MULTI-OBJECTIVE OPTIMIZATION ON SUPERCRITICAL CO_2 RECOMPRESSION BRAYTON CYCLE USING KRIGING SURROGATE MODEL

by

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Supercritical CO_2 cycle has become one of the most popular research fields of thermal science. The selection of operation parameters on thermodynamic cycle process is an important task. The computational model of supercritical CO_2 recompression cycle is built to solve the multi-objective problem in this paper. Then, the optimization of parameters is performed based on genetic algorithm. Several Kriging models are also used to reduce the quantity of samples. According to the calculation, the influence of sample quantity on the result and the time cost is obtained. The results show that it is required to improve the heat transfer when improvement of the cycle efficiency is desired.

Key words: *supercritical CO_2, Brayton cycle, recompression, multi-objective optimization problem, Kriging surrogate model*

Introduction

With the development of society and economy, the demand of electric power is growing sharply. The supercritical carbon dioxide (SCO_2) Brayton cycle can achieve higher power efficiency under the same inlet temperature. It is shown [1] that the efficiency of refrigerant cycle power generation system using supercritical water is about 45%, and the efficiency of SCO_2 Brayton cycle can reach about 48% when the turbine entrance refrigerant temperature is 650 °C. In addition, the size of the turbine and compressor in the power generation system using SCO_2 is much smaller than that of the steam power generation system because of the high energy density of SCO_2. Therefore, in the past twenty years, studies on SCO_2 Brayton cycle have attracted the attention of scholars both at home and abroad.

Iverson *et al.* [2] studied the SCO_2 Brayton cycle solar power generation system in the experimental system of 780 kW. The results show that the cycle efficiency of the system can be improved by SCO_2 Brayton cycle, especially when the turbine inlet refrigerant temperature is higher than 600 °C. Harvego and McKellar [3] used Unisim software to study SCO_2 Brayton cycle with split and recompression in nuclear power system. The results show that the system cycle efficiency reaches about 40% ~ 52% when the reactor outlet temperature is 550 ~ 850 °C. Sienicki *et al.* [4] proposed a conceptual design of the SCO_2 Brayton cycle system of 100 MW sodium cold fast reactor, and pointed out that this system provides a cycle efficiency 1% or more higher than that of conventional steam circulation systems, and the turbine and reactor

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size ratio are also smaller. Yin *et al.* [5] studied the application of supercritical/transcritical carbon dioxide mixed refrigerant in geothermal energy and analyzed the effect of sulfur hexafluoride concentration on CO_2 efficiency. Dostal [6] studied the SCO_2 Brayton cycle of the new generation of nuclear reactors and compared it with conventional steam power cycle in terms of cycle efficiency and economy. In addition, Dyreby *et al.* [7], Muto *et al.* [8], Utamura [9], Bae *et al.* [10], Jeong *et al.* [11], Moullec [12], Zhang *et al.* [13], and Ahn *et al.* [14] carried out detailed studies on the SCO_2 Brayton cycle and achieved valuable results.

In this paper, the process of SCO_2 recompression cycle is studied when two optimization targets are selected to optimize four parameters in the cycle. In addition, a method based on Kriging surrogate model is used in the genetic algorithm (GA) multi-objective optimization to reduce the number of samples and the influence of the initial sample number on the computational results, and the time consuming of the algorithm are discussed.

Calculation model

The circulatory system designed in this paper is shown in fig. 1. The calculation model consists of the main part, the compressor subroutine, the turbine subroutine, and the heat exchanger subroutine. The physical properties of CO_2 are directly retrieved by the database of National Institute of Standards and Technology.

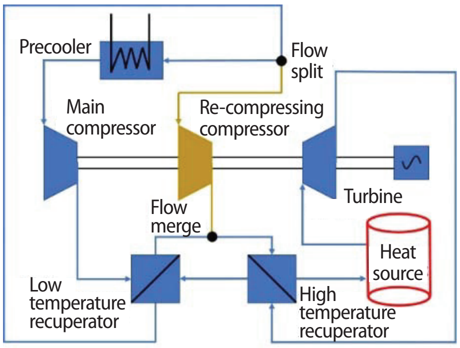

Figure 1. The SCO_2 recompression cycle layout

Fixed parameter: The fixed parameters of the calculation model are shown in table. 1. [2]

Table 1. Parameter value table

Fixed parameters	Value
Inlet temperature of the turbine [K]	773.15
Power of the turbine [MW]	5
Efficiency of the turbine	0.85
Efficiency of the main compressor	0.671
Efficiency of the re-compressor	0.683

Model inputs and outputs

The values are shown in tab. 2. In this paper, inlet pressure, outlet pressure of the turbine, inlet temperature of the main compressor, and flow coefficient (the flow coefficient is defined as the ratio of the mass flow of the working fluid into the main compressor to the total mass flow of the system) are chosen as the inputs of the model. At the same time, the cycle efficiency and the recuperator, *UA*, (overall heat transfer rate times heat transfer area) required by cycle are selected as the outputs of the model.

