MODELING AND SIMULATION OF NATURAL CONVECTION FLOW ALONG A ROUGH SURFACE OF SINUSOIDAL NATURE WITH VARIABLE HEAT FLUX: Using Keller Box Scheme

by

Abuzar GHAFFARI[a*], Tariq JAVED[b], Irfan MUSTAFA[c], and Fotini LABROPULU[d]

[a] Department of Mathematics, University of Education, Lahore, Pakistan
[b] Department of Mathematics, FBAS, International Islamic University, Islamabad, Pakistan
[c] Department of Mathematics, Allama Iqbal Open University, Islamabad, Pakistan
[d] Luther College – Mathematics, University of Regina, Regina, Canada

Original scientific paper
https://doi.org/10.2298/TSCI170929106G

In this study, natural convection flow along a vertical wavy surface has been investigated with variable heat flux. The governing equations are transformed into dimensionless PDE by using the non-dimensional variables and then solved numerically by using an implicit finite difference scheme known as Keller Box method. The effects of the parameters amplitude of the wavy surface, α, exponent of the variable heat flux, m, and Prandtl number on the local skin friction coefficient and local Nusselt number are shown graphically. It is found that for the negative value of exponent of the variable heat flux, m, the local skin friction coefficient increases and Nusselt number decreases but the opposite behavior is observed for the positive values of m. The comparison of limiting case with the previous study is shown through table and it is found that the solution obtained is in excellent agreement with the previous studies.

Key words: *natural convection, variable heat flux, vertical wavy surface, numerical solution*

Introduction

In natural convection flow, heat is transformed from solid to liquid due to the density difference of the fluid without considering any external source. Natural convection flow has significant importance due to its several applications in industries and engineering problems. Applications include cooling of electronic components, crystal growth, geothermal systems, heat exchangers, nuclear reactors, metallurgical processes, *etc*. To enhance the heat transfer rate, rough and irregular surfaces are intentionally considered rather than plane surfaces. Flow over an irregular surfaces has become very important in industrial problem due to its many applications, which are solar collectors, condensers, cavity wall insulating system, grain storage containers, and industrial heat radiation.

Natural convection flow along a vertical wavy surface has received the attention of the researchers due to its practical applications. Yao [1] was the first, who studied natural convection flow along a vertical wavy surface. After that, Moulic and Yao [2] studied the mixed con-

* Corresponding author, e-mail: abuzar.ghaffari@ue.edu.pk

vection flow along a wavy surface. Bhavnani and Bergles [3] studied natural convection heat transfer from sinusoidal wavy surface. Rees and Pop [4] investigated free convection induced by horizontal wavy surface in a porous medium. They considered these cases in which waves have an $O(\mathrm{Ra}^{-1/3})$ amplitude and found that the reverse flow occurs at the heated surface when amplitude was considered greater than $0.95\mathrm{Ra}^{-1/3}$. Kim [5] studied natural convection flow along a wavy vertical plate to non-Newtonian fluids. Chiu and Chou [6] presented transient analysis of natural convection along a vertical wavy surface in micropolar fluid. They solved the governing PDE by a simple transposition theorem and cubic spline collocation method. Notable studies [7-14] on vertical wavy surface have been carried out by considering different flow phenomena. Recently, Golra and Kumari [15] studied the natural convection flow along vertical surface f wavy nature. They modeled the problem in non-similar form and used Keller box scheme to analyze it numerically. The effects of surface waviness amplitude on heat and mass transfer rate are presented through graphs. Mustafa *et al.* [16] studied the natural convection flow of nanofluid along the vertical surface of wavy texture. They performed numerical simulation and results are presented through table and graph. Srinivasacharya and Kumar [17] studied radiation influence on nanofluid over an inclined wavy surface saturated with non-Darcy pours medium. They use similarity transformation to reduce governing PDE into ODE and used successive linearization method for the solution purpose. Mahdy and Ahmed [18], Habiba *et al.* [19], Mustafa and Javed [20] also performed interesting studies along vertical wavy surface.

Merkin and Mahmood [21] studied the free convection boundary-layer flow on a vertical plate with prescribed surface heat flux. They considered a variable wall heat flux which is proportional to $(1+x^2)^\mu$. They investigated that for $\mu > -1/2$, similarity solution valid for x small to the one valid for large and for $\mu \le -1/2$ the similarity equation are not solvable for large value of x. Moulic and Yao [22] investigated natural convection flow along a wavy surface with uniform heat flux. Rees and Pop [23] studied the free convection boundary flow induced by a vertical wavy surface with uniform heat flux in porous medium. Pop *et al.* [24] investigated the free convection flow along a vertical surface with prescribed surface heat flux in a micropolar fluid. They showed that wall temperature increases with the increase of wave amplitude parameter, whether the fluid is Newtonian or micropolar. Tashtoush and Irshaid [25, 26] studied heat and fluid flow from a wavy surface subjected to a variable heat flux. They found a separation point when the amplitude of the wavy surface was considered 0.2 and they showed that the solutions exist for small and large values of x when the exponent of the variable heat flux was considered greater than –0.5.

