A GENERALIZED FOURIER AND FICK'S PERSPECTIVE FOR STRETCHING FLOW OF BURGERS FLUID WITH TEMPERATURE-DEPENDENT THERMAL CONDUCTIVITY

by

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This research addresses heat generation and mixed convection characteristics in Burgers fluid-flow induced by moving surface considering temperature-dependent conductivity. The novel revised Fourier-Fick relations covering heat/mass paradoxes are introduced simultaneously. Boundary-layer concept is implemented for simplification of mathematical model of considered physical problem. Compatible transformations are utilized to transform partial differential system into ordinary ones. The idea of homotopic scheme is employed to establish convergent series solutions. The mechanisms of heat-mass transportation are elaborated graphically by constructing graphs for distinct values of physical constraints. We noticed higher temperature and concentration for Fourier-Fick situations when compared with revised Fourier-Fick situations. Furthermore, an increment in variable conductivity factor yields higher temperature and related thickness of thermal layer. The obtained results are compared with available literature in a limiting manner and reasonable agreement is found.

Key words: revised Fourier-Fick relations, heat generation, Burgers fluid, mixed convection, temperature-dependent conductivity

Introduction

Non-Newtonian liquids like toothpaste, paint, animal blood, milk, grease are naturally omnipresent and extensively utilized in oil exploration, food processing, medical, chemical and bio-chemical engineering [1-5]. The non-Newtonian liquids in general are categorized in rate, differential and integral types. Rate type models elaborates relaxation/retardation times characteristics. Burgers liquid among rate-type liquids is the generalization of Maxwell [6] and Oldroyd-B [7] models predominantly effective for polymers, asphalt concrete and reaction of asphalt [8]. Burgers model is considered and modeled utilizing distinct aspects. For illustration, Alsaedi *et al.* [9] formulated chemically reacted flow in the stagnation region of Burgers liquid. Thermal radiation impact in chemically reacted Burgers liquid flow towards heated surface is

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presented by Khan *et al.* [10]. Hayat *et al.* [11-14] established homotopic solutions for Burgers liquid considering various aspects. Recently, heat generation and gyrotactic microorganisms effects in magneto mixed convective Burgers liquid are addressed by Khan *et al.* [15].

The traditional heat conduction relation, Fourier relation [16], communicates heat flux precisely to temperature gradient utilizing coefficient of thermal conductivity. Fourier relation is not effective for the problems which comprise high thermal gradient, absolute zero temperatures, small variations in temperature and nano/micro scales in space and time [17]. Several theories regarding improvement in Fourier heat conduction relation have been introduced. The situations in microelectronic devices like high frequency heating laser pulse, combined circuit chips, high flux for cutting and melting of objects and in few non-homogeneous objects, the heat conduction through revised Fourier relation is very consequential [18, 19]. Christov [20] revisited the analysis of Cattaneo [18] for material-invariant formulation by including the relaxation time contribution comprising upper-convected Oldroyd's derivatives. Afterwards, several researches in this direction have been reported, for detail see [21-30].

Keeping aforestated analyses in mind, it is noticed that energy expression through revised Fourier relation is reported extensively. However concentration expression by revised Fick relation is not yet studied. Thus our objective here in this investigation is to venture further in this regime by considering mixed convective Burger liquid flow bounded by moving sur-



Figure 1. Physical configuration

face. Heat generation and variable conductivity aspects are retained in energy expression. Homotopy scheme [31-44] is opted for computations of non-linear systems. The outcomes of presented analysis are displayed and discussed.

