NUMERICAL SOLUTION OF THERMAL ELASTIC-PLASTIC FUNCTIONALLY GRADED THIN ROTATING DISK WITH EXPONENTIALLY VARIABLE THICKNESS AND VARIABLE DENSITY

by

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Original scientific paper
https://doi.org/10.2298/TSCI131001136S

Thermal elastic-plastic stresses and strains have been obtained for rotating annular disk by using finite difference method with Von-Mises' yield criterion and non-linear strain hardening measure. The compressibility of the disk is assumed to be varying in the radial direction. From the numerical results, we can conclude that thermal rotating disk made of functionally graded material whose thickness decreases exponentially and density increases exponentially with non-linear strain hardening measure (m = 0.2) is on the safe side of the design as compared to disk made of homogenous material. This is because of the reason that circumferential stress is less for functionally graded disk as compared to homogenous disk. Also, plastic strains are high for functionally graded disk as compared to homogenous disk. It means that disk made of functionally graded material reduces the possibility of fracture at the bore as compared to the disk made of homogeneous material which leads to the idea of stress saving.

Key words: *disk, rotation, thin, thickness, density, thermal, functionally graded*

Introduction

The study of stress distribution at high angular velocity and temperature in rotating disk made of non-homogenous material is an active topic due to large number of industrial and mechanical applications. Some analytical solutions to rotating disk in elastic state and plastic state are available in [1, 2] and research on them is always an important topic. Eraslan [3] investigated inelastic stresses and displacements in rotating solid disks of exponentially varying thickness using Tresca's and Von-Mises' yield criterion with linear strain hardening. Plastic limit angular velocities have been calculated for different disk profiles. Further, Eraslan and Akgul [4] extended his work to find the numerical solution for elastic-plastic stresses in a rotating disk with Von-Mises' yield criterion using general non-linear strain hardening rule. The previous work mainly concentrates on the material whose mechanical and physical properties are constant. In contrast, non-homogenous materials have different spatial distribution of material properties which can be designed according to different engineering applications. Gupta and Shukla [5] studied the effect of non-homogeneity on elastic-plastic stresses in a rotating disk using transition theory and concluded that non-homogeneous thin rotating disk require high angular speed for initial yielding as compared to homogenous disk. Further, Gupta *et al.* [6] studied rotating disk with variable thickness and variable density to analyze creep stresses and concluded that a rotating disk with variable

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thickness and density is on the safer side of design in comparison to flat disk with variable density. Yuriy *et al.* [7] discussed the control problem of thermal stresses in axissymmetrical infinite cylinder using technique of integral transform. Sharma and Sahni [8] investigated elastic-plastic stresses for rotating disk made of transversely isotropic material and concluded that rotating disk made of transversely isotropic material is on the safer side of design as compared to rotating disc made of isotropic material. Reza *et al.* [9] discussed finite element simulation of residual stresses during the quenching process. A 3-D non-linear stress analysis model is used to estimate stress fields of UIC60. You *et al.* [10] calculated the elastic-plastic stresses with polynomial non-linear strain hardening model for rotating solid disk using perturbation technique. Further, with this polynomial non-linear strain hardening model You *et al.* [11] investigated elastic-plastic stresses for rotating disk having arbitrary variable thickness and density using Runge-Kutta method. Zhanling *et al.* [12] used finite element analysis of thermal-structure coupling to investigate stress and temperature when material properties are temperature dependent in a drag disk break application. Sharma and Yadav [13] investigated elastic-plastic stresses for rotating disk made up of isotropic material having exponentially variable thickness and exponentially variable density with non-linear strain hardening using finite difference method. They observed that disk whose thickness decreases radially and density increases radially is on the safer side of the design as compared to disk whose thickness and density varying exponentially as well as flat disk. Deepak *et al.* [14] studied creep stresses in a rotating disk made up of composite of silicon carbide particles in a matrix of pure aluminums and their study indicates that with the increase in particle gradient in the disk, the radial stress increases throughout the disk, whereas the tangential and effective stresses increase near the inner radius but decrease near the outer radius.

In this paper, we investigated thermal elastic-plastic stresses and strains for rotating disk made up of functionally graded material using finite difference method. The thickness and density are assumed to vary exponentially along the radius. Results have been discussed numerically and propounded graphically.

Mathematical formulation

Distribution of material properties and thickness profile with basic equations

A thermal annular axi-symmetrical disk has been considered with inner radius, a, and external radius, b, rotating with angular velocity, ω. The disk is made up of functionally graded material having exponentially varying thickness and density with plane stress condition *i. e.* $T_{zz} = 0$.

The coefficient of thermal expansion, compressibility, density, temperature distribution of the material and thickness profile of the rotating annular disks with radius, r, are expressed:

$$\alpha(r) = \alpha_0 \left(\frac{r}{b}\right)^{-\alpha_1}, \quad C(r) = C_0 \left(\frac{r}{b}\right)^{-k}, \quad \rho(r) = \rho_0 \mathrm{e}^{\left(\frac{r}{b}\right)^d},$$
$$\theta(r) = \frac{\theta_0}{\log\frac{a}{b}} \log\frac{r}{b}, \quad h(r) = h_0 \exp^{-\left(\frac{r}{b}\right)^n} \tag{1}$$

where α_0, C_0, ρ_0, θ_0, and h_0 are material constants and k, d, α_1, and n are the geometric parameters.

