REDUCED DIFFERENTIAL TRANSFORM AND VARIATIONAL ITERATION METHODS FOR 3-D DIFFUSION MODEL IN FRACTAL HEAT TRANSFER WITHIN LOCAL FRACTIONAL OPERATORS

by

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Original scientific paper
https://doi.org/10.2298/TSCI170707033J

The analytical solutions of the 3-D diffusion equation in fractal heat transfer is found. The reduced differential transform and variational iteration methods are considered in the local fractional operator sense. In order to show the power and robustness of the proposed techniques, illustrative example is presented. The results reveal that the presented methods is very effective and simple, and can be used for other problems in mathematical physics.

Key words: *local fractional operator, reduced differential transform method, diffusion equation, variational iteration method*

Introduction

Diffusion problems of the important equations in mathematics which plays a major role in all areas of the engineering and physical sciences. Recently, the diffusion problems in 1-, 2-, and 3-D were considered by several authors by using local fractional decomposition method [1-3], variational iteration method [3-5], Laplace decomposition method [6], Laplace variational iteration method [7], function method [8], differential transform method [9, 10], variational iteration transform method [11], and local radial point interpolation method [12]. In this paper, the local fractional reduced differential transform method (LFRDTM) and local fractional variational iteration method (LFVIM) are used to find the analytical approximate solutions of the 3-D diffusion equation in fractal heat transfer within local fractional derivatives (LFD) was proposed [1]:

$$\frac{1}{\sigma^{\alpha}}\frac{\partial^{\alpha}\omega(\eta_1,\eta_2,\eta_3,\xi)}{\partial\xi^{\alpha}}=\nabla^{2\alpha}\omega(\eta_1,\eta_2,\eta_3,\xi) \tag{1}$$

subject to initial condition:

$$\omega(\eta_1,\eta_2,\eta_3,0)=g(\eta_1,\eta_2,\eta_3) \tag{2}$$

where σ^{α} is a non-differentiable diffusion coefficient, $\omega(\eta_1,\eta_2,\eta_3,\xi)$ – satisfied with the non-differentiable temperature distribution, and $\nabla^{2\alpha}$ – the local fractional Laplace operator defined [1, 7]:

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$$\nabla^{2\alpha} = \frac{\partial^{2\alpha}}{\partial {\eta_1}^{2\alpha}} + \frac{\partial^{2\alpha}}{\partial {\eta_2}^{2\alpha}} + \frac{\partial^{2\alpha}}{\partial {\eta_3}^{2\alpha}} \tag{3}$$

The local fractional reduced differential transform technique [13] is an iterative procedure for getting Taylor series solution of PDE. This method reduces the size of computational work and easily applicable to many physical problems. The LFRDTM is a powerful tool for solving linear and non-linear PDE, it can be applied very easily and it has less computational work than other existing methods like local fractional differential transform method [14].

The LFRDTM for 3-D diffusion model in fractal heat transfer

In this section, we will present the definition and theorems of 3-D reduced differential transform via LFD:

Definition 1. If $\omega(\eta_1,\eta_2,\eta_3,\xi)$ is local fractional analytic function in the domain of interest, then the local fractional spectrum function:

$$\Omega_\mu(\eta_1,\eta_2,\eta_3) = \frac{1}{\Gamma(1+\mu\alpha)}\left[\frac{\partial^{\mu\alpha}\omega(\eta_1,\eta_2,\eta_3,\xi)}{\partial\xi^{\mu\alpha}}\right]_{\xi=\xi_0} \tag{4}$$

is the reduced differential transformed of the function $\omega(\eta_1,\eta_2,\eta_3,\xi)$ via local fractional operator, where $\mu = 0,1,2,....,n$ and $0<\alpha\le 1$.

Definition 2. The differential inverse reduced transform of $\Omega_\mu(\eta_1,\eta_2,\eta_3)$ with local fractional operator is defined:

$$\omega(\eta_1,\eta_2,\eta_3,\xi) = \sum_{\mu=0}^{\infty}\Omega_\mu(\eta_1,\eta_2,\eta_3)(\xi-\xi_0)^{\mu\alpha} \tag{5}$$

By using eqs. (4) and (5), the theorems of the local fractional transform method are deduced:

$$\omega(\eta_1,\eta_2,\eta_3,\xi) = \sum_{\mu=0}^{\infty}\frac{1}{\Gamma(1+\mu\alpha)}\left[\frac{\partial^{\mu\alpha}\omega(\eta_1,\eta_2,\eta_3,\xi)}{\partial\xi^{\mu\alpha}}\right](\xi-\xi_0)^{\mu\alpha} \tag{6}$$

From eq. (6), it is obvious that the local fractional reduced differential transform is derived from the local fractional Taylor theorems.