Calculation process

The calculation process of the calculation model is shown in fig. 2. After entering the input parameters and the fixed parameters of the model, the compressor subroutine and the turbine subroutine are used to calculate the outlet working parameters of each device when the inlet temperature of the re-compressor is assumed. Then, according to the heat balance principle and the heat exchanger subroutine, the inlet parameters of the hot side of the low temperature regenerator and the outlet parameters of the high temperature regenerator are calculated. The total heat absorption of the heat source is calculated according to the heat source inlet parameters. Finally, the minimum heat transfer temperature difference of each regenerator

Table 2. Input and output parameter table

Inputs	Outputs
Inlet pressure of the turbine	
Outlet pressure of the turbine	Cycle efficiency
Inlet temperature of the main compressor	Recuperator *UA* required by cycle
Flow coefficient	

is calculated. If it does not exceed 5 °C, the model is recalculated after changing the compressor inlet temperature. Calculation will not finish until it exceeds 5 °C and the cycle efficiency and the recuperator *UA* required by cycle will be output.

The turbine subroutine: the input parameters of the subroutine include inlet pressure, inlet temperature, outlet pressure, turbine entropy efficiency and turbine power, the output parameters include the outlet temperature and mass flow. Isentropic expansion multiplying by adiabatic efficiency is taken as calculation method and the formula used:

$$S'_{t\text{out}} = S_{t\text{in}}\left(P_{t\text{in}}, T_{t\text{in}}\right) \tag{1}$$

where $S'_{t\text{out}}$ [JK^{-1}] is the turbine export entropy in isentropic expansion, $S_{t\text{in}}$ [JK^{-1}] – the turbine entrance entropy, $P_{t\text{in}}$ [Pa] – the turbine inlet pressure, and $T_{t\text{in}}$ [K] – the turbine inlet temperature.

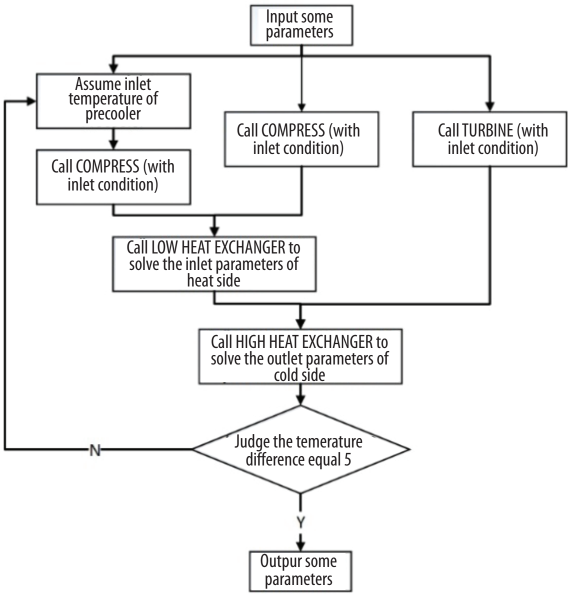

Figure 2. Program for re-compression Brayton cycle

$$M_t = \frac{W_t + H_{t\text{in}}\left(P_{t\text{in}}, T_{t\text{in}}\right)}{\eta_t H'_{t\text{out}}\left(P_{t\text{in}}, S'_{t\text{out}}\right)} \tag{2}$$

where M_t [kgs^{-1}] is the mass flow, W_t [W] – the turbine power, $H'_{t\text{out}}$ [Jkg^{-1}] – the turbine outlet enthalpy in isentropic compression, $H_{t\text{in}}$ [Jkg^{-1}] – the turbine inlet enthalpy, and η_t – the turbine efficiency.

$$H_{t\text{out}} = H_{t\text{in}} - \frac{W_t}{M_t} \tag{3}$$

where $H_{t\text{out}}$ [Jkg^{-1}] – turbine outlet enthalpy.

The turbine power can be solved by eq. (5) and turbine outlet temperature can be obtained according to the outlet pressure by eq. (6). Other physical parameters can be gained by the database of National Institute of Standards and Technology.