The equation of equilibrium for the rotating disk is:

$$\frac{\mathrm{d}}{\mathrm{d}r}(hrT_{rr}) - hT_{\theta\theta} + h\rho\omega^2 r^2 = 0 \tag{2}$$

The relation between strains and radial displacements are:

$$e_r = \frac{\mathrm{d}u}{\mathrm{d}r}, \quad e_\theta = \frac{u}{r}, \quad e_z = t \tag{3}$$

where u is the radial displacement and t is a constant.

The equation of compatibility can be derived from eq. (3):

$$r\frac{\mathrm{d}e_\theta}{\mathrm{d}r} + e_\theta - e_r = 0 \tag{4}$$

The total radial and circumferential strains in rotating annular disks are:

$$e_r = e_r^e + e_r^p + \alpha\theta, \qquad e_\theta = e_\theta^e + e_\theta^p + \alpha\theta \tag{5}$$

The relationship between stresses and strains can be represented by Hooke's Law in elasticity:

$$e_r^e = \frac{1}{3}\left[(2-C)T_{rr} - (1-C)T_{\theta\theta}\right], \quad e_\theta^e = \frac{1}{3}\left[(2-C)T_{\theta\theta} - (1-C)T_{rr}\right] \tag{6}$$

where C is the compressibility, T_{rr} and $T_{\theta\theta}$ are radial and circumferential stresses, respectively.

The temperature field satisfying Laplace heat equation is with:

$$\nabla^2\theta = 0 \ \text{ with } \ \theta = \theta_0 \ \text{ at } \ r = a, \ \theta = 0 \ \text{ at } \ r = b, \ \theta(r) = \frac{\theta_0}{\log\dfrac{a}{b}}\log\frac{r}{b}, \ \theta_0 \ \text{ is a constant} \tag{7}$$

Let us define radial and circumferential stress in terms of stress function:

$$T_{rr} = \frac{\phi}{hr}, \quad T_{\theta\theta} = \frac{1}{h}\frac{\mathrm{d}\phi}{\mathrm{d}r} + \rho\omega^2 r^2 \tag{8}$$

Substituting eq. (8) into eq. (6) and expressing strain components of eq. (5) in terms of stress function:

$$\begin{bmatrix} e_r \\ e_\theta \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2-C & -(1-C) \\ -(1-C) & 2-C \end{bmatrix}\begin{bmatrix} \dfrac{\phi}{hr} \\ \dfrac{1}{h}\dfrac{\mathrm{d}\phi}{\mathrm{d}r} + \rho\omega^2 r^2 \end{bmatrix} + \begin{bmatrix} e_r^p \\ e_\theta^p \end{bmatrix} + \begin{bmatrix} \alpha\theta \\ \alpha\theta \end{bmatrix} \tag{9}$$

Substituting eq. (9) into eq. (4), we have a governing differential equation for thermal elastic-plastic strain hardening rotating disks:

$$\begin{aligned} &r^2\phi'' + r\phi'\left[1 - r\frac{h'}{h} - r\left(\frac{C'}{2-C}\right)\right] - \phi\left[1 - \left(\frac{1-C}{2-C}\right)r\frac{h'}{h} - r\left(\frac{C'}{2-C}\right)\right] + \frac{3}{2-C}hr^2 \cdot \\ &\cdot\left(e_\theta^{p'} + \theta\alpha' + \alpha\theta'\right) = h\omega^2 r^4\left[\rho\left(\frac{C'}{2-C}\right) - \rho'\right] - h\rho\omega^2 r^3\left(\frac{7-4C}{2-C}\right) + \frac{3}{2-C}hr\left(e_r^p - e_\theta^p\right) \end{aligned} \tag{10}$$

where

$$\phi'' = \frac{\mathrm{d}^2\phi}{\mathrm{d}r^2}, \quad \phi' = \frac{\mathrm{d}\phi}{\mathrm{d}r}, \quad C' = \frac{\mathrm{d}C}{\mathrm{d}r}, \quad e_\theta^{p'} = \frac{\mathrm{d}e_\theta^p}{\mathrm{d}r}, \quad \theta' = \frac{\mathrm{d}\theta}{\mathrm{d}r}, \quad \alpha' = \frac{\mathrm{d}\alpha}{\mathrm{d}r}, \quad h' = \frac{\mathrm{d}h}{\mathrm{d}r}, \quad \rho' = \frac{\mathrm{d}\rho}{\mathrm{d}r}$$

In the elastic region ($e_r^p = e_\theta^p = 0$), eq. (10) reduces to:

$$\begin{aligned} &r^2\phi'' + r\phi'\left[1 - r\frac{h'}{h} - r\left(\frac{C'}{2-C}\right)\right] - \phi\left[1 - \left(\frac{1-C}{2-C}\right)r\frac{h'}{h} - r\left(\frac{C'}{2-C}\right)\right] + \\ &+ \frac{3}{2-C}hr^2\left(\theta\alpha' + \alpha\theta'\right) = h\omega^2 r^4\left[\rho\left(\frac{C'}{2-C}\right) - \rho'\right] - h\rho\omega^2 r^3\left(\frac{7-4C}{2-C}\right) \end{aligned} \tag{11}$$

For the plastic deformation, the relations between stresses and plastic strains can be determined according to the deformation theory in plasticity [2]:

$$e_r^p = \frac{e_e^p}{T_{ee}}\left(T_{rr} - \frac{1}{2}T_{\theta\theta}\right), \quad e_\theta^p = \frac{e_e^p}{T_{ee}}\left(T_{\theta\theta} - \frac{1}{2}T_{rr}\right) \tag{12}$$

where e_r^p and e_θ^p are the plastic radial and circumferential strains, e_e^p is the equivalent plastic strain which depends on the material model used and T_{ee} is the equivalent stress.