Whenever $\xi_0 = 0$, the eqs. (4) and (5) become:

$$\Omega_\mu(\eta_1,\eta_2,\eta_3) = \frac{1}{\Gamma(1+\mu\alpha)}\left[\frac{\partial^{\mu\alpha}\omega(\eta_1,\eta_2,\eta_3,\xi)}{\partial\xi^{\mu\alpha}}\right]_{\xi=0} \tag{7}$$

and

$$\omega(\eta_1,\eta_2,\eta_3,\xi) = \sum_{\mu=0}^{\infty}\Omega_\mu(\eta_1,\eta_2,\eta_3)\,\xi^{\mu\alpha} \tag{8}$$

In view of eqs. (4) and (5), the theorems of LFRDTM in 3-D are deduced:

Theorem 1. If $\omega(\eta_1,\eta_2,\eta_3,\xi) = \phi(\eta_1,\eta_2,\eta_3,\xi) + \theta(\eta_1,\eta_2,\eta_3,\xi)$, then:

$$\Omega_\mu(\eta_1,\eta_2,\eta_3) = \Phi_\mu(\eta_1,\eta_2,\eta_3) + \Theta_\mu(\eta_1,\eta_2,\eta_3). \tag{9}$$

Proof. Immediately from eq. (5).

Theorem 2. If $\omega(\eta_1,\eta_2,\eta_3,\xi)=\lambda\phi(\eta_1,\eta_2,\eta_3,\xi)$, where λ is a constant, then:

$$\Omega_\mu(\eta_1,\eta_2,\eta_3)=\lambda\Phi_\mu(\eta_1,\eta_2,\eta_3) \tag{10}$$

Proof. Immediately from eq. (4).

Theorem 3. If $\omega(\eta_1,\eta_2,\eta_3,\xi)=\phi(\eta_1,\eta_2,\eta_3,\xi)\theta(\eta_1,\eta_2,\eta_3,\xi)$, then:

$$\Omega_\mu(\eta_1,\eta_2,\eta_3)=\sum_{l=0}^{\mu}\Phi_l(\eta_1,\eta_2,\eta_3)\Theta_{\mu-l}(\eta_1,\eta_2,\eta_3) \tag{11}$$

Proof. From the definition of $\omega(\eta_1,\eta_2,\eta_3,\xi)$, we get:

$$\omega(\eta_1,\eta_2,\eta_3,\xi)=\left[\sum_{\mu=0}^{\infty}\Phi_\mu(\eta_1,\eta_2,\eta_3)(\xi-\xi_0)^{\mu\alpha}\right]\left[\sum_{\mu=0}^{\infty}\Theta_\mu(\eta_1,\eta_2,\eta_3)(\xi-\xi_0)^{\mu\alpha}\right]=$$

$$=\left[\Phi_0+\Phi_1(\xi-\xi_0)^{\alpha}+\Phi_2(\xi-\xi_0)^{2\alpha}+\cdots\right]\left[\Theta_0+\Theta_1(\xi-\xi_0)^{\alpha}+\Theta_2(\xi-\xi_0)^{2\alpha}+\cdots\right]=$$

$$=\Phi_0\Theta_0+\left(\Phi_1\Theta_0+\Phi_0\Theta_1\right)\left(\xi-\xi_0\right)^{\alpha}+\left(\Phi_2\Theta_0+\Phi_1\Theta_1+\Phi_0\Theta_2\right)\left(\xi-\xi_0\right)^{2\alpha}+$$

$$+\left(\Phi_0\Theta_3+\Phi_1\Theta_2+\Phi_2\Theta_1+\Phi_3\Theta_0\right)\left(\xi-\xi_0\right)^{3\alpha}+\left(\Phi_0\Theta_4+\Phi_1\Theta_3+\Phi_2\Theta_2+\right.$$

$$\left.+\Phi_3\Theta_1+\Phi_4\Theta_0\right)\left(\xi-\xi_0\right)^{4\alpha}+\cdots=$$

$$=\sum_{\mu=0}^{\infty}\sum_{l=0}^{\mu}\Phi_l(\eta_1,\eta_2,\eta_3)\Theta_{\mu-l}(\eta_1,\eta_2,\eta_3)(\xi-\xi_0)^{\mu\alpha}$$

