FRACTAL ANALYSIS FOR HEAT EXTRACTION IN GEOTHERMAL SYSTEM

by

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Heat conduction and convection play a key role in geothermal development. These two processes are coupled and influenced by fluid seepage in hot porous rock. A number of integer dimension thermal fluid models have been proposed to describe this coupling mechanism. However, fluid flow, heat conduction and convection in porous rock are usually non-linear, tortuous and fractal, thus the integer dimension thermal fluid flow models can not well describe these phenomena. In this study, a fractal thermal fluid coupling model is proposed to describe the heat conduction and flow behaviors in fractal hot porous rock in terms of local fractional time and space derivatives. This coupling equation is analytically solved through the fractal travelling wave transformation method. Analytical solutions of Darcy's velocity, fluid temperature with fractal time and space are obtained. The solutions show that the introduction of fractional parameters is essential to describe the mechanism of heat conduction and convection.

Key words: *heat conduction, heat convection, fractal, Darcy's velocity, thermal fluid coupling model, local fractional operator, fractal travelling wave transformation method*

Introduction

High temperature geothermal resource is a kind of new clean energy [1]. Geothermal energy has extremely broad development prospects [2] such as heating buildings, generating electricity, and reducing CO_2 emissions but has not been developed in large scale utilizations. A scheme of geothermal resource development is shown in fig.1, which describes the cycle of heat extraction. Low temperature fluid is injected at an injection well, and high temperature fluid flows out from a production well. The heat exchange through heat conduction and convection occurs between the fluid and the hot fractured rock in the flow path from the injection well to the production well.

Theoretical and numerical simulation studies have been carried out for the development of geothermal resources. Among these studies, one important task is to establish a mathematical model for the heat extraction along the flow path [3-7]. A simple heat conduction model without convection, fluid flow models, hydro-thermal coupling models [3, 4], thermal hydro-mechanical coupling models [5-7] have been proposed so far. However, these models did

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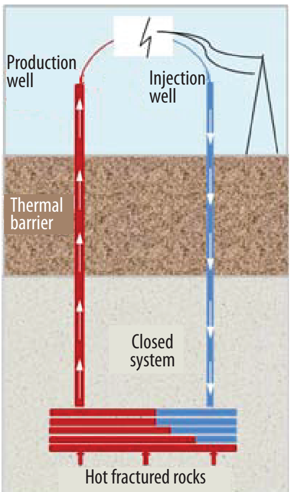

Figure 1. Sketch of geothermal development

not well consider the heterogeneous pores and tortuous fractures in the real stratigraphic structure of the rock. The fluid flows in porous rock matrix are usually non-linear, tortuous and fractal. Therefore, anomalous flow rather than normal Darcy flow may dominate the process of fluid flow in hot rock matrix in such a complex geothermal flow system [8-10]. Simultaneously, the pathway of heat conduction and convection is also anomalous. How these anomalous flow and properties affect the heat extraction in geothermal system is not clear.

In this condition, a fractional derivative model is a good choice for the simulation of the real fluid flow and heat conduction and convection. This kind of models is to replace the derivatives of an integer order with fractional order as the foundation of a classical differential equation in the constitutive equations [9, 10]. The theory of local fractional derivatives has been successfully applied to many problems in fluid mechanics [11-13] and others [14-17]. The fractal travelling wave transformation was introduced to solve the local fractional models [18-20]. In the aspect of the heat conduction, a local fractional heat conduction equation has been solved by the local variational iteration method [21]. However, the heat convection is not taken into account in this model although heat convection is non-negligible in the heat exchange of geothermal system [22].

Mathematical formulation of thermal fluid coupling model

Before any further development, following assumptions were made.

– The fractured rock is heterogeneous, anisotropic, rigorous, porous continuum.
– The density and viscosity of fluid is independent of temperature.
– Fluid flows in both rock matrix and fracture network, and obeys fractal Darcy's law.

Governing equations for fluid flow with fractional derivatives

The classical Darcy law for the normal flow in porous rock is:

$$v_f = -\frac{k}{\mu}\nabla p \tag{1}$$

where v_f is the Darcy's velocity of fluid, k – the permeability of the fractured hot rock, μ – the dynamic viscosity of the pore fluid, and p – the fluid pressure.