The von Mises' equivalent stress is given by the expression:

$$T_{ee} = \frac{1}{\sqrt{2}}\sqrt{\left(T_{rr} - T_{\theta\theta}\right)^2 + \left(T_{\theta\theta} - T_{zz}\right)^2 + \left(T_{zz} - T_{rr}\right)^2} \tag{13}$$

The stress-strain relationship for Swift's hardening law can be written:

$$e_e = \frac{T_{ee}}{E}, \quad e_e \le e_0, \quad e_e^p = \frac{1}{\eta}\left[\left(\frac{T_{ee}}{T_0}\right)^m - 1\right], \quad e_e > e_0 \tag{14}$$

where T_0 is the yield limit, e_e – the equivalent total strain, e_0 – the yield strain and T_{ee} – the equivalent stress.

Substituting e_e^p from eq. (14) into eq. (12) results in:

$$e_r^p = \frac{\frac{1}{\eta}\left[\left(\frac{T_{ee}}{T_0}\right)^m - 1\right]}{T_{ee}}\left(T_{rr} - \frac{1}{2}T_{\theta\theta}\right), \quad e_\theta^p = \frac{\frac{1}{\eta}\left[\left(\frac{T_{ee}}{T_0}\right)^m - 1\right]}{T_{ee}}\left(T_{\theta\theta} - \frac{1}{2}T_{rr}\right) \tag{15}$$

The boundary conditions for the rotating annular disks are:

$$T_{rr} = 0 \quad \text{at} \quad r = a, \quad T_{rr} = 0 \quad \text{at} \quad r = b \tag{16}$$

Substituting eq. (15) into eq. (10), we have a non-linear differential equation in terms of ϕ as:

$$r^2\phi''\left[1 + \frac{3}{2-C}\frac{1}{2\eta T_{ee}^2}\left\{2T_{ee}\left[\left(\frac{T_{ee}}{T_0}\right)^m - 1\right] + \frac{1}{2T_{ee}}\left(T_{rr} - 2T_{\theta\theta}\right)^2\left(1 + (m-1)\left(\frac{T_{ee}}{T_0}\right)^m\right)\right\}\right] -$$

$$-\frac{3}{2-C}\frac{1}{2\eta T_{ee}^2}\left[\begin{array}{l} T_{ee}\left[\left(\frac{T_{ee}}{T_0}\right)^m-1\right]\left[\left(1+2r\frac{h'}{h}\right)r\phi'-\left(1+r\frac{h'}{h}\right)\phi-2h\rho'\omega^2r^4-4h\rho\omega^2r^3\right]+ \\ +\frac{1}{2T_{ee}}\left[1+(m-1)\left(\frac{T_{ee}}{T_0}\right)^m\right](T_{rr}-2T_{\theta\theta})\left\{\begin{array}{l}(2T_{\theta\theta}-T_{rr})\left[-r^2\frac{h'}{h}\phi'+h\rho'\omega^2r^4+\right. \\ \left.+2h\rho\omega^2r^3\right]+(2T_{rr}-T_{\theta\theta})\cdot \\ \cdot\left[r\phi'-\left(1+r\frac{h'}{h}\right)\phi\right]\end{array}\right\}\end{array}\right]-$$

$$-\frac{9}{2\eta T_{ee}}\frac{hr}{(2-C)}\left[\left(\frac{T_{ee}}{T_0}\right)^m-1\right](T_{rr}-T_{\theta\theta})+r\phi'\left[1-r\frac{h'}{h}-r\left(\frac{C'}{2-C}\right)\right]-$$

$$-\phi\left[1-\left(\frac{1-C}{2-C}\right)r\frac{h'}{h}-r\left(\frac{C'}{2-C}\right)\right]+\frac{3}{2-C}hr^2\left[\theta\alpha'+\alpha\theta'\right]=$$

$$=h\omega^2r^4\left[\rho\left(\frac{C'}{2-C}\right)-\rho'\right]-h\rho\omega^2r^3\left(\frac{7-4C}{2-C}\right) \tag{17}$$

Equation (17) is a non-linear differential equation in the plastic region for rotating disk made up of functionally graded material with Swift's non-linear strain hardening measure having non-uniform thickness and whose material properties subjected to thermal loading. When compressibility is assumed to be constants without thermal effects, eq. (17) becomes same as that obtained by Sharma and Yadav [13].

