems which incorporate relatively higher temperature gradient, nano and micro scales in space and time, absolute null temperatures and small variation in temperature [21, 22]. Several non-Fourier conduction concepts have been presented to determine these issues. The situations in microelectronic mechanisms including combined circuit chips, laser pulses heating of high rate or heat flux via cutting and melting of materials and in few non-homogeneous materials, the non-Fourier conduction is very significant [23-26]. Thus some numerical and analytical investigations regarding non-Fourier conduction have been presented in literature. For instance Straughan [27] explored thermal convection in non-Fourier flux model. Uniqueness of non-Fourier flux for incompressible flow liquids is investigated by Tibulle *et al.* [28]. Han *et al.* [29] reported the non-Fourier flux features in stretched flow of Maxwell material. Analysis presented in [29] is further extended by Hayat *et al.* [30] by considering non-Fourier flux and variable thickness of sheet. Khan *et al.* [31] established analytical solutions for 3-D stretched flow of Burgers material in presence of different chemical processes and non-Fourier flux. Effectiveness of thermal stratification in variable thermal conductivity stretched flow of viscous and Eyring-Powell materials is addressed by Hayat *et al.* [32, 33]. Ali *et al.* [34] examined radiation and MHD characteristics in Casson ferrofluid. Non-Fourier flux consideration in Maxwell nanoliquid flow with slip aspect is presented by Sui *et al.* [35]. Nadeem *et al.* [36] considered non-Fourier flux and stratification phenomenon in flow by Maxwell nanomaterial.

Thermal stratification is regarded a renowned device of biogeochemical series in lakes [37-39]. Strong stratification decelerates the avoids of transfer of elements within surface and ground waters. Consequently, the regulating methods and subsequent water excellence in small citified pond bionetworks would possibly be influenced by the time and strength of temperature stratification. However communication between physical and biogeochemical procedures (for example temperature stratification) in citified pond bionetworks remain badly studied. Our intention here is to explore stratification characteristics in variable thermal conductivity stretched flow of hyperbolic tangent material. Flow caused here is due to stretching cylinder. Some investigations regarding laminar and turbulence flow can be mentioned through [40-47]. Solutions of non-linear system are obtained numerically through bvp4c technique. Graphical demonstrations for diverse influential variables against velocity and temperature distributions are further motivation of this investigation.

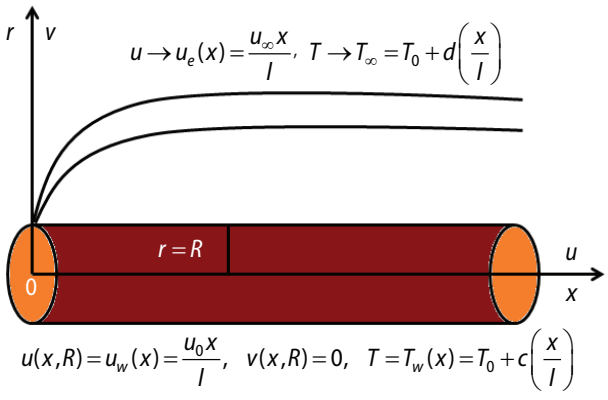

Figure 1. Geometry of flow problem

Formulation

Flow equations

Here 2-D stagnation point flow of incompressible hyperbolic tangent material towards stretched cylinder with velocity $u_w(x) = u_0(x/l)$ is investigated, see fig. 1.

The free stream velocity is denoted by u_e. Heat generation/absorption and dissipation effect are neglected. Heat transfer process is in the absence of heat source/sink and viscous dissipation effect. Heat transfer process is based using non-Fourier heat flux. Governing flow expressions for hyperbolic tangent liquid [27]:

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\nu(1-n)\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)+n\sqrt{2}\Gamma\nu\frac{\partial u}{\partial r}\frac{\partial^2 u}{\partial r^2}+$$
$$+\frac{n\Gamma}{2}\nu\left(\frac{\partial u}{\partial r}\right)^2+u_e(x)\frac{\mathrm{d}u_e(x)}{\mathrm{d}x} \tag{2}$$

with

$$u(x,R)=u_w(x)=\frac{u_0 x}{l},\quad v(x,R)=0$$
$$u(x,r)\to u_e(x)=\frac{u_\infty x}{l}\quad \text{as}\quad r\to\infty \tag{3}$$