The compressor subroutine: the input parameters of the subroutine include inlet pressure, inlet temperature, outlet pressure, compressor entropy efficiency and mass flow, the output parameters include the outlet temperature and compression power consumption. The formula used:

$$S'_{\text{cout}} = S_{\text{cin}}\left(P_{\text{cin}}, T_{\text{cin}}\right) \tag{4}$$

where S'_{cout} [JK^{-1}] is the compressor export entropy in isentropic compression, S_{cin} [JK^{-1}] – the compressor entrance entropy, P_{cin} [Pa] – the compressor inlet pressure, and T_{cin} [K] – the compressor inlet temperature.

$$W_c = M_c \frac{cinH'_{\text{cout}}\left(P_{cin}, S'_{\text{cout}}\right) - H'_{\text{cin}}\left(P_{cin}, T_{cin}\right)}{\eta_c} \tag{5}$$

where W_c [W] is the compressor power consumption, M_c [kgs^{-1}] – the mass flow, H'_{cout} [Jkg^{-1}] – the compressor outlet enthalpy in isentropic compression, H_{cin} [Jkg^{-1}] – the compressor inlet enthalpy, and η_c – the compressor efficiency.

$$H_{\text{cout}} = H_{\text{cin}} + \frac{W_c}{M_c} \tag{6}$$

where H_{cout} [Jkg^{-1}] is the compressor outlet enthalpy.

The compressor power consumption can be solved by eq. (1) and compressor outlet temperature can be obtained according to the outlet pressure by eq. (2). Other physical parameters can be gained by the database of National Institute of Standards and Technology.

The heat exchanger subroutine: in this paper, the SCO_2 recompression Brayton cycle is considered as ideal calculation, so the actual structure and heat transfer efficiency are not considered in the heat exchanger subroutine. In addition, in order to improve the effectiveness of heat transfer and take into account the minimum temperature difference of the heat exchanger, the minimum temperature difference of the heat exchanger between the hot and cold side is set as 5 °C. The formula used:

$$H_{\text{heat\_in}} - H_{\text{heat\_out}} = H_{\text{cold\_out}} - H_{\text{cold\_in}} \tag{7}$$

where $H_{\text{heat\_in}}$ [Jkg^{-1}] is the heat fluid inlet enthalpy of heat exchanger, $H_{\text{heat\_out}}$ [Jkg^{-1}] – the heat fluid outlet enthalpy of heat exchanger, $H_{\text{cold\_out}}$ [Jkg^{-1}] – the cold fluid outlet enthalpy of heat exchanger, $H_{\text{cold\_in}}$ [Jkg^{-1}] – the cold fluid inlet enthalpy of heat exchanger.

Output of the model: the first objective of the optimization is the cycle efficiency, η. The formula used:

$$\eta = 1 - \frac{x(H_{\text{cout}\_r} - H_{\text{cout}\_m})}{(H_{t\text{in}} - H_{\text{cold\_out}\_H})} \tag{8}$$

where η is cycle efficiency, x – flow coefficient, H_{cout_r} [Jkg^{-1}] – the re-compressor outlet enthalpy, H_{cout_m} [Jkg^{-1}] – the main compressor outlet enthalpy, $H_{t\text{in}}$ [Jkg^{-1}] – the turbine inlet enthalpy, and $H_{\text{cold\_out}\_H}$ [Jkg^{-1}] – the heat fluid inlet enthalpy of high temperature regenerator.

The second objective of the optimization is the recuperator *UA* required by cycle. It is not appropriate to use the heat exchanger power as the optimization target because it is related to the mass flow. The recuperator *UA* required by the cycle process is used as an evaluation of the heat exchanger economy. The formula used:

$$UA = \frac{W_{\text{heat\_transfer}}}{1000 \dfrac{(\Delta T_{\text{heat}} - \Delta T_{\text{cold}})}{\ln\left(\dfrac{\Delta T_{\text{heat}}}{\Delta T_{\text{cold}}}\right)}} \tag{9}$$

where *UA* [kWK^{-1}] is the overall heat transfer rate times heat transfer area, $W_{heat\_transfer}$ [W] – the heat exchanger power, ΔT_{heat} [K] – the heat fluid temperature difference in heat exchanger, and ΔT_{cold} [K] – the cold fluid temperature difference in heat exchanger.

Results and discussion

Multi-objective optimization based on genetic algorithm

The multi-objective optimization based on genetic algorithm is calculated after the establishment of the calculation model. Since the goal of this research is to maximize the efficiency and minimize the recuperator *UA* required by the cycle, we firstly evaluate the reciprocal of the efficiency to modify the problem to be a multi-objective optimization for minimization. The population of each group contains 100 individuals and the Pareto set is obtained after 100 iterations. The objectives scatter plot with the Pareto is showed in fig. 3. The improvement of the cycle efficiency is realized at the expense of the recuperator *UA* required by the cycle and the cost is higher when the circulation efficiency is higher.