Finite difference algorithm

To determine thermal elastic-plastic non-homogeneous stresses and strains in thin rotating disks with non-linear strain hardening material, we have to solve the second order non-linear differential eq. (17) under the boundary conditions (16). The general form of eq. (17) can be written:

$$\phi''=f\left(r,\ \phi,\phi'\right) \tag{18}$$

– First partitioned the disk domain $r=\left[a\ b\right]$ into p subintervals of length Δr and express the differential operator ϕ' and ϕ'' in finite difference form:

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}r^2}=\frac{\phi_{i+1}-2\phi_i+\phi_{i-1}}{\left(\Delta r\right)^2}\quad\text{and}\quad\frac{\mathrm{d}\phi}{\mathrm{d}r}=\frac{\phi_{i+1}-\phi_{i-1}}{2\Delta r}$$

– With $h = 1/p$, we have $p + 1$ nodal points $\phi_1,\phi_2,\ldots\phi_{p+1}$. The values at the end points are given by eq. (16), *i. e.* $\phi_1=0,\ \phi_{p+1}=0$. Using the finite difference approximation, we get the following system of equations:

$$\frac{\phi_{i+1}-2\phi_i+\phi_{i-1}}{\left(\Delta r\right)^2}=f\left(r,\phi_i,\frac{\phi_{i+1}-\phi_{i-1}}{2\Delta r}\right),\quad i=2,3,\ldots p$$

Stress function, stresses and strains can be obtained from the aforementioned system of $\left(p-1\right)$ non-linear equations using Newton-Raphson method.

Numerical discussion

The material properties of the annular disks made of functionally graded material with angular velocity $(\omega = 300, 700\ \text{rad/s})$ and thermal effects $(\theta_0 = 0, 400, 700\ ^\circ\text{C})$ are defined as: material density $\rho_0 = 7850\ \text{kg/m}^3$, compressibility coefficient $C_0 = 0.5$, thermal expansion coefficient $\alpha_0 = 17.8 \cdot 10^{-6}\ ^\circ\text{C}^{-1}$, Poisson's ratio $\nu = 0.3$. The inner and outer radii of the disk are taken as $a = 0.1$ m and $b = 0.5$ m, respectively. The geometric parameters of the disk are taken as $n = 0.5, 1$ in thickness function, $d = 0.7, 1$ in density function, $\alpha_1 = 0$ in coefficient of thermal expansion and $k = 0, -0.5, -1, -1.5$ in compressibility function.

In order to explain the effect of rotation and compressibility on stresses in a disk made up of homogenous and functionally graded materials, tab. 1 and curve have been drawn in figs*. 1-3 between stresses and radii $r = 0.1{:}0.1{:}0.5$.

Table 1 has made for circumferential (hoop) stresses with different parameters of compressibility and angular speed for non-linear strain hardening measure $m = 0.2$. From tab. 1, it is observed that without thermal effects circumferential stresses are maximum at the internal surface and increases significantly with the increase in angular speed. For functionally graded disk, circumferential stresses increases with the increase in compressibility radially. It is also noticed that circumferential stress is high for homogenous disk ($k = 0$) as compared to functionally graded disk ($k = -0.5, -1, -1.5$). Also, circumferential stresses are high for highly functionally graded disk ($k = -5$) as compared to homogenous disk and these stresses increases with the increase in compressibility $(k = -5, -6, -7, etc.)$. With the introduction of thermal effects, circumferential stresses decrease and these stresses further decrease with the increase in temperature. With the increase in Swift's strain hardening measure, circumferential stress increases whereas these stresses decrease with the exponential decrease in thickness and increase in density.

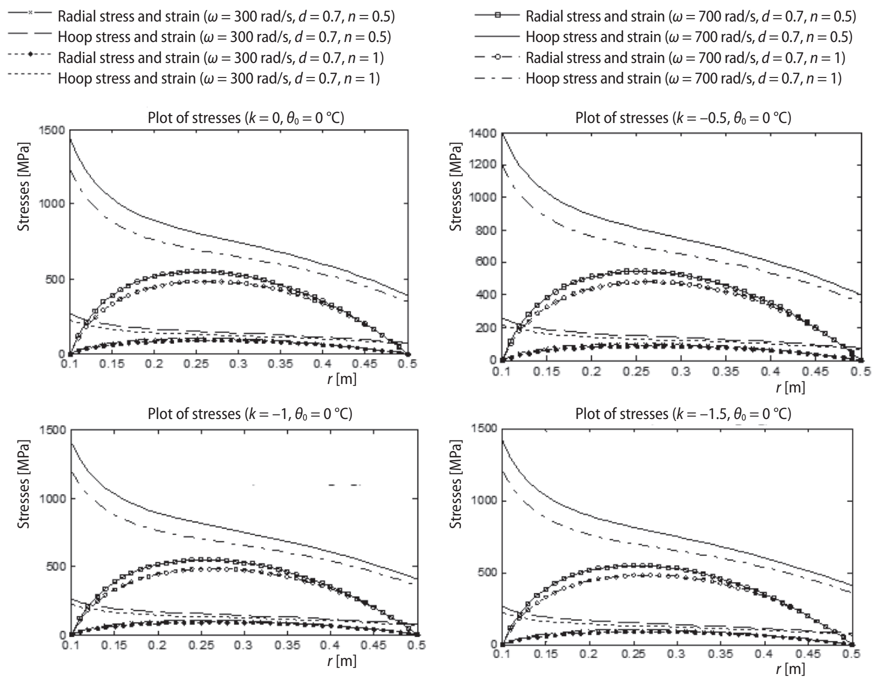

Figure 1. Elastic-plastic stresses in a rotating disk with parameters $(k = 0, -0.5, -1, -1.5)$