The above literature witnessed that the numerical study of natural convection flow along vertical surface of sinusoidal nature with variable heat flux has not been considered before. The aim of present study to put our efforts for finding the numerical solution of natural convection flow along a vertical wavy surface with variable heat flux. For this purpose, we chose a heat flux model at the wall as proposed by Merkin and Mahmood [21] which is proportional to $(1+x^2)^m$. In this expression if we chose $m=0$, the case reduced to the constant heat flux model as done by Mustafa and Javed [20], and for $m=1,2,3\ldots$, we get highly non-linear model for wall heat flux.

Mathematical formulation

The natural convection viscous, incompressible boundary-layer flow along a vertical wavy surface with variable heat flux is considered. The variable wall heat flux $q_w = q_0(1+x^2)^m$ assumed by Merkin and Mahmood [21] is applied behind the plate, where x is the dimensionless variable and q_0 is constant. The surface of the plate is considered as a wavy surface, which

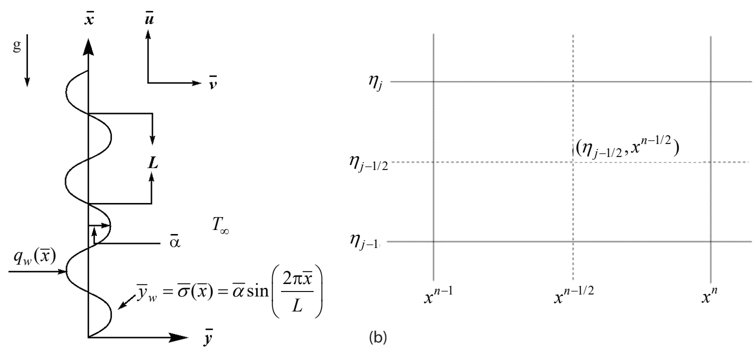

Figure 1. (a) The physical model and co-ordinate system, (b) net rectangle of difference approximations for the Keller box scheme

is defined as $\bar{\sigma}(\bar{x}) = \bar{\alpha}\sin(2\pi\bar{x}/L)$, where $\bar{\alpha}$ is the amplitude and L is the characteristic length of the sinusoidal wave. The geometry of the wavy surface is shown in fig. 1. The governing equations of the considered problem with Boussinesq approximations can be written:

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0 \tag{1}$$

$$\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}} = -\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{x}} + \nu\nabla^2\bar{u} + g\beta\left(T - T_\infty\right) \tag{2}$$

$$\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{v}}{\partial\bar{y}} = -\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{y}} + \nu\nabla^2\bar{v} \tag{3}$$

$$\bar{u}\frac{\partial\bar{T}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{T}}{\partial\bar{y}} = \frac{k}{\rho C_p}\nabla^2 T \tag{4}$$

where $(\bar{x},\bar{y})$ are the Cartesian co-ordinate system along and normal to the tangent of wavy surface and $(\bar{u},\bar{v})$ are the components of velocity in the $\bar{x}$- and $\bar{y}$-directions, respectively, p – the pressure, ν – the kinematic viscosity, ρ – the density of the fluid, T – the temperature, T_∞ – the uniform ambient temperature, g – the acceleration, β – the thermal expansion coefficient, k – the thermal conductivity, and C_p – the specific heat. The boundary conditions for the problem are given:

$$\begin{gathered}\bar{u} = 0, \quad \bar{v} = 0, \quad -k(\bar{n}\bar{\nabla}T) = q_w(x) = q_0(1+x^2)^m \quad \text{at} \quad \bar{y} = \bar{\sigma}(\bar{x}) \\ \bar{u} = 0, \quad T = T_\infty, \quad \bar{p} = p_\infty \quad \text{as} \quad \bar{y} \to \infty\end{gathered} \tag{5}$$

where q_0 is the constant, $\bar{n}$ – the unit normal to the wavy surface, and p_∞– the outside boundary-layer pressure of the fluid. After using the following non-dimensional variable as used by Pop *et al.* [24]:

$$\begin{gathered}x = \frac{\bar{x}}{L}, \quad y = \frac{\bar{y} - \bar{\sigma}(\bar{x})}{L}\mathrm{Gr}^{\frac{1}{5}}, \quad p = \frac{L^2}{\rho\nu^2}\mathrm{Gr}^{-\frac{4}{5}}\left(\bar{p} - p_\infty\right), \quad u = \frac{\rho L}{\mu}\mathrm{Gr}^{-\frac{2}{5}}\bar{u} \\ v = \frac{\rho L}{\mu}\mathrm{Gr}^{-\frac{1}{5}}\left(\bar{v} - \sigma_x\bar{u}\right), \quad \theta = \left[\frac{T - T_\infty}{q_w(x)L}\right]k\mathrm{Gr}^{\frac{1}{5}}, \quad \sigma(x) = \frac{\bar{\sigma}(\bar{x})}{L}, \quad \sigma_x = \frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}\bar{x}} = \frac{\mathrm{d}\sigma}{\mathrm{d}x}, \quad \alpha = \frac{\bar{\alpha}}{L}\end{gathered} \tag{6}$$