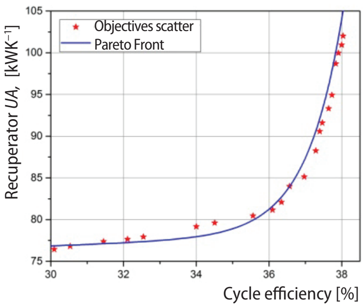

Figure 3. Objectives scatter plot with the Pareto Front

Some data obtained from the optimization are shown in tab. 3 when the optimized cycle efficiency is between 37% and 38%.

Table 3. Optimization result parameter table

Inlet pressure of the turbine [MPa]	Inlet temperature of the main compressor [K]	Outlet pressure of the turbine [MPa]	Flow coefficient	Cycle efficiency	Recuperator *UA* required by cycle [kWK^{-1}]
28.825	33.420	8.351	0.681	37.247	88.778
27.839	33.373	8.336	0.676	37.424	91.184
25.610	33.319	8.166	0.661	37.595	95.552
25.246	33.281	8.383	0.659	37.856	100.031
24.030	33.289	8.399	0.651	37.998	105.142

Multi-objective optimization based on genetic algorithm and Kriging surrogate model

Although the corresponding optimization results can be obtained through the previous process, the calculation of the model in the actual process is more complicated. The Kriging models are used to reduce the quantity of samples. The Latin hypercube sampling [15] is used to select the initial samples and the models are calculated as examples of 250, 500, 1000, 2500, and 5000. After fitting the corresponding function, the Pareto sets obtained based on Kriging model are compared with the previous set in figs. 4(a)-4(e). Besides, the range of the efficiency and the heat exchanger *UA* after optimization are more extensive than those obtained by the original optimization process. Only the cycle efficiencies of 32% to 38% are selected for the comparison for each example.

Then the accuracy of the solution set obtained by the previous samples is evaluated. Firstly, the fitting curve of each Pareto solution set is obtained by using the exponential function, and recuperator *UA* required by cycle is calculated in the cycle efficiency from 32% to 38%. The comparison between the calculated data and the original data is given in fig. 5. The

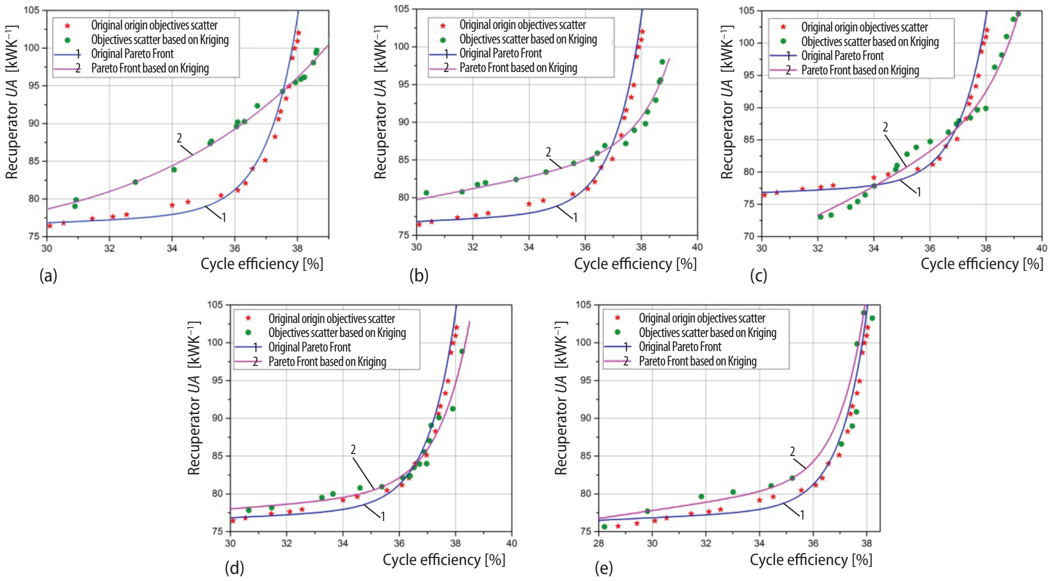

Figure 4. Compared graph of objectives scatter plot with the Pareto Front
(*for color image see journal web site*)

results show that the error of the function is decreased compared with the original model as the number of samples increases. A more detailed comparison is shown in tab. 4 from which such trend is fully indicated.