* Legend is valid for all figures (figs. 1-6)

From fig. 1, it is observed that circumferential stresses are maximum at internal surface for the disks made up of both homogenous and functionally graded material. Also, circumferential stresses are maximum for homogenous disk as compared to functionally graded disk. As compressibility measure changes from linear to non-linear, circumferential stresses decreases. With the increase in non-linearity, stresses are again increases *i. e.* $\sigma_\theta = 257.843\,\text{MPa}$ at $k = -0.5$, $\sigma_\theta = 258.195\,\text{MPa}$ at $k = -1$, and $\sigma_\theta = 260.190\,\text{MPa}$ at $k = -1.5$. From fig. 2, it has also been observed that with the increase in angular velocity from $\omega = 300$ rad/s to $\omega = 700$ rad/s, circumferential stresses increase significantly and maximum at internal surface of the disk. With the increase in thickness measure from $n = 0.5$ to $n = 1$ and density measure from $d = 0.7$ to $d = 1$, circumferential stress decreases. It can be observed from fig. 2 that circumferential stresses with thermal effects are maximum at internal surface for disk made of homogenous and functionally graded material. These circumferential stresses are maximum for homogenous disk as compared to functionally graded disk. With the increase in angular velocity circumferential stress increases significantly while with the increase in temperature, thickness and density, circumferential stress decreases as can be seen from fig. 3.

In order to explain the effect of rotation and compressibility on strains in a disk made up of homogenous and functionally graded materials, tab. 2 and graphs have been drawn in figs. 4-6 between strains and radii r = 0.1:0.1:0.5.

It has been noticed from tab. 2 that without thermal effects, circumferential strains are maximum at external surface of the disk. Also, these circumferential strains are compressive in nature. With the increase in angular speed, circumferential strains decrease significantly. Also, these circumferential strains increase when the thickness of disk decreases while density of the disk increases exponentially. These strains decreases with the increase in temperature while increases with the increase in angular speed. It has also been noted that cir-

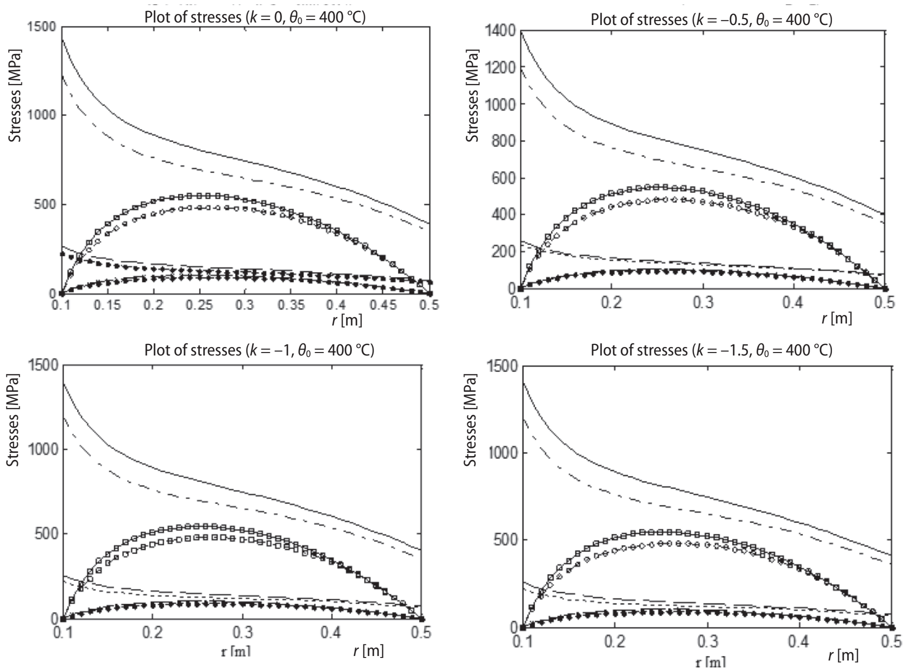

Figure 2. Thermal elastic-plastic stresses in a rotating disk with thermal effects ($\theta_0 = 400\,°\text{C}$) with parameters ($k$ = 0, – 0.5, – 1, – 1.5)

Table 1. Hoop stress in rotating annular disk having different geometric parameters of compressibility (k = 0, – 0.5, – 1, – 1.5) with non-linear strain hardening measure m = 0.2, angular velocity ω [rads^{-1}], and temperature θ_0 [°C]