Therefore, we obtain:

$$\Omega_\mu(\eta_1,\eta_2,\eta_3)=\sum_{l=0}^{\mu}\Phi_l(\eta_1,\eta_2,\eta_3)\Theta_{\mu-l}(\eta_1,\eta_2,\eta_3)$$

Theorem 4. If $\omega(\eta_1,\eta_2,\eta_3,\xi)=(\partial^\alpha/\partial\xi^\alpha)\phi(\eta_1,\eta_2,\eta_3,\xi)$, then:

$$\Omega_\mu(\eta_1,\eta_2,\eta_3)=\frac{\Gamma[1+(\mu+1)\alpha]}{\Gamma(1+\mu\alpha)}\Phi_{\mu+1}(\eta_1,\eta_2,\eta_3) \tag{12}$$

Proof. Using the definition of $\Omega_\mu(\eta_1,\eta_2,\eta_3)$, we can conclude that:

$$\Omega_\mu(\eta_1,\eta_2,\eta_3)=\frac{1}{\Gamma(1+\mu\alpha)}\left[\frac{\partial^{\mu\alpha}\omega(\eta_1,\eta_2,\eta_3,\xi)}{\partial\xi^{\mu\alpha}}\right]_{\xi=\xi_0}=$$

$$=\frac{1}{\Gamma(1+\mu\alpha)}\left[\frac{\partial^{(\mu+1)\alpha}\phi(\eta_1,\eta_2,\eta_3,\xi)}{\partial\xi^{(\mu+1)\alpha}}\right]_{\xi=\xi_0}=$$

$$=\frac{\Gamma[1+(\mu+1)\alpha]}{\Gamma(1+\mu\alpha)}\Phi_{\mu+1}(\eta_1,\eta_2,\eta_3)$$

Theorem 5. If $\omega(\eta_1,\eta_2,\eta_3,\xi)=(\partial^{n\alpha}/\partial\eta_i^{\,n\alpha})\phi(\eta_1,\eta_2,\eta_3,\xi)$, where $i=1,2,3,\ n\in N$, then:

$$\Omega_\mu(\eta_1,\eta_2,\eta_3)=\frac{\partial^{n\alpha}}{\partial\eta_i^{\,n\alpha}}\Phi_\mu(\eta_1,\eta_2,\eta_3) \tag{13}$$

Proof. From the expression (4), we obtain:

$$\Omega_\mu(\eta_1,\eta_2,\eta_3)=\frac{1}{\Gamma(1+\mu\alpha)}\left[\frac{\partial^{\mu\alpha}\omega(\eta_1,\eta_2,\eta_3,\xi)}{\partial\xi^{\mu\alpha}}\right]_{\xi=\xi_0}=$$

$$=\frac{1}{\Gamma(1+\mu\alpha)}\left\{\frac{\partial^{\mu\alpha}}{\partial\xi^{\mu\alpha}}\left[\frac{\partial^{n\alpha}\phi(\eta_1,\eta_2,\eta_3,\xi)}{\partial\eta_i^{n\alpha}}\right]\right\}_{\xi=\xi_0}=$$

$$=\frac{1}{\Gamma(1+\mu\alpha)}\left\{\frac{\partial^{n\alpha}}{\partial\eta_i^{n\alpha}}\left[\frac{\partial^{\mu\alpha}\phi(\eta_1,\eta_2,\eta_3,\xi)}{\partial\xi^{\mu\alpha}}\right]\right\}_{\xi=\xi_0}=$$

$$=\frac{\partial^{n\alpha}}{\partial\eta_i^{n\alpha}}\Phi_\mu(\eta_1,\eta_2,\eta_3)$$

The LFVIM for 3-D diffusion model in fractal heat transfer

The LFVIM [4, 15] presents a correction local fractional functional for eq. (1):

$$\omega_{n+1}(\eta_1,\eta_2,\eta_3,\xi)=\omega_n(\eta_1,\eta_2,\eta_3,\xi)+$$
$$+\frac{1}{\Gamma(1+\alpha)}\int_0^\xi\frac{\gamma^\alpha}{\Gamma(1+\alpha)}\left[\frac{\partial^\alpha\omega_n(\eta_1,\eta_2,\eta_3,\kappa)}{\partial\kappa^\alpha}-\sigma^\alpha\nabla^{2\alpha}\tilde{\omega}_n(\eta_1,\eta_2,\eta_3,\kappa)\right](\mathrm{d}\kappa)^\alpha \tag{14}$$

where $\gamma^\alpha/\Gamma(1+\alpha)$ is a fractal Lagrange multiplier.