Formulation

We aim to formulate mixed convective laminar flow of incompressible Burgers liquid towards moving surface subject to revised Fourier-Fick relations. Heat generation and thermal dependence choice of conductivity aspects are considered for energy expression formulation. Whole analysis is addressed by ignoring viscous dissipation and thermal radiation contributions. Detailed flow assumptions can be understood through fig. 1. We have the following governing expressions [1]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda_{1} \left(u^{2} \frac{\partial^{2} u}{\partial x^{2}} + v^{2} \frac{\partial^{2} u}{\partial y^{2}} + 2uv \frac{\partial^{2} u}{\partial x \partial y} \right) +$$

$$= \left[u^{3} \frac{\partial^{3} u}{\partial x^{3}} + v^{3} \frac{\partial^{3} u}{\partial y^{3}} + u^{2} \left(\frac{\partial^{2} u}{\partial x^{2}} \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \frac{\partial^{2} v}{\partial x^{2}} + 2 \frac{\partial v}{\partial x} \frac{\partial^{2} u}{\partial x \partial y} \right) +$$

$$+ \lambda_{2} \left[+ 3v^{2} \left(\frac{\partial v}{\partial y} \frac{\partial^{2} u}{\partial y^{2}} + \frac{\partial u}{\partial y} \frac{\partial^{2} u}{\partial x \partial y} \right) + 3uv \left(u \frac{\partial^{3} u}{\partial x^{2} \partial y} + v \frac{\partial^{3} u}{\partial x \partial y^{2}} \right) +$$

$$+ 2uv \left(\frac{\partial u}{\partial y} \frac{\partial^{2} u}{\partial x^{2}} + \frac{\partial v}{\partial x} \frac{\partial^{2} u}{\partial y^{2}} + \frac{\partial v}{\partial y} \frac{\partial^{2} u}{\partial x \partial y} - \frac{\partial u}{\partial y} \frac{\partial^{2} v}{\partial x \partial y} \right) \right]$$

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$$= v \left[\frac{\partial^2 u}{\partial y^2} + \lambda_3 \left(u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} \right) \right] + g \left[\Lambda_1 (T - T_{\infty}) + \Lambda_2 (C - C_{\infty}) \right]$$
(2)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \lambda_{T} \begin{bmatrix} u\frac{\partial u}{\partial x}\frac{\partial T}{\partial x} + v\frac{\partial v}{\partial y}\frac{\partial T}{\partial y} + v\frac{\partial u}{\partial y}\frac{\partial T}{\partial x} + 2uv\frac{\partial^{2}T}{\partial x\partial y} + u^{2}\frac{\partial^{2}T}{\partial x^{2}} + v^{2}\frac{\partial^{2}T}{\partial x^{2}} + v^{2}\frac{\partial^{2}T}{\partial y^{2}} - \frac{Q}{\rho c_{p}}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) \end{bmatrix} = \frac{1}{\rho c_{p}}\frac{\partial}{\partial y}\left[K\left(T\right)\frac{\partial T}{\partial y}\right] + \frac{Q}{\rho c_{p}}\left(T - T_{\infty}\right)$$
(3)

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + \lambda_C \left(u\frac{\partial u}{\partial x}\frac{\partial C}{\partial x} + v\frac{\partial v}{\partial y}\frac{\partial C}{\partial y} + v\frac{\partial u}{\partial y}\frac{\partial C}{\partial x} + u\frac{\partial v}{\partial y}\frac{\partial C}{\partial x} + u\frac{\partial v}{\partial y}\frac{\partial C}{\partial x} + u\frac{\partial v}{\partial y}\frac{\partial C}{\partial x} + v\frac{\partial v}{\partial y}\frac{\partial C}{\partial y}\frac{\partial C}{\partial x} + v\frac{\partial v}{\partial y}\frac{\partial C}{\partial y}\frac{\partial C}{\partial y}\frac{\partial C}{\partial x} + v\frac{\partial v}{\partial y}\frac{\partial C}{\partial y}\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}\frac{\partial C}{\partial y}\frac{\partial C}{\partial y}\frac{\partial v}{\partial y}\frac{\partial C}{\partial y}\frac{\partial v}{\partial y}\frac{\partial C}{\partial y}\frac{\partial v}{\partial y}\frac{\partial v}{\partial y}\frac{\partial C}{\partial y}\frac{\partial v}{\partial y}\frac{\partial v}{\partial$$

with conditions [10]:

$$u = U_w(x) = cx, \ v = 0, \ T = T_w, \ C = C_w \text{ at } y = 0,$$

$$u \to 0, \ T \to T_{\infty}, \ C \to C_{\infty} \text{ when } y \to \infty$$
(5)