θ_0 [°C]	$T_{\theta\theta}$ [MPa]	ω = 300 rad/s, d = 0.7, n = 0.5			ω = 300 rad/s, d = 1, n = 1			ω = 700 rad/s, d = 0.7, n = 0.5			ω = 700 rad/s, d = 1, n = 1		
	k \ r	**0.1**	**0.3**	**0.5**	**0.1**	**0.3**	**0.5**	**0.1**	**0.3**	**0.5**	**0.1**	**0.3**	**0.5**
0	**0**	265.0684	136.5472	71.7840	225.9867	118.5655	63.1811	1442.7691	743.4235	390.8258	1230.0670	645.5231	343.9862
	-0.5	257.8431	137.4753	73.4469	219.7997	119.5076	64.6199	1403.4647	748.4767	399.8788	1196.4099	650.6527	351.8196
	-1	258.1946	137.4663	74.3184	220.0074	119.5696	65.3629	1405.3682	748.4275	404.6234	1197.5330	650.9898	355.8642
	-1.5	260.1903	137.1896	74.7894	221.6020	119.3645	65.7561	1416.2223	746.9213	407.1871	1206.2053	649.8734	358.0047
400	**0**	265.0684	136.5472	71.7840	225.9867	118.5655	63.1811	1442.7691	743.4235	390.8258	1230.0670	645.5231	343.9863
	-0.5	257.8431	137.4753	73.4469	219.7997	119.5076	64.6199	1403.4647	748.4767	399.8788	1196.4099	650.6527	351.8196
	-1	258.1946	137.4663	74.3184	220.0074	119.5696	65.3629	1405.3682	748.4275	404.6233	1197.5330	650.9898	355.8642
	-1.5	260.1903	137.1896	74.7894	221.6020	119.3645	65.7561	1416.2223	746.9213	407.1871	1206.2053	649.8734	358.0048
700	**0**	265.0684	136.5472	71.7840	225.9867	118.5655	63.1811	1442.7691	743.4235	390.8258	1230.0670	645.5231	343.9862
	-0.5	257.8431	137.4753	73.4469	219.7997	119.5076	64.6199	1403.4647	748.4767	399.8788	1196.4099	650.6527	351.8196
	-1	258.1946	137.4663	74.3184	220.0074	119.5696	65.3629	1405.3682	748.4275	404.6234	1197.5330	650.9898	355.8642
	-1.5	260.1903	137.1896	74.7894	221.6020	119.3645	65.7561	1416.2224	746.9213	407.1872	1206.2053	649.8734	358.0047

Table 2. Hoop strain in rotating annular disk having different geometric parameters of compressibility (k = 0, – 0.5, – 1, – 1.5) with non-linear strain hardening measure m = 0.2, angular velocity ω [rads^{-1}], and temperature θ_0 [°C]

θ_0 [°C]	e_θ^p	ω = 300 rad/s, d = 0.7, n = 0.5			ω = 300 rad/s, d = 1, n = 1			ω = 700 rad/s, d = 0.7, n = 0.5			ω = 700 rad/s, d = 1, n = 1		
	k \ r	**0.1**	**0.3**	**0.5**	**0.1**	**0.3**	**0.5**	**0.1**	**0.3**	**0.5**	**0.1**	**0.3**	**0.5**
0	**0**	-0.00648	-0.00475	-0.00658	-0.00649	-0.00464	-0.00659	-0.0063	-0.00465	-0.00644	-0.00632	-0.00453	-0.00645
	-0.5	-0.00648	-0.0048	-0.00658	-0.0065	-0.00469	-0.00659	-0.0063	-0.00468	-0.00644	-0.00632	-0.00458	-0.00645
	-1	-0.00648	-0.0048	-0.00658	-0.0065	-0.00469	-0.00659	-0.0063	-0.00468	-0.00644	-0.00632	-0.00458	-0.00645
	-1.5	-0.00648	-0.00478	-0.00658	-0.00649	-0.00468	-0.00659	-0.0063	-0.00467	-0.00644	-0.00632	-0.00457	-0.00645
400	**0**	-0.00648	-0.00475	-0.00658	-0.00649	-0.00464	-0.00659	-0.0063	-0.00464	-0.00644	-0.00632	-0.00453	-0.00645
	-0.5	-0.00648	-0.0048	-0.00658	-0.0065	-0.00469	-0.00659	-0.0063	-0.00468	-0.00644	-0.00632	-0.00458	-0.00645
	-1	-0.00648	-0.0048	-0.00658	-0.0065	-0.00469	-0.00659	-0.0063	-0.00468	-0.00644	-0.00632	-0.00458	-0.00645
	-1.5	-0.00648	-0.00478	-0.00658	-0.00649	-0.00468	-0.00659	-0.0063	-0.00467	-0.00644	-0.00632	-0.00457	-0.00645
700	**0**	-0.00648	-0.00475	-0.00658	-0.00649	-0.00464	-0.00659	-0.0063	-0.00464	-0.00644	-0.00632	-0.00453	-0.00645
	-0.5	-0.00648	-0.0048	-0.00658	-0.0065	-0.00469	-0.00659	-0.0063	-0.00468	-0.00644	-0.00632	-0.00458	-0.00645
	-1	-0.00648	-0.0048	-0.00658	-0.0065	-0.00469	-0.00659	-0.0063	-0.00468	-0.00644	-0.00632	-0.00458	-0.00645
	-1.5	-0.00648	-0.00478	-0.00658	-0.00649	-0.00468	-0.00659	-0.0063	-0.00467	-0.00644	-0.00632	-0.00457	-0.00645

cumferential strains are less for homogenous disk (k = 0) as compared to disk made of functionally graded material and these strains decreases with the increase in compressibility measure ($k = -0.5, -1, -1.5, -2$), *etc.*

From fig. 4, it is observed that plastic strains are maximum at external surface for homogenous as well as functionally graded disk. Plastic strains decreases significantly for functionally graded disk when compressibility measure changes from $k = -1$ to $k = -1.5$ as