Making the local fractional variation of eq. (14), we obtain:

$$\delta_\alpha\omega_{n+1}(\eta_1,\eta_2,\eta_3,\xi)=\delta_\alpha\omega_n(\eta_1,\eta_2,\eta_3,\xi)+$$
$$+\frac{\delta_\alpha}{\Gamma(1+\alpha)}\int_0^\xi\frac{\gamma^\alpha}{\Gamma(1+\alpha)}\left[\frac{\partial^\alpha\omega_n(\eta_1,\eta_2,\eta_3,\kappa)}{\partial\kappa^\alpha}-\sigma^\alpha\nabla^{2\alpha}\tilde{\omega}_n(\eta_1,\eta_2,\eta_3,\kappa)\right](\mathrm{d}\kappa)^\alpha \tag{15}$$

The extremum condition of $\omega_{n+1}(\eta_1,\eta_2,\eta_3,\xi)$ requires that $\delta_\alpha\omega_{n+1}(\eta_1,\eta_2,\eta_3,\xi)=0$. This yields the stationary conditions:

$$1+\left.\frac{\gamma^\alpha}{\Gamma(1+\alpha)}\right|_{\kappa=\xi}=0,\quad \left.\left[\frac{\gamma^\alpha}{\Gamma(1+\alpha)}\right]^{(\alpha)}\right|_{\kappa=\xi}=0 \tag{16}$$

This in turn presents Lagrange multiplier:

$$\frac{\gamma^\alpha}{\Gamma(1+\alpha)}=-1 \tag{17}$$

Substituting eq. (17) into eq. (14), we obtain the iteration formula:

$$\omega_{n+1}(\eta_1,\eta_2,\eta_3,\xi)=\omega_n(\eta_1,\eta_2,\eta_3,\xi)-$$
$$-\frac{1}{\Gamma(1+\alpha)}\int_0^\xi\left[\frac{\partial^\alpha\omega_n(\eta_1,\eta_2,\eta_3,\kappa)}{\partial\kappa^\alpha}-\sigma^\alpha\nabla^{2\alpha}\omega_n(\eta_1,\eta_2,\eta_3,\kappa)\right](\mathrm{d}\kappa)^\alpha \tag{18}$$

Consequently, from eq. (18), we have the solution of eq. (1):

$$\omega(\eta_1,\eta_2,\eta_3,\xi)=\lim_{n\to\infty}\omega_n(\eta_1,\eta_2,\eta_3,\xi) \tag{19}$$

Illustrative example

The following local fractional diffusion equation:

$$\frac{\partial^\alpha}{\partial \xi^\alpha}\omega(\eta_1,\eta_2,\eta_3,\xi)=\nabla^{2\alpha}\omega(\eta_1,\eta_2,\eta_3,\xi) \tag{20}$$

is presented and its initial values are defined:

$$\omega(\eta_1,\eta_2,\eta_3,0)=\sin_\alpha(\eta_1{}^\alpha)\cos_\alpha(\eta_2{}^\alpha)\cos_\alpha(\eta_3{}^\alpha) \tag{21}$$

We first solve eq. (20) by using LFRDTM.

Implementing the RDTM via local fractional derivative to eq. (20), we have the following relation:

$$\frac{\Gamma\left[1+(\mu+1)\alpha\right]}{\Gamma(1+\mu\alpha)}\Omega_{\mu+1}(\eta_1,\eta_2,\eta_3)=\nabla^{2\alpha}\Omega_\mu(\eta_1,\eta_2,\eta_3) \tag{22}$$

which equivalent to the following formula:

$$\Omega_{\mu+1}(\eta_1,\eta_2,\eta_3)=\frac{\Gamma(1+\mu\alpha)}{\Gamma\left[1+(\mu+1)\alpha\right]}\left[\nabla^{2\alpha}\Omega_\mu(\eta_1,\eta_2,\eta_3)\right] \tag{23}$$

Using the initial condition eq. (21), we get:

$$\Omega_0(\eta_1,\eta_2,\eta_3)=\sin_\alpha(\eta_1{}^\alpha)\cos_\alpha(\eta_2{}^\alpha)\cos_\alpha(\eta_3{}^\alpha) \tag{24}$$