The real flow pathway of fluid is along the tortuous fractures of the hot fractured rock. Hence, the flow is fractal. The fractional Darcy velocity of fluid is [12]:

$$v(x,t) = -\frac{k}{\mu}\frac{\partial^\alpha p(x,t)}{\partial x^\alpha} \tag{2}$$

where $\partial^\alpha(\cdot)/\partial x^\alpha$ is the local fractional derivative of fractal order α with the space local fractional derivative, and $0 < \alpha \le 1$, and t is the time.

Correspondingly, the fractal mass conservation equation is [12]:

$$\frac{\partial^\alpha m(x,t)}{\partial t^\alpha} + \frac{\partial^\alpha [\rho v(x,t)]}{\partial x^\alpha} = Q_s \tag{3}$$

where m is the fluid mass, ρ – the fluid density, and Q_s – the fluid source.

For steady-state flow, eq. (3), can be simplified:

$$\frac{\partial^\alpha [v(x,t)]}{\partial x^\alpha} = \frac{Q_s}{\rho} \tag{4}$$

The local fractional integral of $f(x)$ is defined [11]:

$${}_aJ_x^\alpha f(x) = \frac{1}{\Gamma(1+\alpha)} \int_a^x f(s)\left(\mathrm{d}s\right)^\alpha, \quad 0 \le \alpha < 1 \tag{5}$$

Integrating eq. (5) into eq. (4) yields:

$$v(x,t) = v_0 - \frac{Q_s}{\rho} \frac{x^\alpha}{\Gamma(1+\alpha)} \tag{6}$$

Governing equations for heat conduction and convection

Both heat conduction and convection are coupled and influenced by fluid seepage within hot porous rock in geothermal development. They obey the energy conservation law in classical form:

$$\frac{\partial (C_{eq}T)}{\partial t} + \nabla(-K_{eq}\nabla T) + K_f \alpha_f T \nabla v = Q_T \tag{7}$$

where C_{eq} is the specific heat capacity, T – the temperature, K_{eq} – the effective thermal conductivity, K_f – the bulk modulus of fluid, α_f – the thermal expansion coefficient of fluid, and Q_T – the heat source.

In fractal porous media, an anomalous form is:

$$\frac{C_{eq}\partial^\alpha [T(x,t)]}{\partial t^\alpha} - K_{eq} \frac{\partial^{2\alpha} [T(x,t)]}{\partial x^{2\alpha}} + K_f \alpha_f \frac{\partial^\alpha [v(x,t)]}{\partial x^\alpha} T(x,t) = Q_T \tag{8}$$

Integrating eq. (4) into eq. (8) yields:

$$\frac{C_{eq}\partial^\alpha [T(x,t)]}{\partial t^\alpha} - K_{eq} \frac{\partial^{2\alpha} [T(x,t)]}{\partial x^{2\alpha}} + \frac{K_f \alpha_f Q_s}{\rho} T(x,t) = Q_T \tag{9}$$

For the convenience of calculation, eq. (9) is simplified into:

$$A\frac{\partial^\alpha [T(x,t)]}{\partial t^\alpha} - \frac{\partial^{2\alpha} [T(x,t)]}{\partial x^{2\alpha}} + BT(x,t) = 0 \tag{10}$$

where

$$A = \frac{C_{eq}}{K_{eq}}, \quad B = \frac{K_f \alpha_f Q_s}{\rho K_{eq}}, \quad Q_T = 0\,, \tag{11}$$

The initial and boundary conditions are given:

$$T(x,0) = T_0 \tag{12}$$

$$T(0,t) = T_0, \quad \frac{\partial^\alpha T(R,t)}{\partial x^\alpha} = q_0 \tag{13}$$

Therefore, the PDE of the temperature in heat conduction and convection can be written:

$$A\frac{\partial^{\alpha}[T(x,t)]}{\partial t^{\alpha}}-\frac{\partial^{2\alpha}[T(x,t)]}{\partial x^{2\alpha}}+BT(x,t)=0 \qquad (t>0, \quad 0<x<R) \tag{14}$$

Subject to the initial-boundary value conditions:

$$T(x,0)=T_0 \tag{15}$$

$$T(0,t)=T_0, \quad \frac{\partial^{\alpha}T(R,t)}{\partial x^{\alpha}}=q_0 \tag{16}$$

However, we mainly consider the PDE of the temperature in heat conduction and convection in the form:

$$A\frac{\partial^{\alpha}[T(x,t)]}{\partial t^{\alpha}}-\frac{\partial^{2\alpha}[T(x,t)]}{\partial x^{2\alpha}}+BT(x,t)=0 \tag{17}$$

Analytical solutions for temperature in time and space

In this section, we find the travelling wave solutions for the PDE of the temperature in heat conduction and convection.