In the aforestated expressions (u,v) signify the velocity components in (x,r) directions, respectively, ν – the kinematic viscosity, ρ – the density of the liquid, Γ – the material parameter, n – the power index, and (u_w,u_e) the stretching and free stream velocities, respectively. In view of the transformations:

$$\eta=\sqrt{\frac{u_0}{\nu l}}\left(\frac{r^2-R^2}{2R}\right),\quad u=\frac{u_0 x}{l}f'(\eta),\quad v=-\frac{R}{r}\sqrt{\frac{u_0\nu}{l}}f(\eta) \tag{4}$$

Equation (1) is justified automatically whereas eqs. (2) and (3) are converted to the following forms:

$$(1-n)(1+2\gamma\eta)f'''+2\gamma(1-n)f''+ff''-f'^2+$$
$$+2\lambda n(1+2\gamma\eta)^{3/2}f''f'''+3\lambda\gamma(1+2\gamma\eta)^{1/2}(f'')^2+A^2=0 \tag{5}$$

$$f(0)=0,\quad f'(0)=1\quad\text{and}\quad f'(\infty)=A \tag{6}$$

where $\lambda=(\Gamma u_0^{3/2}x)/[(2\nu)^{1/2}l^{3/2}]$ denotes the Weissenberg number, $A=u_\infty/u_0$ the ratio of velocities, and $\alpha=[(\nu l)/(u_0R^2)]^{1/2}$ the curvature parameter.

Energy equation

The energy equation for this case is:

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=-\vec{\nabla}\vec{\mathbf{q}} \tag{7}$$

where T is the temperature, c_p – the specific heat, and $\vec{\mathbf{q}}$ – the heat flux. Heat flux in view of Cattaneo-Christov expression satisfies:

$$\vec{\mathbf{q}}+\lambda_1\left(\frac{\partial\vec{\mathbf{q}}}{\partial t}+\vec{\mathbf{v}}\vec{\nabla}\vec{\mathbf{q}}-\vec{\mathbf{q}}\vec{\nabla}\vec{\mathbf{v}}+\left(\vec{\nabla}\vec{\mathbf{v}}\right)\vec{\mathbf{q}}\right)=-k(T)\vec{\nabla}T \tag{8}$$

where λ_1 denotes the thermal relaxation time of heat flux and $k(T)$ the temperature dependent thermal conductivity. For $\lambda_1=0$, eq. (8) reduces to Fourier's law. Equation (8) for incompressibility condition yields:

$$\vec{\mathbf{q}} + \lambda_1\left(\vec{\mathbf{v}}\vec{\nabla}\vec{\mathbf{q}} - \vec{\mathbf{q}}\vec{\nabla}\vec{\mathbf{v}}\right) = -k\left(T\right)\vec{\nabla}T \tag{9}$$

Eliminating $\vec{\mathbf{q}}$ from eqs. (7) and (9) we obtain:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} + \lambda_1\left(\begin{array}{c} u^2\dfrac{\partial^2 T}{\partial x^2} + v^2\dfrac{\partial^2 T}{\partial r^2} + 2uv\dfrac{\partial^2 T}{\partial r\partial x} + u\dfrac{\partial u}{\partial x}\dfrac{\partial T}{\partial x} + \\ +u\dfrac{\partial v}{\partial x}\dfrac{\partial T}{\partial r} + v\dfrac{\partial u}{\partial r}\dfrac{\partial T}{\partial x} + v\dfrac{\partial v}{\partial r}\dfrac{\partial T}{\partial r} \end{array}\right) = \frac{1}{\rho c_p r}\frac{\partial}{\partial r}\left[k\left(T\right)r\frac{\partial T}{\partial r}\right] \tag{10}$$

with the conditions:

$$\begin{aligned} T &= T_w\left(x\right) = T_0 + c\left(\frac{x}{l}\right) \quad \text{at} \quad r = a \\ T &= T_\infty\left(x\right) = T_0 + d\left(\frac{x}{l}\right) \quad \text{when} \quad r \to \infty \end{aligned} \tag{11}$$

where (c, d) denotes the dimensional constants and (T, T_0, T_∞) the fluid, reference and ambient temperatures, respectively. The temperature dependent thermal conductivity $k(T)$ is [24]:

$$k\left(T\right) = k_\infty\left(1 + \varepsilon\frac{T - T_\infty}{\Delta T}\right) \tag{12}$$

in which ε represents the small scalar parameter and k_∞ the thermal conductivity of the ambient fluid and $\Delta T = T_w - T_0$.

Employing:

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_0} \tag{13}$$

together with eq. (11) we have from eq. (10):

$$\left(1 + 2\gamma\eta\right)\theta'' + 2\gamma\theta' + \Pr(f\theta' - f'\theta) + \varepsilon\left[\begin{array}{c}\left(1 + 2\gamma\eta\right) \\ (\theta'^2 + \theta\theta'') + 2\gamma\theta\theta'\end{array}\right] - \Pr\delta ff'\theta' - \Pr\delta f^2\theta'' = 0 \tag{14}$$

$$\theta(0) = 1, \quad \theta(\infty) = 0 \tag{15}$$

where $\Pr = \mu c_p/k$ shows the Prandtl number and $\gamma = \lambda_1 u_0/l$ the thermal relaxation factor.

Physical quantity

Here skin friction coefficient is:

$$C_f = \frac{2\tau_w}{\rho u_w^2} \tag{16}$$

The wall shear stress at $r = R$ is:

$$\tau_w = \mu\left[\left(1 - n\right)\frac{\partial u}{\partial r} + \frac{n}{\sqrt{2}}\Gamma\left(\frac{\partial u}{\partial r}\right)^2\right]_{r=R} \tag{17}$$

From eqs. (16) and (17) one has:

$$\frac{C_f \operatorname{Re}_x^{1/2}}{2} = (1-n) f''(0) + n\lambda f''^2(0) \tag{18}$$

In which Re_x denotes Reynolds number.

Solution methodology and analysis of results

Here bvp4c technique is employed in order to solve eqs. (5) and (14) together with boundary conditions (6) and (15). Our intention here is to analyze the characteristics of distinct physical variables like curvature parameter, γ, power index, n, Weissenberg number, λ, Prandtl number, thermal stratification parameter, S, variable thermal conductivity parameter, ε, and thermal relaxation parameter, δ, on dimensionless velocity, $f'(\eta)$, and temperature $\theta(\eta)$. Such objective is achieved via plots in figs. 2-11.

Figure 2 disclosed the impact of γ on $f'(\eta)$. Larger values of γ enhance the velocity. Physically radius of cylinder reduces when α is enhanced which offers little resistance to the fluid motion and consequently the fluid velocity increases. Effect of n on $f'(\eta)$ is depicted through fig. 3. Clearly larger n decay $f'(\eta)$ and related thickness of momentum layer. Analysis of characteristics of λ on $f'(\eta)$ is presented through fig. 4. Since Weissenberg number is the ratio of relaxation time of the fluid and a specific process time. It increases the thickness of the fluid due to which the velocity decays for larger λ.

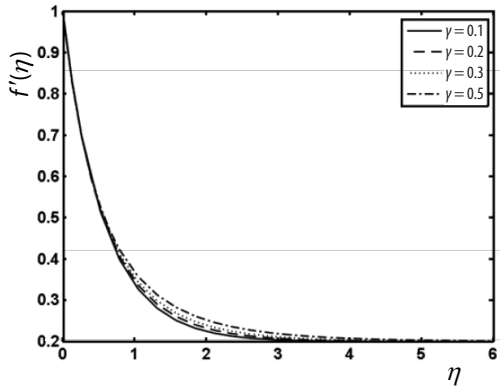

Figure 2. The $f'(\eta)$ via γ when $S = \delta = \varepsilon = 0.2$, $n = A = \lambda = 0.1$ and Pr = 2.0