In view of eqs. (23) and (24), we obtain:

$$\Omega_1(\eta_1,\eta_2,\eta_3)=\frac{1}{\Gamma(1+\alpha)}\left[\nabla^{2\alpha}\Omega_0(\eta_1,\eta_2,\eta_3)\right]=$$
$$=\frac{1}{\Gamma(1+\alpha)}\left[-3\sin_\alpha(\eta_1{}^\alpha)\cos_\alpha(\eta_2{}^\alpha)\cos_\alpha(\eta_3{}^\alpha)\right]=$$
$$=-\frac{3}{\Gamma(1+\alpha)}\sin_\alpha(\eta_1{}^\alpha)\cos_\alpha(\eta_2{}^\alpha)\cos_\alpha(\eta_3{}^\alpha) \tag{25}$$

$$\Omega_2(\eta_1,\eta_2,\eta_3)=\frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}\left[\nabla^{2\alpha}\Omega_1(\eta_1,\eta_2,\eta_3)\right]=$$
$$=\frac{9}{\Gamma(1+2\alpha)}\sin_\alpha(\eta_1{}^\alpha)\cos_\alpha(\eta_2{}^\alpha)\cos_\alpha(\eta_3{}^\alpha) \tag{26}$$

$$\Omega_3(\eta_1,\eta_2,\eta_3)=\frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}\left[\nabla^{2\alpha}\Omega_2(\eta_1,\eta_2,\eta_3)\right]=$$
$$=-\frac{27}{\Gamma(1+3\alpha)}\sin_\alpha(\eta_1{}^\alpha)\cos_\alpha(\eta_2{}^\alpha)\cos_\alpha(\eta_3{}^\alpha) \tag{27}$$

and so on.

Finally, the reduced differential inverse transform of $\Omega_\mu(\eta_1,\eta_2,\eta_3)$ gives:

$$\omega(\eta_1,\eta_2,\eta_3,\xi)=\sum_{\mu=0}^{\infty}\Omega_\mu(\eta_1,\eta_2,\eta_3)\,\xi^{\mu\alpha}=$$

$$=\left[1-\frac{3\xi^{\alpha}}{\Gamma(1+\alpha)}+\frac{9\xi^{2\alpha}}{\Gamma(1+2\alpha)}-\frac{27\xi^{3\alpha}}{\Gamma(1+3\alpha)}+\cdots\right]\sin_{\alpha}(\eta_1^{\alpha})\cos_{\alpha}(\eta_2^{\alpha})\cos_{\alpha}(\eta_3^{\alpha})=$$

$$=E_{\alpha}(-3\xi^{\alpha})\sin_{\alpha}(\eta_1^{\alpha})\cos_{\alpha}(\eta_2^{\alpha})\cos_{\alpha}(\eta_3^{\alpha}) \tag{28}$$

Now, we solve eq. (20) by using LFVIM.
From eq. (21) we have:

$$\omega_0(\eta_1,\eta_2,\eta_3)=\sin_{\alpha}(\eta_1^{\alpha})\cos_{\alpha}(\eta_2^{\alpha})\cos_{\alpha}(\eta_3^{\alpha}) \tag{29}$$

By using eq. (18) we structure a local fractional iteration procedure:

$$\omega_{n+1}(\eta_1,\eta_2,\eta_3,\xi)=\omega_n(\eta_1,\eta_2,\eta_3,\xi)-$$

$$-\frac{1}{\Gamma(1+\alpha)}\int_0^{\xi}\left[\frac{\partial^{\alpha}\omega_n(\eta_1,\eta_2,\eta_3,\kappa)}{\partial\kappa^{\alpha}}-\nabla^{2\alpha}\omega_n(\eta_1,\eta_2,\eta_3,\kappa)\right](\mathrm{d}\kappa)^{\alpha},\quad n\ge 0 \tag{30}$$

Hence, from eqs. (29) and (30) we can derive the first approximation term:

$$\omega_1(\eta_1,\eta_2,\eta_3,\xi)=\omega_0(\eta_1,\eta_2,\eta_3,\xi)-$$

$$-\frac{1}{\Gamma(1+\alpha)}\int_0^{\xi}\left[\frac{\partial^{\alpha}\omega_0(\eta_1,\eta_2,\eta_3,\kappa)}{\partial\kappa^{\alpha}}-\nabla^{2\alpha}\omega_0(\eta_1,\eta_2,\eta_3,\kappa)\right](\mathrm{d}\kappa)^{\alpha}=$$

$$=\sin_{\alpha}(\eta_1^{\alpha})\cos_{\alpha}(\eta_2^{\alpha})\cos_{\alpha}(\eta_3^{\alpha})\left[1-\frac{3\xi^{\alpha}}{\Gamma(1+\alpha)}\right] \tag{31}$$