into the eqs. (2)-(4) and ignoring the small order terms in Grashof number, the following non-dimensional form of the governing equations are obtained:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\sigma_x \mathrm{Gr}^{\frac{1}{5}}\frac{\partial p}{\partial y}+\left(1+\sigma_x^2\right)\frac{\partial^2 u}{\partial y^2}+\theta \tag{7}$$

$$\sigma_x\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\mathrm{Gr}^{\frac{1}{5}}\frac{\partial p}{\partial y}+\sigma_x\left(1+\sigma_x^2\right)\frac{\partial^2 u}{\partial y^2}-\sigma_{xx}u^2 \tag{8}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}+\frac{2mx}{1+x^2}u\theta=\frac{1+\sigma_x^2}{\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2} \tag{9}$$

where Grashof number is $\mathrm{Gr}=\mathrm{g}\beta q_w(x)L^4/kv^2$. The pressure gradient along the x-axis is determined by using the inviscid flow solution, which gives that $\partial p/\partial x=0$. Equation (8) shows that the term on the left hand is of $O(1)$ that's why the pressure gradient along y-axis $\partial p/\partial y$ should be of $O(\mathrm{Gr}^{-1/5})$. After eliminating the pressure gradient $\partial p/\partial x$ from eqs. (7) and (8) leads to:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(1+\sigma_x^2\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_x\sigma_{xx}}{1+\sigma_x^2}u^2+\frac{\theta}{1+\sigma_x^2} \tag{10}$$

The boundary conditions will reduce to the form:

$$\left.\begin{array}{l} u=0, \quad v=0, \quad \dfrac{\partial\theta}{\partial y}=-\dfrac{1}{\sqrt{1+\sigma_x^2}} \quad \text{at} \quad y=0 \\ u=0, \quad \theta=0, \quad p=0 \quad \text{as} \quad y\to\infty \end{array}\right\} \tag{11}$$

For the numerical solution of the governing eqs. (9) and (10), we introduce the following transformations:

$$\psi=(5x)^{\frac{4}{5}}f(x,\eta), \quad \theta=(5x)^{\frac{1}{5}}\theta(x,\eta), \quad \eta=y(5x)^{-\frac{1}{5}} \tag{12}$$

where ψ is the stream function, which is defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$. The above equations become:

$$\left(1+\sigma_x^2\right)f'''-\left(\frac{5x\sigma_x\sigma_{xx}}{1+\sigma_x^2}+3\right)f'^2+4ff''+\frac{1}{1+\sigma_x^2}\theta=5x\left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right) \tag{13}$$

$$\frac{\left(1+\sigma_x^2\right)}{\mathrm{Pr}}\theta''-\left(\frac{10mx^2}{1+x^2}+1\right)\theta f'+4f\theta'=5x\left(f'\frac{\partial\theta}{\partial x}-\theta'\frac{\partial f}{\partial x}\right) \tag{14}$$

and boundary conditions take the new form:

$$\begin{array}{c} f(x,0)=f'(x,0)=0, \quad \theta'(x,0)=-\dfrac{1}{\sqrt{1+\sigma_x^2}} \\ f'(x,\infty)=0, \quad \theta(x,\infty)=0 \end{array} \tag{15}$$

where prime denotes the partial derivative with respect to η. The physical quantities of interest are the skin friction coefficient and Nusselt number which are defined:

$$C_{fx}=\frac{\tau_w}{\rho U^2}\,,\quad \mathrm{Nu}_x=\frac{\overline{x}q_w(\overline{x})}{k(T_w-T_\infty)} \tag{16}$$

where k is the thermal conductivity of the fluid, τ_w – the wall shear stress, and $q_w(\overline{x})$ – the variable heat flux from the surface, which are defined:

$$\tau=\mu\left(\overline{n}\,\overline{\nabla}\overline{u}\right)_{y=0}\,,\quad q_w=-k\left(\overline{n}\,\overline{\nabla}T\right)_{y=0} \tag{17}$$

After using the eqs. (12) and (17) into the eq. (16), the skin friction coefficient and Nusselt number take the form:

$$C_{fx}\left[\frac{\mathrm{Gr}}{(5x)^2}\right]^{1/5}=\sqrt{1+\sigma_x^2}\,f''(x,0),\quad \mathrm{Nu}_x\left(\mathrm{Gr}\frac{x^4}{5}\right)^{-1/5}=\frac{1}{\theta(x,0)} \tag{18}$$