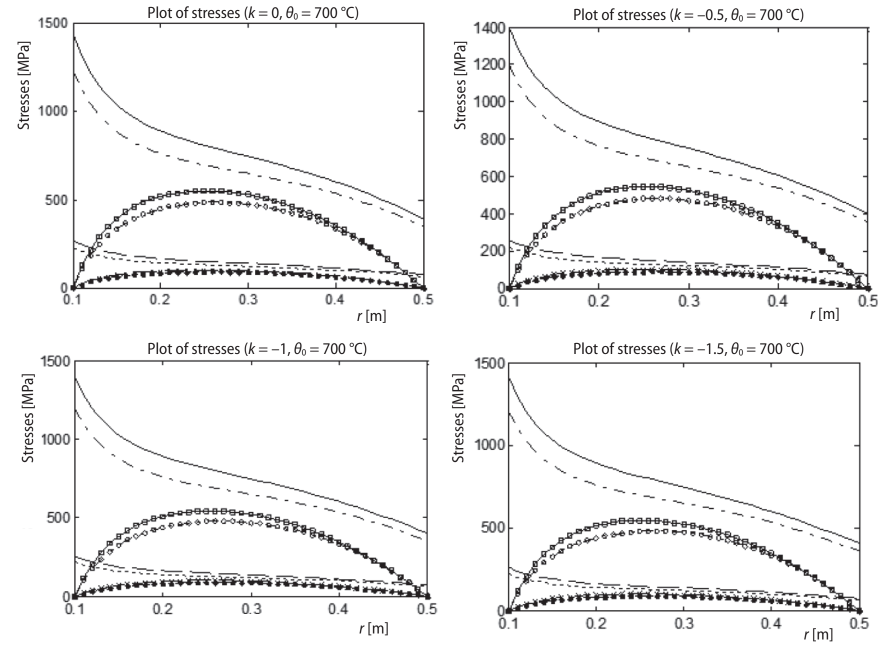

Figure 3. Thermal elastic-plastic stresses in a rotating disk with thermal effects (θ_0 = 700 °C) with parameters (k = 0, − 0.5, − 1, − 1.5)

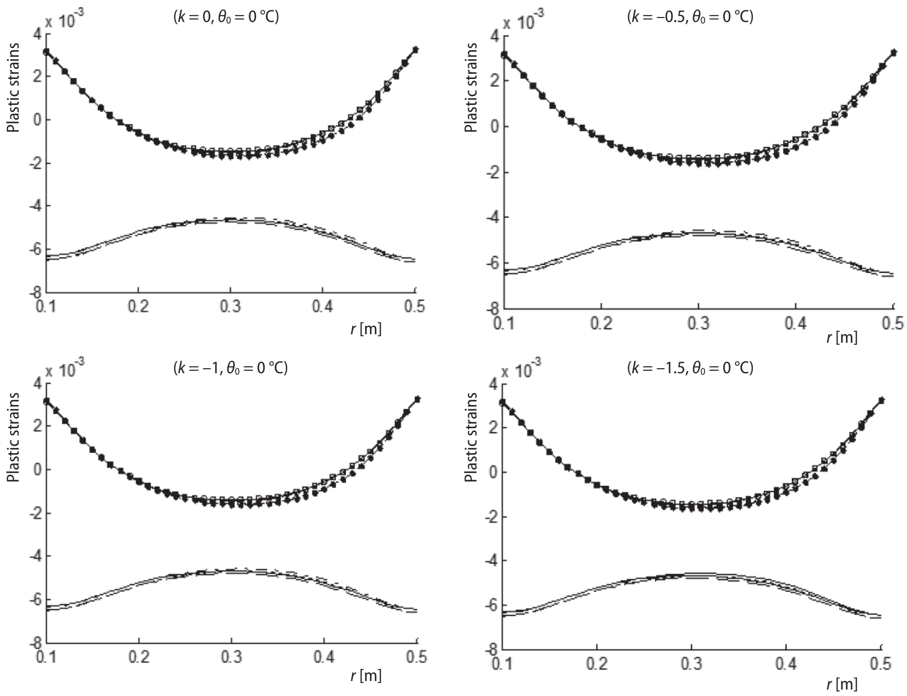

Figure 4. Plastic strains in a rotating disk with parameters (k = 0, − 0.5, − 1, − 1.5)

can be seen from fig. 4. Plastic strains decreases significantly when angular velocity changes from ω = 300 rad/s to ω = 700 rad/s, but as thickness measure changes from $h = 0.5$ to $h = 1$ and density measure from $d = 0.7$ to $d = 1$, these strains increases significantly. With the increase in temperature, plastic strains increases remarkably. From fig. 5, it has been observed that circumferential strains increases with the introduction of temperature. These strains further increases with the increase in temperature as can be seen from fig. 6. Also circumferential strains decrease significantly with the increase in angular velocity while increases with the increase in thickness and density.