The second approximation can be calculated in the similar way, which is:

$$\omega_2(\eta_1,\eta_2,\eta_3,\xi)=\omega_1(\eta_1,\eta_2,\eta_3,\xi)-$$

$$-\frac{1}{\Gamma(1+\alpha)}\int_0^{\xi}\left[\frac{\partial^{\alpha}\omega_1(\eta_1,\eta_2,\eta_3,\kappa)}{\partial\kappa^{\alpha}}-\nabla^{2\alpha}\omega_1(\eta_1,\eta_2,\eta_3,\kappa)\right](\mathrm{d}\kappa)^{\alpha}=$$

$$=\sin_{\alpha}(\eta_1^{\alpha})\cos_{\alpha}(\eta_2^{\alpha})\cos_{\alpha}(\eta_3^{\alpha})\left[1-\frac{3\xi^{\alpha}}{\Gamma(1+\alpha)}+\frac{9\xi^{2\alpha}}{\Gamma(1+2\alpha)}\right] \tag{32}$$

Proceeding in this manner, we get the third approximation:

$$\omega_3(\eta_1,\eta_2,\eta_3,\xi)=\omega_2(\eta_1,\eta_2,\eta_3,\xi)-$$

$$-\frac{1}{\Gamma(1+\alpha)}\int_0^{\xi}\left[\frac{\partial^{\alpha}\omega_2(\eta_1,\eta_2,\eta_3,\kappa)}{\partial\kappa^{\alpha}}-\nabla^{2\alpha}\omega_2(\eta_1,\eta_2,\eta_3,\kappa)\right](\mathrm{d}\kappa)^{\alpha}=$$

$$=\sin_{\alpha}(\eta_1^{\alpha})\cos_{\alpha}(\eta_2^{\alpha})\cos_{\alpha}(\eta_3^{\alpha})\left[1-\frac{3\xi^{\alpha}}{\Gamma(1+\alpha)}+\frac{9\xi^{2\alpha}}{\Gamma(1+2\alpha)}-\frac{27\xi^{3\alpha}}{\Gamma(1+3\alpha)}\right] \tag{33}$$

$$\vdots$$

$$\omega_n(\eta_1,\eta_2,\eta_3,\xi)=\omega_{n-1}(\eta_1,\eta_2,\eta_3,\xi)-$$

$$-\frac{1}{\Gamma(1+\alpha)}\int_0^{\xi}\left[\frac{\partial^{\alpha}\omega_{n-1}(\eta_1,\eta_2,\eta_3,\kappa)}{\partial\kappa^{\alpha}}-\nabla^{2\alpha}\omega_{n-1}(\eta_1,\eta_2,\eta_3,\kappa)\right](\mathrm{d}\kappa)^{\alpha}=$$

$$=\left[\sum_{k=0}^{n}\frac{(-3)^k\xi^{k\alpha}}{\Gamma(1+k\alpha)}\right]\sin_{\alpha}(\eta_1^{\alpha})\cos_{\alpha}(\eta_2^{\alpha})\cos_{\alpha}(\eta_3^{\alpha}) \tag{34}$$

Thus, we have the final solution reads:

$$\omega(\eta_1,\eta_2,\eta_3,\xi)=\lim_{n\to\infty}\omega_n(\eta_1,\eta_2,\eta_3,\xi)=E_\alpha(-3\xi^\alpha)\sin_\alpha(\eta_1{}^\alpha)\cos_\alpha(\eta_2{}^\alpha)\cos_\alpha(\eta_3{}^\alpha) \tag{35}$$

Remark. From eqs. (28) and (35), the analytical solution of the given problem eq. (20) by using reduced differential transformation method is the same results as that obtained by the variational iteration method.

Conclusion

In this work, the local fractional reduced differential transform and variational iteration methods have been successfully used to solve the 3-D diffusion equation in fractal heat transfer within LFD and LFIO. The reduced differential transform method reduces significantly the numerical computations compare with the variational iteration method and differential transform method.

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Paper submitted: July 7, 2017
Paper revised: November 24, 2017
Paper accepted: December 4, 2017