Numerical solution

The non-linear PDE (13) and (14) along with boundary conditions (15) are solved by an implicit finite difference scheme known as Keller box [27]. This method is stable and has second order accuracy. In this method, we first converted differential equations into the system of the first-order form. For this purpose, we introduced new dependent variables $U(x,\eta)$, $V(x,\eta)$, and $Q(x,\eta)$. Letting:

$$f'=U,\quad f''=V,\quad \theta'=Q \tag{19}$$

by using the relation into eqs. (13)-(15) we obtained following reduced form:

$$L_1V'-L_2U^2+4fV+\left(\frac{1}{L_1}\right)\theta-5x\left(U\frac{\partial U}{\partial x}-V\frac{\partial f}{\partial x}\right)=0 \tag{20}$$

$$\frac{L_1}{\mathrm{Pr}}Q'-L_3QU+4fQ-5x\left(U\frac{\partial\theta}{\partial x}-Q\frac{\partial f}{\partial x}\right)=0 \tag{21}$$

and the boundary conditions are:

$$\begin{gathered}f(x,0)=0,\quad U(x,0)=0,\quad Q(x,0)=-L_4\\ U(x,\infty)=0,\quad \theta(x,\infty)=0\end{gathered} \tag{22}$$

where

$$L_1=1+\sigma_x^2,\quad L_2=\frac{5x\sigma_x\sigma_{xx}}{1+\sigma_x^2}+3,\quad L_3=1+\frac{10mx^2}{1+x^2},\quad L_4=\frac{1}{\sqrt{1+\sigma_x^2}} \tag{23}$$

Now the next step is to approximate the quantities f, U, V, θ, and Q at the points $(x_n,\ \eta_j)$. The values of the functions are replaced by its mean value like:

$$(\square)_{j-\frac{1}{2}}^{\boldsymbol{n}}=\frac{1}{2}\left[(\square)_j^{\boldsymbol{n}}+(\square)_{j-1}^{\boldsymbol{n}}\right],\quad (\square)_j^{\boldsymbol{n}-1/2}=\frac{1}{2}\left[(\square)_j^{\boldsymbol{n}}+(\square)_j^{\boldsymbol{n}-1}\right]$$

and derivatives in η and x-direction are replaced by central differences like:

$$(\square)_{j-\frac{1}{2}}^{\boldsymbol{n}}=\frac{1}{h}\left[(\square)_j^{\boldsymbol{n}}-(\square)_{j-1}^{\boldsymbol{n}}\right],\quad (\square)_j^{\boldsymbol{n}-1/2}=\frac{1}{k}\left[(\square)_j^{\boldsymbol{n}}-(\square)_j^{\boldsymbol{n}-1}\right]$$

To handle the non-linearity of resulting algebraic equations, Newton's linearization process is implemented. For $(i+1)^{\text{th}}$ iterations we write:

$$f_j^{i+1} = f_j^i + \delta f_j^i, \quad U_j^{i+1} = U_j^i + \delta U_j^i, \quad V_j^{i+1} = V_j^i + \delta V_j^i$$

$$\theta_j^{i+1} = \theta_j^i + \delta\theta_j^i \quad \text{and} \quad Q_j^{i+1} = Q_j^i + \delta Q_j^i$$

The solution of the obtained system of linear equations is obtained by block tri-diagonal scheme over the rectangular nodes as shown in fig. 1(b). The node points over the rectangular net are defined:

$$x^0 = 0, \quad x^n = x^{n-1} + k_n, \quad n = 1,2,3\ldots N$$

$$\eta^0 = 0, \quad \eta_j = \eta_{j-1} + h_j, \quad j = 1,2,3\ldots J$$

where n and j represent the position of the node point in x- and η-direction, respectively, and k_n and h_j are the step size in x- and η-direction.

The employed technique is validated by comparison of $f''(0)$ with Pop *et al.* [24] and Sadiqa *et al.* [28] for a limiting case given in tab. 1. These results are in good agreement that gives us a confidence for the accuracy of the employed numerical technique.

Result and discussion

A numerical solution of coupled non-linear PDE (13) and (14) subject to the boundary conditions (15) is obtained by using an implicit finite difference scheme as previously described. The effects of various parameters, namely amplitude of the wavy surface, α, exponent of variable heat flux, m, and Prandtl number on the local skin friction coefficient C_{fx} and local Nusselt number, Nu_x, are shown graphically.