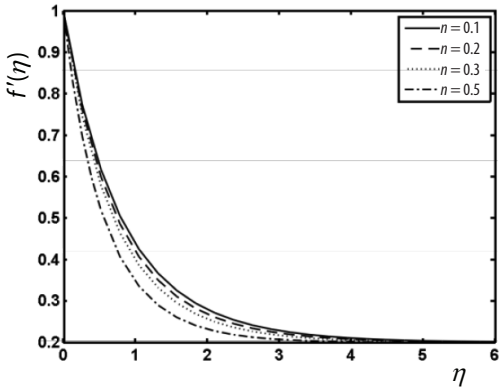

Figure 3. The $f'(\eta)$ via n when $S = \delta = \varepsilon = \gamma = 0.2$, $A = \lambda = 0.1$ and Pr = 2.0

Figure 5 reports the behavior of γ on $\theta(\eta)$. Clearly $\theta(\eta)$ decays near stretching cylinder however it increases far away from the surface. In fact, rise in γ causes an enhancement in thermal thickness of thermal layer. As a consequence, the heat transport rate reduces and therefore $\theta(\eta)$ of the liquid is higher. Salient features of n and λ on $\theta(\eta)$ are described via figs. 6 and 7. Here $\theta(\eta)$ is via enhancement of n. When n is enhanced however opposite behavior is noted for higher λ. Figure 8 signifies the impact of Prandtl number on $\theta(\eta)$. As expected $\theta(\eta)$ diminishes for higher values of Prandtl number since there is an inverse relationship between Prandtl number and thermal diffusivity. Due to this reason the temperature decays. Figure 9 elucidates the variation of S on $\theta(\eta)$. It is reported that $\theta(\eta)$ and related thermal layer are reduced when S increases. Physically larger S reduce the convective flow between ambient fluid and heated cylinder and therefore $\theta(\eta)$ reduces. Behavior of ε on $\theta(\eta)$ is scrutinized through

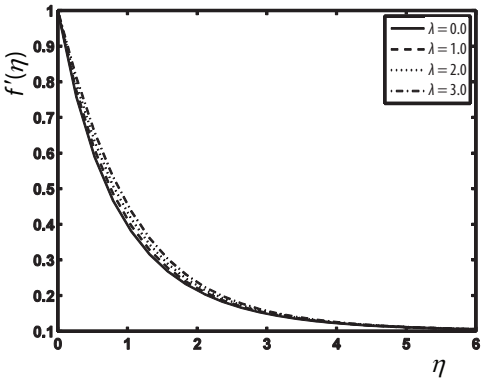

Figure 4. The $f'(\eta)$ via λ when $S = \delta = \varepsilon = \gamma = 0.2$, $A = n = 0.1$ and Pr = 2.0

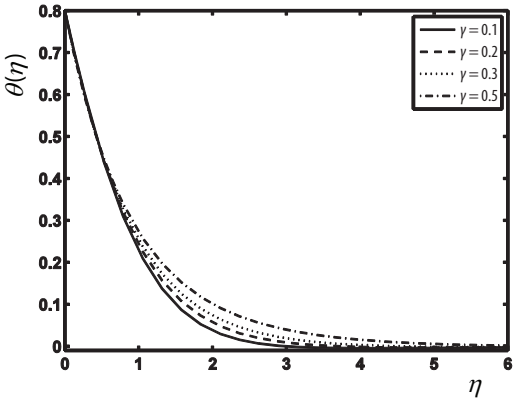

Figure 5. The $\theta(\eta)$ via γ when $S = \delta = \varepsilon = 0.2$, $A = n = \lambda = 0.1$ and Pr = 2.0