Note that (u, v) illustrate liquid velocities in (horizontal, vertical) directions, respectively, v – the kinematic viscosity, ρ – the liquid density, (λ_1, λ_3) – the relaxation/retardation times, λ_2 – the material variable of Burgers fluid, g – the gravitational acceleration, (Λ_1, Λ_2) – the (thermal, solutal) expansion coefficients, (T, C) – the fluid (temperature, concentration), (T_{∞}, C_{∞}) – the ambient fluid (temperature, concentration), (λ_T, λ_C) – the (heat, mass) flux relaxation times, Q – the heat absorption/generation coefficient, D – the mass diffusion, and c – the stretching rate. Variable conductivity in mathematical form is [7]:

$$K(T) = K_{\infty} \left(1 + \varepsilon \frac{T - T_{\infty}}{\Delta T} \right)$$
(6)

in which $\Delta T = T_w - T_\infty$, K_∞ – the ambient liquid conductivity, and ε – the small variable which identifies the characteristic of temperature for thermal dependence conductivity.

Letting [7]:

$$\eta = y \sqrt{\frac{c}{\nu}}, \quad u = cxf'(\eta), \quad v = -\sqrt{c\nu} f(\eta),$$

$$\theta(\eta) = \frac{T - T_{\infty}}{T_{w} - T_{\infty}}, \quad \varphi(\eta) = \frac{C - C_{\infty}}{C_{w} - C_{\infty}}$$
(7)

equation (1) is justified automatically whereas eqs. (2)-(15) are reduced to the following forms:

$$f''' + ff'' - f'^{2} + \beta_{1} \left(2 ff f'' - f^{2} f''' \right) + \beta_{2} \left(f^{3} f^{iv} - 2 ff'^{2} f'' - 3 f^{2} f''^{2} \right) + \beta_{3} \left(f''^{2} - ff^{iv} \right) + \lambda \left(\theta + N \phi \right) = 0$$
(8)

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$$(1 + \varepsilon\theta)\theta'' + \varepsilon\theta'^{2} + \Pr f\theta' + \Pr \delta\theta - \Pr \delta\gamma_{1}f\theta' - -\Pr \gamma_{1}(ff'\theta' + f^{2}\theta'') = 0$$
(9)

$$\phi'' + \operatorname{Sc} f \phi' - \operatorname{Sc} \gamma_2 \left(f f' \phi' + f^2 \phi'' \right) = 0$$
⁽¹⁰⁾

$$f(0) = 0, f'(0) = 1, f'(\infty) \to 0$$

$$\tag{11}$$

$$\theta(0) = 1, \ \theta(\infty) \to 0 \tag{12}$$

$$\phi(0) = 1, \ \phi(\infty) \to 0 \tag{13}$$

where prime (') designates differentiation with respect to η . The parameters occurring in eqs. (8)-(10) in non-dimensional forms can be described:

$$\beta_{1} = \lambda_{1}c, \quad \beta_{2} = \lambda_{2}c^{2}, \quad \beta_{3} = \lambda_{3}c, \quad \lambda = \frac{\mathrm{Gr}_{x}}{\mathrm{Re}_{x}^{2}}, \quad N = \frac{\mathrm{Gr}_{x}}{\mathrm{Gr}_{x}}, \quad \mathrm{Gr}_{x} = \frac{\mathrm{gA}_{1}(T_{w} - T_{w})x^{3}}{v^{2}},$$
$$\mathrm{Gr}_{x}^{*} = \frac{\mathrm{gA}_{2}(C_{w} - C_{w})x^{3}}{v^{2}}, \quad \mathrm{Re}_{x} = \frac{xU_{w}(x)}{v}, \quad \mathrm{Pr} = \frac{\mu c_{p}}{K_{w}}, \quad \delta = \frac{Q}{c\rho c_{p}},$$
$$\gamma_{1} = \lambda_{T}c, \quad \gamma_{2} = \lambda_{C}c, \quad \mathrm{Sc} = \frac{v}{D}$$