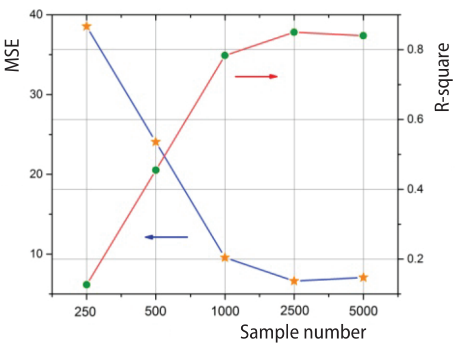

Figure 5. Transformation diagram of MSE and R-square

Then the time required for solving the models of different initial samples is evaluated. Because of the large difference between computation times, the natural logarithm of the value is obtained. The comparison of the total time consuming is shown in fig. 6(a) and the comparison of time consuming for each part in the optimization algorithm based on kriging interpolation is shown in fig. 6(b). It can be seen from fig. 6(a) that the time consuming of the optimization based on Kriging model is much less than the direct GA optimization in low sample numbers. When the initial number of samples increases, the time-consuming gradually increases but still retains the advantage over the direct GA optimization and such advantage disappears when

Table 4. Comparison table of calculation precision

Sample number	250	500	1000	2500	5000
SSE	2120.04	1323.09	525.70	364.23	388.64
MSE	38.55	24.06	9.56	6.62	7.07
RMSE	6.21	4.90	3.09	2.57	2.66
R-square	0.13	0.45	0.78	0.85	0.84
Average error [%]	6.79	5.23	2.65	2.22	3.02

the number of samples exceeds 2500. It is seen from fig. 6(b) that the total time consuming of the optimization based on Kriging model consists of different time consuming. With the increase of the number of samples, the rise of sampling time is not prominent, but the fitting time and the optimization time develop quite sharply.

Finally, due to the sample number of the original optimization process is 10000 which is higher than the sample number of the optimization based on Kriging model, the method has a greater advantage in the case where the sample selection is more difficult.

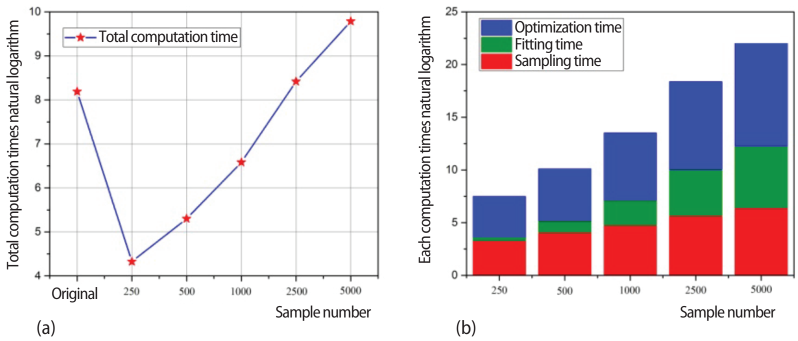

Figure 6. Comparison chart of computation time

Conclusions

The multi-objective optimization on supercritical recompression CO_2 Brayton cycle using Kriging surrogate model is studied in this paper. The results obtained from the original optimization, the comparison of results and time-consuming between the optimization obtained based on Kriging model and the original optimization is gained. The following conclusions are obtained.

- The improvement of the cycle efficiency is achieved at the expense of the recuperator *UA* required by the cycle and the cost is higher when the circulation efficiency rises. So, it is necessary to select the appropriate input parameters to improve the efficiency.
- The error of the results obtained by the GA multi-objective optimization based on the Kriging model and the original GA multi-objective optimization is small. The error of the former is decreased as the number of samples increases. However, the surrogate model is more extensive than the original model. So it is necessary to consider the rationality of the optimization results;
- The time consuming of the optimization based on Kriging model is much less than that of the direct GA optimization. Nevertheless, the time consuming gradually increases and then exceeds the time consuming of the direct GA optimization when the initial number of samples increases. Besides, the sample number of the original optimization process is higher than the sample number of the optimization based on Kriging model.

In conclusion, the GA multi-objective optimization based on Kriging model can effectively optimize the SCO_2 recompression cycle. It is necessary to make a comprehensive consideration of the calculation results and the computational cost.

Nomenclature

H – enthalpy, [Jkg^{-1}]
M – mass flow, [kgs^{-1}]
P – pressure, [Pa]
S – entropy, [JK^{-1}]

T – temperature, [K]
W – power, [W]
x – split-flow coefficient, [–]

Greek symbol

η – efficiency, [–]

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