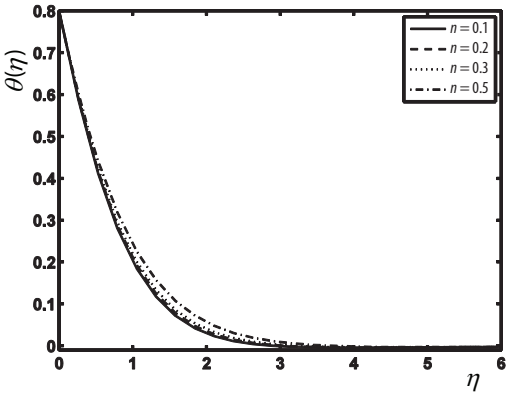

Figure 6. The $\theta(\eta)$ via n when $S = \delta = \varepsilon = \gamma = 0.2$, $A = \lambda = 0.1$ and Pr = 2.0

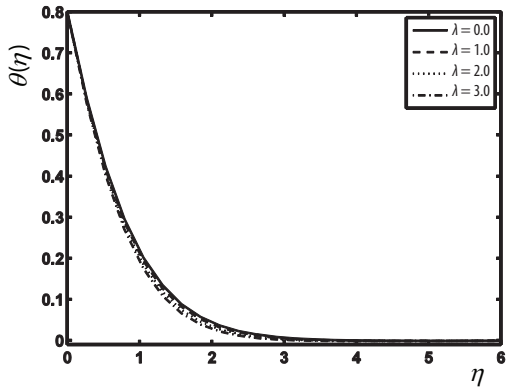

Figure 7. The $\theta(\eta)$ via λ when $S = \delta = \varepsilon = \gamma = 0.2$, $A = n = 0.1$ and Pr = 2.0

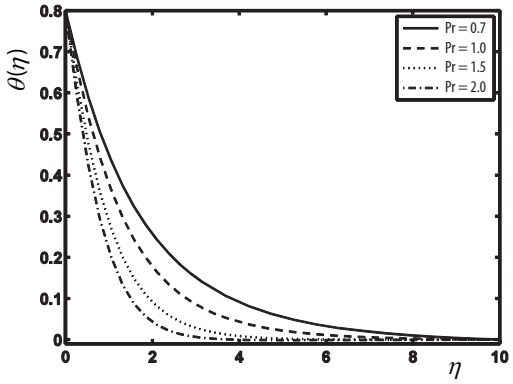

Figure 8. The $\theta(\eta)$ via Pr when $S = \delta = \varepsilon = \gamma = 0.2$, and $A = n = \lambda = 0.1$

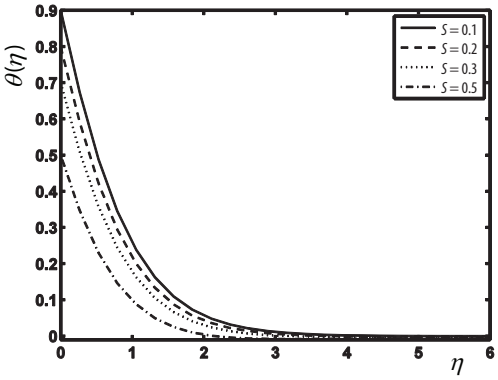

Figure 9. The $\theta(\eta)$ via S when $\delta = \varepsilon = \gamma = 0.2$, and $A = n = \lambda = 0.1$, and Pr = 2.0

fig. 10. Here $\theta(\eta)$ is an increasing function of ε. Physically thermal conductivity of the liquid increases for larger ε due to which more amount of heat is transferred from sheet to the fluid and thus temperature enhances. Figure 11 highlights the impact of δ on $\theta(\eta)$. Here temperature enhances via δ. Physically when we increase δ then material particles need extra time to transfer heat to its adjacent particles and so temperature reduces. For $\delta = 0$ the heat transfers promptly throughout the material. Hence temperature distribution is higher for $\delta = 0$ *i. e.* for Fourier's law when compared with Cattaneo-Christov heat flux model.