Convergence analysis

We utilized homotopy scheme [31-44] for the development of convergent solutions. No doubt \hbar -curve (s) are crucial to ensure convergence of non-linear differential systems. Therefore we portrayed \hbar -curves in fig. 1 at 16th-order approximation for such objective. Flat portions of these curves help to achieve admissible values of $(\hbar_f, \hbar_\theta, \hbar_\phi)$. From fig. 2 we found $-1.25 \le h_f \le -0.40$, $-1.48 \le h_\theta \le -0.50$, and $-1.50 \le h_\phi \le -0.50$ with $\beta_1 = 0.5$, $\beta_2 = 0.2$, $\beta_3 = 0.45$, $\lambda = N = \delta = 0.1$, $\varepsilon = \gamma_1 = \gamma_2 = 0.2$, Sc = 1.1, and Pr = 1.2. Furthermore, conver-



Figure 2. The \hbar – curves for f, θ , and ϕ

gence is also assured numerically, see tab. 1. Clearly eqs. (8)-(10) converge at 20th order approximation, respectively. Besides, presented analysis is also compared with [8] and reasonable agreement is found, see tab. 2.

Analysis

This portion highlights the significant features of various variables vs. temperature, θ , and concentration, ϕ . For such interest, figs. 3-8 are constructed and described in detail. Figure 3 addresses the characteristics of heat generation $(\delta > 0)$ and heat absorption $(\delta < 0)$ variables against θ . Here θ increments for $\delta > 0$ whereas it illustrates oppo-

site impact when $\delta < 0$. Heat transfers promptly for $\delta > 0$ which yield θ enhancement. Less heat amount is transferred for $\delta < 0$ which corresponds to θ reduction. The contribution of Prandtl number vs. θ is analyzed through fig. 4. Higher Prandtl number estimations results in

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Table 1. Convergence analysis of series solutions for distinct order approximations when $\beta_1 = 0.5$, $\beta_2 = 0.2$, $\beta_3 = 0.45$, $\lambda = N = \delta = 0.1$, $\varepsilon = \gamma_1 = \gamma_2 = 0.2$, Sc = 1.1, and Pr = 1.2

	Order of approximations	-f''(0)	$-\boldsymbol{\theta}'(0)$	- ¢' (0)
	1	0.8985	0.6239	0.7783
	5	0.8953	0.4349	0.6428
	10	0.8940	0.4201	0.6461
	15	0.8940	0.4192	0.6476
	20	0.8940	0.4192	0.6476
	30	0.8940	0.4192	0.6476
l	40	0.8940	0.4192	0.6476

Table 2. Comparative outcomes of f''(0) with [8] for several values of B_1 when $B_2 = 0 = B_2 = \lambda = N$

p_1 when $p_2 = 0 - p_3 - \lambda - N$				
β_1	[8]	Present		
0.0	1.000000	1.000000		
0.2	1.051948	1.051889		
0.4	1.101850	1.101903		
0.6	1.150163	1.150137		
0.8	1.196692	1.196711		
1.2	1.285257	1.285363		
1.6	1.368641	1.368758		
2.0	1 447617	1 447651		



Figure 3. The θ via δ

Figure 4. The θ via Prandtl number

lower diffusivity which consequently diminishes θ . Figure 5 describes change in θ for variable conductivity factor, ε . As expected, larger ε leads to higher θ and associated thickness layer. In fact, liquid conductivity upsurges when ε is incremented. So extra heat amount is exchanged from surface to material and thus θ is enhanced. The role of γ_1 on θ is elaborated in fig. 6. It is seen that larger γ_1 corresponds to non-conducting behavior due to which θ decays. Furthermore, temperature, θ , is higher for $\gamma_1 = 0$ in comparison to $\gamma_1 > 0$. Figure 7 explores Schmidt