Figures 2(a) and 2(b) show the graphical result of local skin friction coefficient and local Nusselt number for various values of the amplitude of the wavy surface, α, along the x-direction with Prandtl number $\text{Pr} = 1$ and exponent of variable heat flux $m = 1$. It is observed that local skin friction coefficient and local Nusselt number decrease with the increase of amplitude of the wavy surface, α. In fig. 2(a) the highest peak is obtained near the leading edge but far away from the leading edge, C_{fx}, decreases monotonically. The effects of exponent of variable heat flux m on C_{fx} and Nu_x are shown in figs. 3(a) and 3(b) with Prandtl number Pr = 1 and $\alpha = 0.2$. It is seen that local skin friction coefficient, C_{fx}, decreases and local Nusselt number increases with the increas-

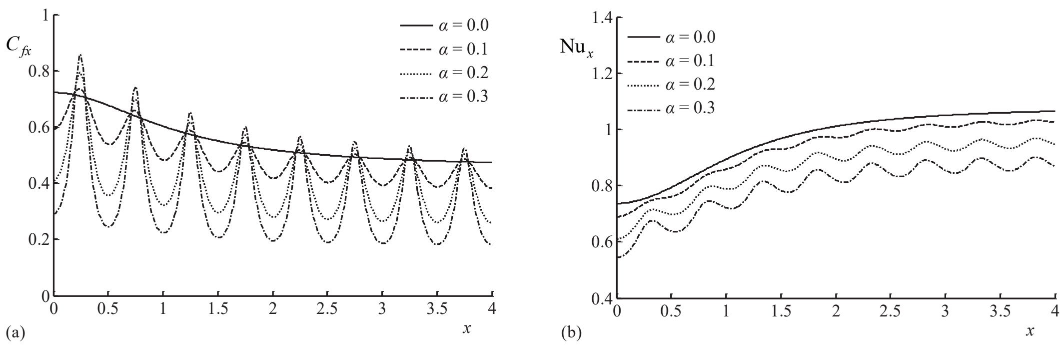

Figure 2. (a) Change in skin friction coefficient along with wavy amplitude α (b) change in Nusselt number along with wavy amplitude α

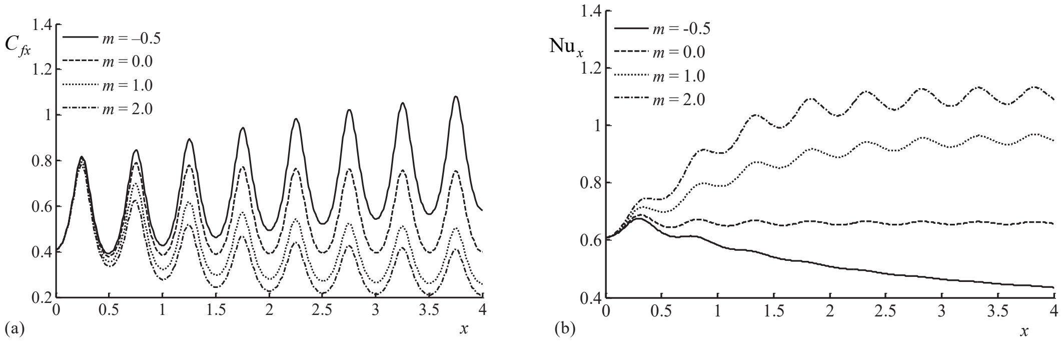

Figure 3. (a) Change in skin friction for different *m* and (b) change in Nusselt number for different *m*

ing of exponent of variable heat flux, m. For negative value of m the highest peak of the wave in skin friction coefficient is obtained far away from the leading edge and skin friction coefficient increases along the upstream direction. For non-negative values of m the highest peak of the wave is obtained at the leading edge. In fig. 3(b) the local Nusselt number decreases along the downstream direction for negative value of m and the highest peak is obtained near the leading edge and far away from the leading edge the amplitude of the wave tends to zero.

For the constant heat flux $m = 0$, the amplitude of the wave decreases far away from the leading edge but for the positive value of m it is seen that the local Nusselt number increases along the upstream direction. The effects of various values of Prandtl number on the local skin friction coefficient and Nusselt number are shown in figs. 4(a) and 4(b) with α and $m = 1$. Figure 4(a) depicts that local skin friction coefficient decreases along the downstream direction with the increasing of Prandtl number also the amplitude of the wave decreases far away from the leading edge. Figure 4(b) describes that local Nusselt number increases with the increasing of Prandtl number, secondly Nusselt number increases along the upstream direction due to the variable heat flux. The isothermal lines and streamlines for different values of exponent of variable heat flux m with amplitude $\alpha = 0.2$ and Pr = 0.5 are shown in figs. 5(a)-5(d) and 6(a)-6(d). In figs. 5(a)-5(d) isothermal lines illustrate that thermal boundary-layer thickness decreases with the increase in exponent of variable heat flux m. In figs. 6(a)-6(d), streamlines illustrate that flow rate decreases within the boundary-layer with the increasing of exponent of variable heat flux m and the values of ψ_{max} reduces which are 8.62 to 5.25 when m is considered from –0.5 to 2.0.