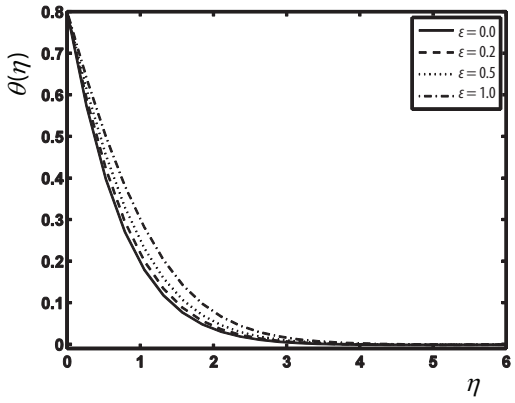

Figure 10. The $\theta(\eta)$ via ε when $\delta = S = \gamma = 0.2$, and $A = n = \lambda = 0.1$ and Pr = 2.0

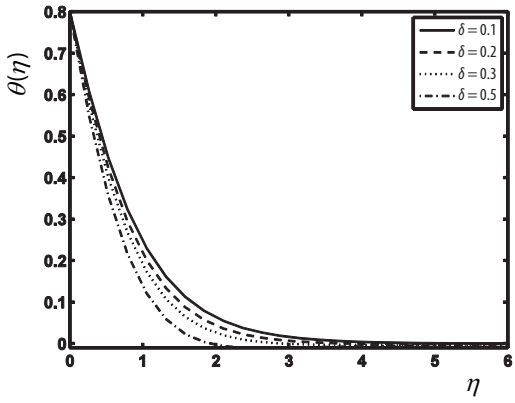

Figure 11. The $\theta(\eta)$ via δ when $\varepsilon = S = \gamma = 0.2$, and $A = n = \lambda = 0.1$ and Pr = 2.0

Table 1 explores characteristics of n, γ, λ, and A on skin friction ($C_f \mathrm{Re}_x^{1/2}$) when $\varepsilon = S = \delta = 0.2$ and Pr = 2.0. Here $C_f \mathrm{Re}_x^{1/2}$ rises via larger γ whereas it decays for n, λ and A.

Table 1. Numerical data for skin friction ($C_f \mathrm{Re}_x^{1/2}$) when $\varepsilon = S = \delta = 0.2$ and Pr = 2.0

n	γ	λ	A	$C_f \mathrm{Re}_x^{1/2}$
0.1	0.2	0.1	0.1	–0.9625799
0.2				–0.8981956
0.3				–0.8287302
0.5				–0.6655272
0.1	0.1			–0.9395822
	0.2			–0.9625799
	0.3			–0.9848631
	0.5			–1.0274700
	0.2	0.0		–0.9839514
		1.0		–0.787906
		2.0		–0.6319903
		3.0		–0.5141813
		0.1	0.1	–0.9625799
			0.2	–0.9122701
			0.3	–0.844517
			0.4	–0.7614328

Conclusions

Main objective of this research is to report the application of non-Fourier heat flux theory for flow of tangent hyperbolic liquid in a thermally stratified medium. The main points are listed below.

- Curvature parameter, γ, magnifies, $f'(\eta)$, and $\theta(\eta)$.
- Opposite behaviors of n and λ on $f'(\eta)$ and $\theta(\eta)$ are observed.
- Temperature is higher for larger ε when compared with γ and Prandtl number.
- Temperature for Fourier's expression is larger than non-Fourier expression.
- Skin friction coefficient is an increasing function of γ.
- The results for flux due to Fourier's expression can be obtained by putting $\delta = 0$.

Nomenclature

A – ratio of velocities
C_f – skin friction coefficient
c_p – specific heat
$f'(\eta)$ – dimensionless velocity
$k(T)$ – variable thermal conductivity
k_∞ – thermal conductivity of ambient fluid
l – characteristics length
n – power index
Pr – Prandtl number
$\vec{\mathbf{q}}$ – heat flux
R – radius of cylinder
Re_x – Reynolds number
T – temperature
T_∞ – ambient temperature
u, v – velocity components
u_e – free stream velocity
u_w – stretching velocity
x, y – space co-ordinates

Greek symbols

γ – curvature parameter
Γ – material parameter
δ – thermal relaxation parameter
ε – variable thermal conductivity parameter
η – non-dimensional parameter
$\theta(\eta)$ – dimensionless temperature
λ_1 – thermal relaxation time of heat flux
μ – dynamic viscosity
ν – kinematic viscosity
ρ – fluid density
τ_w – wall shear stress

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