Figure 5. The θ via ε

Figure 6. The θ via γ_1

number impact on ϕ . Here ϕ dwindles for larger estimation of Schmidt number. In fact, Brownian diffusivity arises in Schmidt number expression which reduces via higher estimation of Schmidt number. Consequently, concentration, ϕ , reduces. Analysis for the γ_2 characteristics is expressed through fig. 8. Here concentration, ϕ , diminishes when γ_2 is increased.



Figure 7. The ϕ via Schmidt number

Figure 8. The ϕ via γ_2

Final remarks

This research describes heat generation, variable conductivity and mixed convection characteristics in non-Newtonian (Burgers) fluid-flow towards moving surface. Revised Fourier-Fick relations are considered for modeling energy and concentration expressions. We obtained following significant points from the aforestated analysis:

- Temperature, θ , rises when heat generation ($\delta > 0$) and variable conductivity, ε , factors are enhanced.
- Larger Prandtl number and thermal relaxation variable, γ_1 , correspond to θ reduction.
- An increment in Schmidt number and solutal relaxation variable, γ₂, yield lower concentration, φ.
- The situations regarding traditional Fourier-Fick relations can be retrieved by setting $\gamma_1 = \gamma_2 = 0$ in eqs. (9) and (10).
- Burgers fluid model corresponds to Oldroyd-B fluid model (β₃ = 0), Maxwell fluid model (β₂ = β₃ = 0) and viscous fluid model (β₁ = β₂ = β₃ = 0).

References

- Hayat, T., et al., Stagnation Point Flow of Burgers' Fluid over a Stretching Surface, Progress in Computational Fluid Dynamics, International Journal, 13 (2013), 1, pp. 48-53
- [2] Maqbool, K., et al., Hall Effect on Falkner-Skan Boundary Layer Flow of FENE-P Fluid over a Stretching Sheet, Communications in Theoretical Physics, 66 (2016), 5, pp. 547-554
- [3] Hayat, T., et al., Stretched Flow of Carreau Nanofluid with Convective Boundary Condition, Pramana Journal of Physics, 86 (2016), 1, pp. 3-17
- [4] Hayat, T., et al., Homogeneous-Heterogeneous Reactions in MHD Flow Of Micropolar Fluid by a Curved Stretching Surface, Journal of Molecular Liquids, 240 (2017), Aug., pp. 209-220
- [5] Waqas, M., et al., Transport of Magnetohydrodynamic Nanomaterial in a Stratified Medium Considering Gyrotactic Microorganisms, Physica B: Condensed Matter, 529 (2018), Jan., pp. 33-40
- [6] Hayat, T., et al., Mixed Convection Radiative Flow of Maxwell Fluid Near a Stagnation Point with Convective Condition, Journal of Mechanics, 29 (2013), 3, pp. 403-409
- [7] Hayat, T., et al., On 2D Stratified Flow of an Oldroyd-B Fluid with Chemical Reaction: An Application of Non-Fourier Heat Flux Theory, *Journal of Molecular Liquids*, 223 (2016), C, pp. 566-571