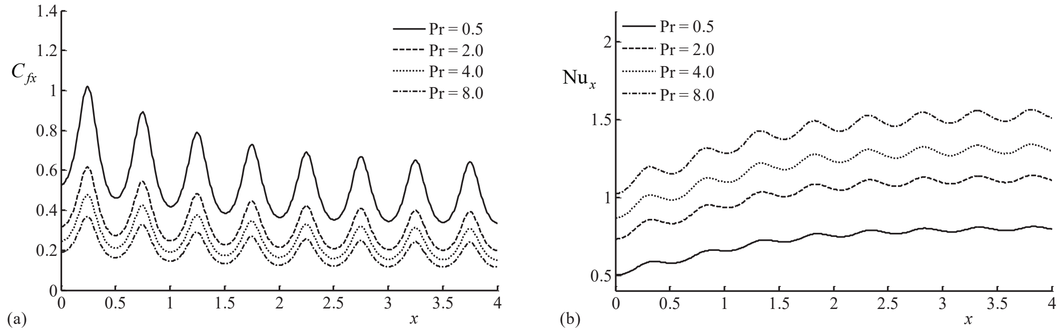

Figure 4. (a) Change in skin friction for different value of Prandtl number and (b) change in Nusselt number for different value of Prandtl number

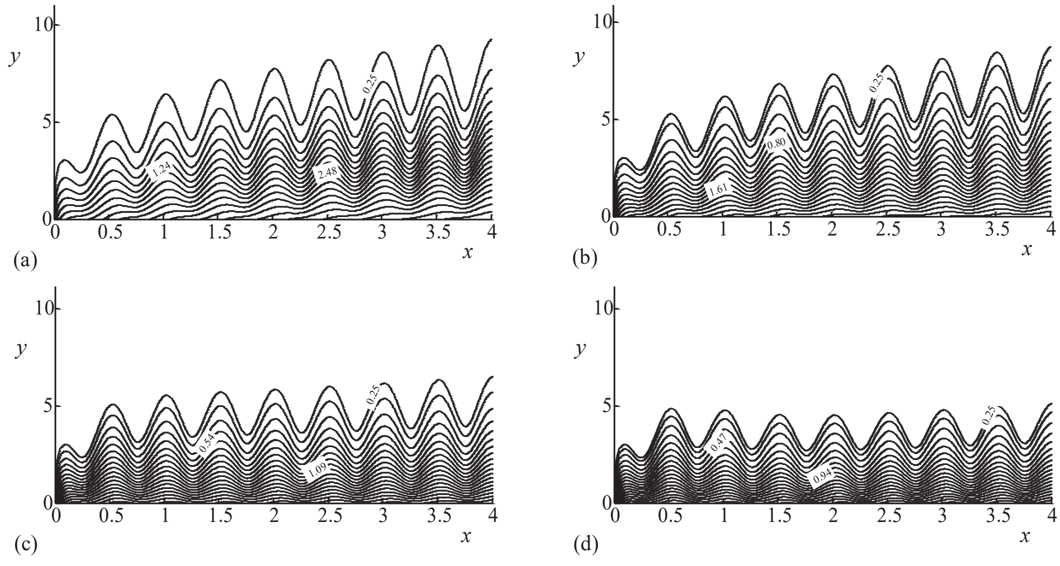

Figure 5. Isothermal lines for (a) m = –0.5, (b) m = 0.0, (c) m = 0.5, (d) m = 2.0 when α = 0.2 and Pr = 0.5