In order to illustrate the non-differentiable travelling wave solutions for the problem, we consider the following travelling wave transformation of the non-differentiable type, θ [18-20]:

$$\theta^{\alpha}=x^{\alpha}-\phi t^{\alpha} \tag{18}$$

where ϕ is a constant.

Thus, we have the function $T(\theta)=T(x,t)$ such that:

$$\frac{\partial^{\alpha}T(x,t)}{\partial x^{\alpha}}=\frac{\partial^{\alpha}T}{\partial\theta^{\alpha}}\left(\frac{\partial\theta}{\partial x}\right)^{\alpha}=\frac{\partial^{\alpha}T}{\partial\theta^{\alpha}} \tag{19}$$

$$\frac{\partial^{2\alpha}T(x,t)}{\partial x^{2\alpha}}=\frac{\partial^{2\alpha}T}{\partial\theta^{2\alpha}} \tag{20}$$

and

$$\frac{\partial^{\alpha}T(x,t)}{\partial t^{\alpha}}=\frac{\partial^{\alpha}T}{\partial\theta^{\alpha}}\left(\frac{\partial\theta}{\partial t}\right)^{\alpha}=-\phi\frac{\partial^{\alpha}T}{\partial\theta^{\alpha}} \tag{21}$$

Substituting eqs. (20) and (21) into eq. (17), we have the equation of the form:

$$A\phi\frac{\mathrm{d}^{\alpha}T(\theta)}{\mathrm{d}\theta^{\alpha}}+\frac{\mathrm{d}^{2\alpha}T(\theta)}{\mathrm{d}\theta^{2\alpha}}-BT(\theta)=0 \tag{22}$$

Following the idea [23], we set the solution of the form:

$$T(\theta)=\gamma E_{\alpha}[\chi\theta^{\alpha}] \tag{23}$$

where γ and χ are constants, and the Mittag-Leffler function defined on fractal sets is defined as [11,17]:

$$E_{\alpha}(\theta^{\alpha})=\sum_{i=0}^{n}\frac{\theta^{i\alpha}}{\Gamma(1+i\alpha)} \tag{24}$$

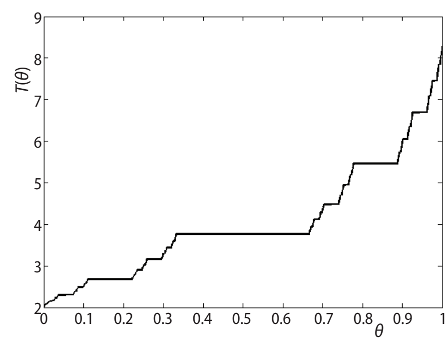

Figure 2. The plot of the function with the different parameters $\gamma = 2$ and $\chi = 1$

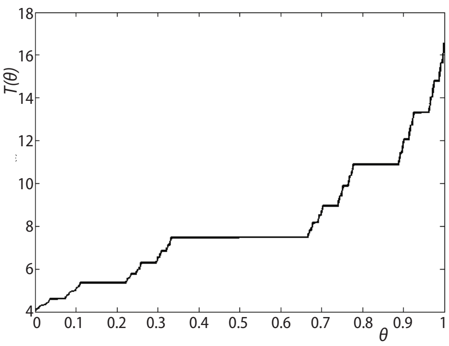

Figure 3. The plot of the function with the different parameters $\gamma = 2$ and $\chi = 2$