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- [8] Lee, A. R., Markwick, A. H. D., The Mechanical Properties of Bituminous Surfacing Materials under Constant Stress, *Journal of the Society of Chemical Industry*, 56 (1937), pp. 146-156
- [9] Alsaedi, A., et al., Stagnation Point Flow of Burgers' Fluid and Mass Transfer with Chemical Reaction and Porosity, *Journal of Mechanics*, 29 (2013), 3, pp. 453-460
- [10] Khan, W. A., et al., Impact of Chemical Processes on Magneto Nanoparticle for the Generalized Burgers Fluid, Journal of Molecular Liquids, 234 (2017), May, pp. 201-208
- [11] Hayat, T., et al., Newtonian Heating in Stagnation Point Flow of Burgers Fluid, Applied Mathematics and Mechanics, 36 (2015), 1, pp. 61-68
- [12] Hayat, T., et al., Joule Heating Effects in MHD Flow of Burgers Fluid, Heat Transfer Research, 47 (2016), 12, pp. 1083-1092
- [13] Hayat, T., et al., Mixed Convection Flow of a Burgers Nanofluid in the Presence of Stratifications and Heat Generation/Absorption, *The European Physical Journal Plus, 131* (2016), Aug., 253
- [14] T. Hayat, et al., Magnetohydrodynamic Flow of Burgers Fluid with Heat Source and Power Law Heat Flux, Chinese Journal of Physics, 55 (2017), 2, pp. 318-330
- [15] Khan, M., et al., Impact of Nonlinear Thermal Radiation and Gyrotactic Microorganisms on the Magneto-Burgers Nanofluid, International Journal of Mechanical Sciences, 130 (2017), Sept., pp. 375-382
- [16] Fourier, J. B. J., *Théorie Analytique De La Chaleur*, Chez Firmin Didot, Paris, 1822
- [17] Daneshjou, K., et al., Non-Fourier Heat Conduction Analysis of Infinite 2D Orthotropic Fg Hollow Cylinders Subjected to Time-Dependent Heat Source, Applied Thermal Engineering, 98 (2016), Apr., pp. 582-590
- [18] Cattaneo, C., A Form of Heat Conduction Equation which Eliminates the Paradox of Instantaneous Propagation, *Comptes Rendus*, 247 (1958), pp. 431-433
- [19] Vernotee, P., Les paradoxes de la theorie continue de 1 equation del la Chaleur, Computes Rendus, 246 (1958), pp. 3154-3155
- [20] Christov, C. I., On Frame Indifferent Formulation of the Maxwell-Cattaneo Model of Finite Speed Heat Conduction, *Mechanics Research Communication*, 36 (2009), 4, pp. 481-486
- [21] Han, S., et al., Coupled Flow and Heat Transfer in Viscoelastic Fluid with Cattaneo-Christov Heat Flux Model, Appl. Math. Lett., 38 (2014), Dec., pp. 87-93
- [22] Nadeem, S., Muhammad, N., Impact of Stratification and Cattaneo-Christov Heat Flux in the Flow Saturated with Porous Medium, *Journal of Molecular Liquids*, 224 (2016), Part A, pp. 423-430
- [23] Waqas, M., et al., Cattaneo-Christov Heat Flux Model for Flow of Variable Thermal Conductivity Generalized Burgers Fluid, Journal of Molecular Liquids, 220 (2016), Aug., pp. 642-648
- [24] Liu, L., et al., Fractional Anomalous Diffusion with Cattaneo-Christov Flux Effects in a Comb-Like Structure, Applied Mathematical Modelling, 40 (2016), 13-14, pp. 6663-6675
- [25] Hayat, T., et al., Impact of Cattaneo-Christov Heat Flux Model in Flow of Variable Thermal Conductivity Fluid over a Variable Thicked Surface, International Journal of Heat and Mass Transfer, 99 (2016), Aug., pp. 702-710
- [26] Khan, W. A., et al., An Improved Heat Conduction and Mass Diffusion Models for Rotating Flow of an Oldroyd-B Fluid, *Results in Physics*, 7 (2017), Sept., pp. 3583-3589
- [27] Waqas, M., et al., On Cattaneo-Christov Double Diffusion Impact for Temperature-Dependent Conductivity of Powell-Eyring Liquid, Chinese Journal of Physics, 55 (2017), 3, pp. 729-737
- [28] Hayat, T., et al., Application of Non-Fourier Heat Flux Theory in Thermally Stratified Flow of Second Grade Liquid with Variable Properties, *Chinese Journal of Physics*, 55 (2017), 2, pp. 230-241
- [29] Hayat, T., et al., On Doubly Stratified Chemically Reactive Flow of Powell-Eyring Liquid Subject to Non-Fourier Heat Flux Theory, Results in Physics, 7 (2017), Dec., pp. 99-106
- [30] Zubair, M., et al., Simulation of Nonlinear Convective Thixotropic Liquid with Cattaneo-Christov Heat Flux, Results in Physics, 8 (2018), Mar., pp. 1023-1027
- [31] Liao, S. J., Beyond Perturbation: Introduction to Homotopy Analysis Method, Chapman and Hall, CRC Press, Boca Raton, Fla., USA, 2003
- [32] Ellahi, R., et al., Series Solutions of Non-Newtonian Nanofluids with Reynolds' Model and Vogel's Model by Means of the Homotopy Analysis Method, *Mathematical and Computer Modelling*, 55 (2012), 7-8, 1876-1891
- [33] Hayat, T., et al., Effects of Joule Heating and Thermophoresis on Stretched Flow with Convective Boundary Conditions, Scientia Iranica, Transaction B, Mechanical Engineering, 21 (2014), 3, pp. 682-692
- [34] Hayat, T., et al., A Model of Solar Radiation and Joule Heating in Magnetohydrodynamic (MHD) Convective Flow of Thixotropic Nanofluid, *Journal of Molecular Liquids*, 215 (2016), Mar., pp. 704-710