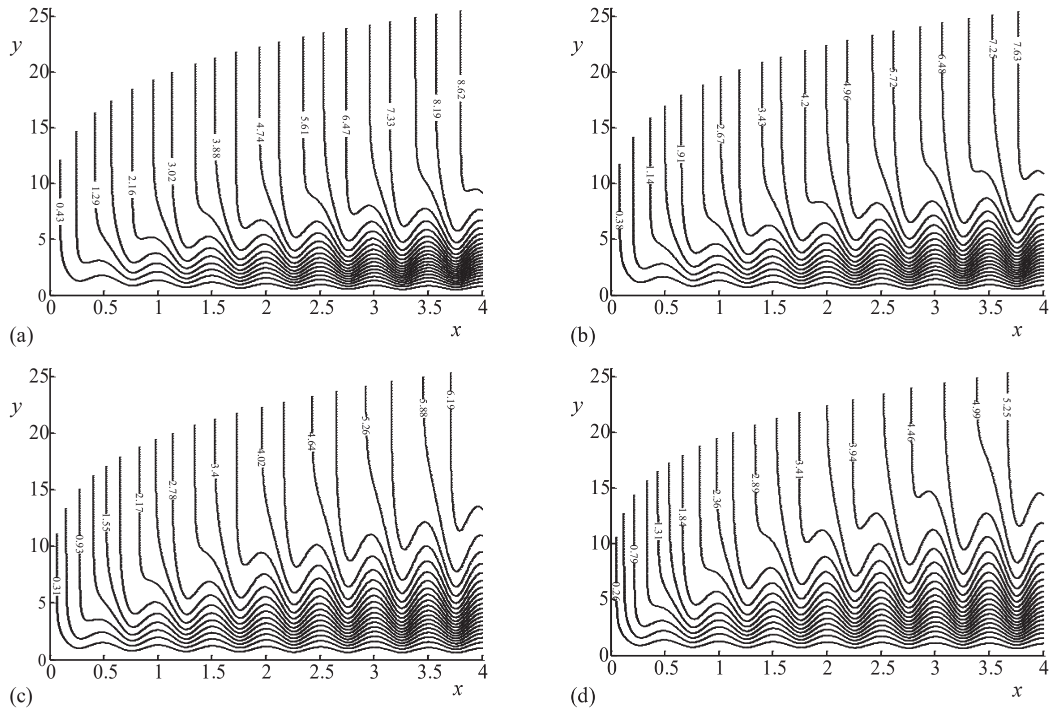

Figure 6. Stream lines for (a) m = –0.5, (b) m = 0.0, (c) m = 0.5, (d) m = 2.0 when α = 0.2 and Pr = 0.5

Table 1. Numerical values of $\theta(x,0)$ for various values of x with Pr = 1.0 and m = 0

	α = 0.0			α = 0.2		
x	Present	[24]	[27]	Present	[24]	[27]
1.50	1.3574	1.3585	1.3574	1.5258	1.5217	1.5309
1.625	1.3574	1.3585	1.3574	1.5334	1.5307	1.5365
1.75	1.3574	1.3585	1.3574	1.5086	1.5137	1.5133
1.875	1.3574	1.3585	1.3574	1.5032	1.5009	1.5103
2.0	1.3574	1.3585	1.3574	1.5242	1.5201	1.5305
2.125	1.3574	1.3585	1.3574	1.5307	1.5279	1.5351
2.250	1.3574	1.3585	1.3574	1.5100	1.5147	1.5158
2.375	1.3574	1.3585	1.3574	1.5053	1.5029	1.5135
2.5	1.3574	1.3585	1.3574	1.5232	1.5191	1.5306
2.625	1.3574	1.3585	1.3574	1.5288	1.5261	1.5346
2.750	1.3574	1.3585	1.3574	1.5109	1.5154	1.5178
2.875	1.3574	1.3585	1.3574	1.5068	1.5043	1.5160
3.0	1.3574	1.3585	1.3574	1.5224	1.5184	1.5310

Concluding remarks

The study of natural convection flow over the surface of vertical wavy texture due to variable heat flux has been performed. The mathematical model of the present problem is developed in form of non-linear PDE. These highly non-linear PDE are transformed into dimensionless form by using suitable transformations. These obtained equations are simulated numerically with help of finite difference scheme and results are presented in form of Nusselt number, skin friction coefficient, stream lines and thermal lines. It is found that

- The local skin friction coefficient increases and local Nusselt number decreases for the negative value of exponent of the variable heat flux m but for the positive values of m opposite behavior is observed.
- Thermal boundary-layer thickness and boundary-layer flow rate decrease with the increasing of the exponent of the variable heat flux m.
- With increase of m random motion in fluid particles near the wall changes rapidly and dispersion in the streamlines is as noted.

References

[1] Yao, L. S., Natural Convection Along a Vertical Wavy Surface, *ASME Journal of Heat Transfer*, *105* (1983), 1-2, pp. 465-468
[2] Moulic, S. G., Yao, L. S., Mixed Convection along Wavy Surface, *ASME Journal of Heat Transfer*, *111* (1989), 4, pp. 974-979
[3] Bhavnani, S. H., Bergles, A. E., Natural Convection Heat Transfer from Sinusoidal Wavy Surfaces, *Heat and Mass Transfer*, *26* (1991), 6, pp. 341-349
[4] Rees, D. A. S., Pop, I., Free Convection Induced by a Horizontal Wavy Surface in a Porous Medium, *Fluid Dynamics Research*, *14* (1994), 4, pp. 151-166
[5] Kim, E., Natural Convection along a Wavy Vertical Plate to Non-Newtonian Fluids, *International Journal of Heat Mass Transfer*, *40* (1997), 13, pp. 3069-3078
[6] Chiu, C. P., Chou, H. M., Transient Analysis of Natural Convection along a Vertical Wavy Surface in Micropolar Fluids, *International Journal of Engineering Science*, *32* (1994), 1, pp. 19-33
[7] Wang, C. C., Chen, C. K., Mixed Convection Boundary Layer Flow of Non-Newtonian Fluids along Vertical Wavy Plates, *International Journal of Heat and Fluid Flow*, *23* (2002), 6, pp. 831-839
[8] Wang, C. C., Chen, C. K., Transient Force and Free Convection along a Vertical Wavy Surface in Micropolar Fluids, *International Journal of Heat and Mass Transfer*, *44* (2001), 17, pp. 3241-3251