The plots of the function with the different parameters are presented in figs. 2 and 3. Substituting

$$\frac{\mathrm{d}^{\alpha}T(\theta)}{\mathrm{d}\theta^{\alpha}} = \frac{\mathrm{d}^{\alpha}}{\mathrm{d}\theta^{\alpha}}\left[\gamma E_{\alpha}\left(\chi\theta^{\alpha}\right)\right] = \gamma\chi E_{\alpha}\left(\chi\theta^{\alpha}\right) \tag{25}$$

and

$$\frac{\mathrm{d}^{2\alpha}T(\theta)}{\mathrm{d}\theta^{2\alpha}} = \frac{\mathrm{d}^{2\alpha}}{\mathrm{d}\theta^{2\alpha}}\left[\gamma E_{\alpha}\left(\chi\theta^{\alpha}\right)\right] = \gamma\chi^{2}E_{\alpha}\left(\chi\theta^{\alpha}\right) \tag{26}$$

into eq. (22) yields:

$$A\phi\gamma\chi E_{\alpha}(\chi\theta^{\alpha}) + \gamma\chi^{2}E_{\alpha}(\chi\theta^{\alpha}) - \gamma E_{\alpha}(\chi\theta^{\alpha}) = 0 \tag{27}$$

which reduces to the equation of the form:

$$\chi^{2} + A\phi\chi - 1 = 0 \tag{28}$$

with the solution:

$$\chi = \frac{-A\phi \pm \sqrt{A^{2}\phi^{2} + 4}}{2} \tag{29}$$

Making use of eqs. (18), (23), and (29), we obtain the travelling wave solutions of the PDE of the temperature in heat conduction and convection:

$$T_{1}(x,t) = \gamma E_{\alpha}\left[-\frac{A\phi - \sqrt{A^{2}\phi^{2} + 4}}{2}\left(x^{\alpha} - \phi t^{\alpha}\right)\right] \tag{30}$$

and

$$T_{2}(x,t) = \gamma E_{\alpha}\left[-\frac{A\phi + \sqrt{A^{2}\phi^{2} + 4}}{2}\left(x^{\alpha} - \phi t^{\alpha}\right)\right] \tag{31}$$

The graphs of the travelling wave solutions for the PDE of the temperature in heat conduction and convection are displayed in fig. 4.

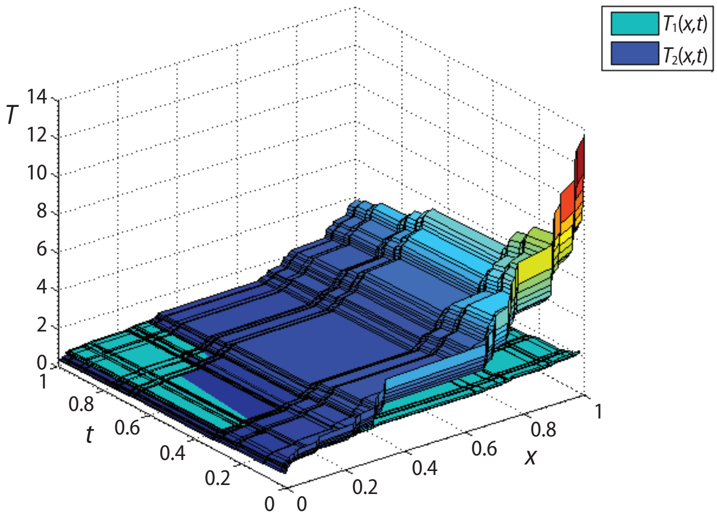

Figure 4. The travelling wave solutions for the parameters $\gamma = A = \phi = 1$ (*for color image see journal web site*)

Conclusion

This study proposed a fractal thermal fluid coupling model to describe the heat conduction and flow behaviors in fractal hot porous rock for the first time. A coupling equation set for fluid flow, heat conduction and convection was obtained by combining fluid seepage equation and heat transfer equation in terms of local fractional time and space derivatives. This coupling equation set was analytically solved through the fractal travelling wave transformation technique. Analytical solutions of Darcy's velocity, fluid temperature with fractal time and space were obtained. The result is successfully applied to model the heat conduction and flow behaviors in fractal hot porous rock.

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Nomenclature

C_{eq} – specific heat capacity, [Jkg^{-1}°C^{-1}]
K_{eq} – effective thermal conductivity, [Wm^{-1}K^{-1}]
K_f – bulk modulus of fluid, [Pa]
k – permeability of the fractured hot rock, [m^2]
p – fluid pressure, [Pa]
Q_T – heat source, [Wm^{-2}K^{-1}]
T – temperature, [°C]
v – Darcy's velocity of fluid, [ms^{-1}]

Greek symbols

α_f – thermal expansion coefficient of fluid, [mm^{-1}K^{-1}]
μ – dynamic viscosity of the pore fluid, [Pa·s]

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