- [35] Waqas, M., et al., Magnetohydrodynamic (MHD) Mixed Convection Flow of Micropolar Liquid Due to Nonlinear Stretched Sheet with Convective Condition, International Journal of Heat and Mass Transfer, 102 (2016), Nov., pp. 766-772
- [36] Shen, B., et al., Bioconvection Heat Transfer of a Nanofluid over a Stretching Sheet with Velocity Slip and Temperature Jump, *Thermal Science*, 21 (2017), 6A, pp. 2347-2356
- [37] Waqas, M., et al., Mixed Convective Stagnation Point Flow of Carreau Fluid with Variable Properties, Journal of the Brazilian Society of Mechanical Sciences and Engineering, 39 (2017), 8, pp. 3005-3017
- [38] Hayat, T., et al., Mixed Convection Flow of Jeffrey Fluid along an Inclined Stretching Cylinder with Double Stratification Effect, *Thermal Science*, 21 (2017), 2, pp. 849-862
- [39] Hayat, T., et al., MHD Flow of Powell-Eyring Fluid by a Stretching Cylinder with Newtonian Heating, *Thermal Science*, 22 (2018), 1B, pp. 371-382
- [40] Hassan, M., et al., Particle Shape Effects on Ferrofuids Flow and Heat Transfer under Influence of Low Oscillating Magnetic Field, Journal of Magnetism and Magnetic Materials, 443 (2017), Dec., pp. 36-44
- [41] Sadiq, M. A., et al., Importance of Darcy-Forchheimer Relation in Chemically Reactive Radiating Flow Towards Convectively Heated Surface, Journal of Molecular Liquids, 248 (2017), Dec., pp. 1071-1077
- [42] Hayat, T., et al., Influence of Thermal Radiation and Chemical Reaction in Mixed Convection Stagnation Point Flow of Carreau Fluid, Results in Physics, 7 (2017), Oct., pp. 4058-4064
- [43] Hayat, T., et al., Significant Consequences of Heat Generation/Absorption and Homogeneous-Heterogeneous Reactions in Second Grade Fluid Due to Rotating Disk, *Results in Physics*, 8 (2018), Mar., pp. 223-230
- [44] Ijaz, N., et al., Analytical Study on Liquid-Solid Particles Interaction in the Presence of Heat and Mass Transfer Through a Wavy Channel, Journal of Molecular Liquids, 250 (2018), Jan., pp. 80-87