[9] Kumari, M., *et al*., Free Convection Boundary Layer Flow of a Non-Newtonian Fluid along a Vertical Wavy Surface, *International Journal of Heat and Fluid Flow*, *18* (1997), 6, pp. 625-631
[10] Rathish Kumar, B. V., Shalini, Non-Darcy Free Convection Induced by a Vertical Wavy Surface in a Thermally Stratified Porous Medium, *International Journal of Heat and Mass Transfer*, *47* (2004), 10-11, pp. 2353-2363
[11] Jang, J. H., Yan, W. M., Mixed Convection Heat and Mass Transfer along a Vertical Wavy Surface, *International Journal of Heat and Mass Transfer, 47* (2004), 3, pp. 419-428
[12] Jang, J. H., *et al*., Natural Convection Heat and Mass Transfer along a Vertical Wavy Surface, *International Journal of Heat and Mass Transfer*, *46* (2003), 6, pp. 1075-1083
[13] Jang, J. H., Yan, W. M., Transient Analysis of Heat and Mass Transfer by Natural Convection over a Vertical Wavy Surface, *International Journal of Heat and Mass Transfer*, *47* (2004), 17-18, pp. 3695-3705
[14] Molla, M. M., *et al*., Natural Convection Flow along a Vertical Wavy Surface with Uniform Surface Temperature in Presence of Heat Generation/Absorption, *International Journal of Thermal Science*, *43* (2004), 2, pp. 157-163
[15] Gorla, R. S., Kumari, M., Free Convection along a Vertical Wavy Surface in a Nanofluid*, Proceedings of the Institution of Mechanical Engineers, Part N: Journal of Nanoengineering and Nanosystems*, *225* (2011), 3, pp. 133-142
[16] Mustafa, I., *et al.*, Hydromagnetic Natural Convection Flow of Water-Based Nanofluid along a Vertical Wavy Surface with Heat Generation, *Journal of Molecular Liquids, 229* (2017), Mar., pp. 246-254
[17] Srinivasacharya, D., Kumar, P. V., Mixed Convection over an Inclined Wavy Surface in a Nanofluid Saturated Non-Darcy Porous Medium with Radiation Effect, *International Journal of Chemical Engineering, 2015* (2015), ID 927508
[18] Mahdy, A., Ahmed, S. E., Laminar Free Convection over a Vertical Wavy Surface Embedded in a Porous Medium Saturated with a Nanofluid, *Transport in Porous Media, 91* (2012), 2, pp. 423-435
[19] Habiba, F., *et al.*, Natural Convection Flow of Cu-H_2O Nanofluid along a Vertical Wavy Surface with Uniform Heat Flux, *Proceedings,* 11th International Conference on Mechanical Engineering, 2016, pp. 18-20
[20] Mustafa, I., Javed, T., Heat Transfer In Natural Convection Flow Of Nanofluid Along A Vertical Wavy Plate With Variable Heat Flux, *Thermal Science, 23* (2019), 1, pp. 179-190
[21] Merkin, J. H., Mahmood, T., On the Free Convection Boundary Layer on a Vertical Plate with Prescribed Surface Heat Flux, *Journal of Engineering Mathematics, 24* (1990), 2, pp. 95-107
[22] Moulic, S. G., Yao, L. S., Natural Convection along a Vertical Wavy Surface with Uniform Heat Flux, *ASME Journal of Heat Transfer*, *111* (1989), 4, pp. 1106-1108
[23] Rees, D. A., Pop, I., Free Convection Induced by a Vertical Wavy Surface with Uniform Heat Flux in a Porous Medium, *ASME Journal of Heat Transfer*, *117* (1995), 2, pp. 547-550
[24] Pop, I., *et al*., Free Convection about a Vertical Wavy Surface with Prescribed Surface Heat Flux in a Micropolar Fluid, *Technische Mechanik*, *18* (1998), 4, pp. 229-237
[25] Tashtoush, B., Abu-Irshaid, E., Heat and Fluid Flow from a Wavy Surface Subjected to a Variable Heat Flux, *Acta Mechanica, 152* (2001), 1-4, pp. 1-8
[26] Javed, T., *et al*., Influence of Radiation on Vertical Wavy Surface with Constant Heat Flux: Using Keller Box Scheme, *Alexandria Engineering Journal, 55* (2016), 3, pp. 2221-2228
[27] Cebeci, T., Bradshaw, P., *Physical and Computational Aspects of Convective Heat Transfer*, Springer, New York, NY., USA, 1984
[28] Siddiqa, S., *et al*., Natural Convection Flow with Surface Radiation along a Vertical Wavy Surface, *Numerical Heat Transfer, Part A, Application, 64* (2013), 5, pp. 400-415

Paper submitted: September 29, 2017
Paper revised: January 8, 2018
Paper accepted: March 6, 2018