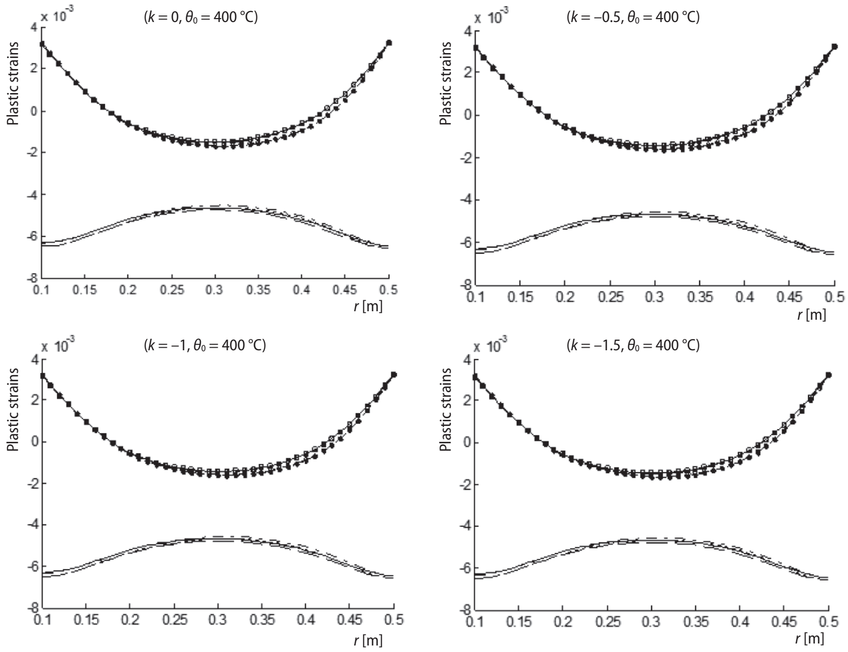

Figure 5. Thermal plastic strains in a rotating disk with thermal effects (θ_0 = 400°C) with parameters (k = 0, – 0.5, – 1, – 1.5)

Conclusion

From the previous analysis, it can be conclude that disk made of functionally graded material having non-linear strain hardening m = 0.2 with thermal effects whose thickness decreases exponentially and density increases exponentially is on the safer side of the design as compared to disk made of homogenous material. This is because of the reason that circumferential stress is less for functionally graded disk as compared to homogenous disk. Also, plastic strains are high for functionally graded disk as compared to homogenous disk. It means that disk made of functionally graded material reduces the possibility of fracture at the bore as compared to the disk made of homogeneous material which leads to the idea of stress saving.

Nomenclature

a – inner radii of the disk
b – outer radii of the disk
C – compressibility of the disk
C_0, h_0 – material constants
d, k, n – geometric parameters
E – Young's modulus

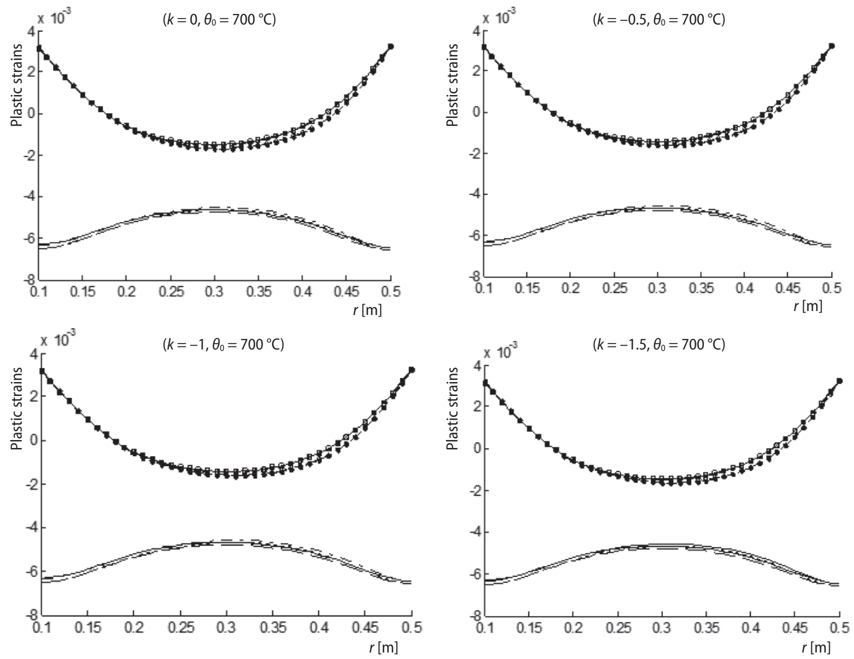

Figure 6. Thermo plastic strains in a rotating disk with thermal effects (θ_0 = 700 °C) with parameters (k = 0, − 0.5, − 1, − 1.5)

e_0 – yield strain
e_e – equivalent strain
e_r^e – radial elastic
e_r^p – plastic strains
e_r – total radial strains
e_z – circumferential strains
e_θ^e – circumferential elastic
e_θ^p – plastic strains
e_θ – axial strains
h – thickness of the disk
r – radius of the disk
T_{ee} – equivalent stress
$T_{rr}, T_{\theta\theta}, T_{zz}$ – radial, circumferential, and axial stresses
u – radial displacement

Greek symbols

α – thermal expansion
α_0, θ_0, ρ_0 – material constants
α_1 – geometric parameter
θ – temperature
ρ – density of the disk
ϕ – stress function
ω – angular velocity of the disk
ν – Possion's ratio

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Paper submitted: October 1, 2013
Paper revised: April 9, 2018
Paper accepted: April 29, 2018