UNSTEADY TRANSPORT OF MHD MIXED CONVECTION INSPIRED BY THERMAL RADIATION AND PARTIAL SLIP PERFORMANCE Finite Difference Approach

by

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Background: In this article mixed convection boundary layer flow of MHD fluid on permeable stretching surface is investigated under the effects of velocity and thermal slip. The physical unsteady problem is examined by considering thermal radiation effects on momentum and thermal boundary-layer flow. Different from available literature, in the present study we consider mix convective flow, thermal radiation, transverse applied magnetic field, velocity, and thermal slip.

Methodology: The transform non-linear system of differential equation is tackled numerically by the aid of finite difference scheme named as Keller-Box. Stable solution is correct up to six decimal places and special cases overlaps with the existing results in literature validating the present analysis.

Conclusion: It is concluded that mixed convection leads to accelerate fluid-flow and reduce temperature profile. Injection contributes in rising magnitude of velocity and temperature when compared with suction effects. Velocity and thermal slip parameter influence in lowering fluid-flow while temperature profile decrease for velocity slip parameter and opposite trend is witness corresponding to thermal slip parameter. Both velocity and temperature are increasing function of thermal radiation. In addition, the skin friction coefficient and the local Nusselt number are tabulated and analyzed.

Novelty: Present study is concerned with fluid-flow applications in plastic films, polymer extrusion, glass fiber, metallurgical processes, and metal spinning.

Key words: finite difference, computational design, thermal and velocity slips, thermal radiation

Introduction

The boundary-layer flow over a stretching surface has a prominent place in engineering and industrial applications such as plastic films, polymer extrusion, glass fiber, metallurgical processes, metal spinning, *etc.* In outlook of these practical aspects, Sakiadis [1] formulated boundary-layer equation due to boundary-layer flow over stretched surface. The stretching flows under various characteristics have been discussed extensively for steady flows [2-4]. The aforementioned studies are associated with stretching surfaces. However, flow over porous surfaces in steady flow has tremendous applications in filtration processes, metal and plastic extrusion, cosmic and geophysical sciences. Attia [5] examined heat transfer effects on a steady flow over a rotating disk in porous medium. Second law of thermodynamics is analyzed for steady flow of a nanofluid towards a rotating porous disk by Rashidi *et al.* [6]. Very recently, Manjunatha *et al.* [7] worked on conducting dusty fluid-flow in porous medium towards

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stretching cylinder. They analyzed effect of curvature parameter on velocity and temperature distributions of fluid.

In practice, we rarely need to have a precise design of problems concerning with unsteady fluid-flows. Acharya *et al.* [8] analyzed effect of heat transfer of an unsteady flow over an infinite vertical porous plate. Similarly, water-gas flow influenced by heat transfer in porous medium is introduced by Gawin *et al.* [9]. Hayat *et al.* [10] carried out analysis on periodic MHD flows due to porous disk. An unsteady flow analysis is considered by Das *et al.* [11] in which mass transfer effects over moving porous plate is solved numerically. A non-Newtonian fluid induced by porous plate is studied by Hameed and Nadeem [12]. Das *et al.* [13] investigated unsteady flow of a viscous fluid over a vertical porous plate with suction for constant magnetic field. Very recently, MHD rheological fluid-flow over a porous medium is inspected by Nadeem *et al.* [14]. Unsteady dusty viscous flow of fluid submerged in parallel plates where upper plate is bounded by porous medium has been discussed by Saxena and Agarwal [15]. They studied effect of magnetic field on flow velocity of both fluid and dust phase. Moreover, Sharma *et al.* [16] researched on influence of inclined magnetic field on an unsteady flow through porous media. They proved that inclination cause a positive change in flow velocity and rate of shear stress at surface.

The MHD flow and thermal analysis in porous medium have become scorching issue for quite a long time, which is replicated in number of publications. Heat transfer and thermal radiation analysis is carried out on an unsteady mixed convection flow towards porous surface by Elbashbeshy and Aldawody [17]. Hayat *et al.* [18] found a series solution of mixed convection stagnation point flow over a porous medium under the influence of thermal radiation. Investigation of a micro-polar fluid with radiation is considered by Rashidi and Abbasbandy [19]. Uwanta and Hamza [20] investigated time dependent convective flow between porous plates with thermal and suction/injection effects. The unsteady nanofluid flow is examined by Kandasamy and Muhaimin [21]. Hiemenz-non-darcy flow has been studied for magnetic field and thermal stratification induced by solar radiation over porous wedge. Introduction to heat exchanger efficiency over porous media is given by Shirvan *et al.* [22]. They gave a numerical solution which describes turbulent fluid flow. Some recent articles in this regard are cited ref. [23-25]

The aim of current study is to confer time-dependent MHD mixed convection flow with partial slip effects. In addition, thermal radiation effects are incorporated. Such analysis has concern with electric power generation, solar power technology, astrophysical flows, nuclear reactors, space vehicle re-entry and in other industrial areas.

Problem formulation and governing equations

We study MHD mixed convection flow of an incompressible viscous fluid bounded by a porous stretching sheet. The fluid is electrically conducting under time dependent magnetic field, B(t), exerted in a transverse influence of direction to flow. Induced magnetic field effect is absent whereas heat transfer is present in presence of thermal radiation. Moreover, sheet possesses velocity and thermal slip conditions. Here x- and y-axes are selected parallel and perpendicular to the stretching sheet. Physical flow diagram and relevant coordinate system are incorporated in fig. 1.

The governing equations of motion (*i.e.* the continuity, momentum, and energy) in vector form for viscous fluid are:

 ∇

$$\overline{V} = 0 \tag{1}$$

f

$$\rho(\overline{V}\nabla)\overline{V} = k \left(\nabla \times \overline{N}\right) + m \left(T - T_{\infty}\right)\overline{g} + \mu \left(1 + \frac{1}{\beta} + k\right)\nabla^{2}\overline{V} - \frac{\mu}{k_{1}}\left(1 + \frac{1}{\beta}\right)\overline{V} - (\overline{J} \times \overline{B})$$

$$(2)$$

$$p_{j}(\overline{V}\nabla)\overline{N} = k \left(\nabla \overline{V} - 2k\overline{N} - (\nabla^{2}\overline{N}) \right)$$
(3)

$$\overline{V}\nabla T = \alpha \nabla^2 T + \tau \nabla \overline{V} + \frac{1}{\rho C_o} (\overline{J} \times \overline{B}) \overline{V}$$
(4)

$$\overline{V}\nabla T = D\nabla^2 C - k_1 C \tag{5}$$

The component form of continuity, motion and energy equations provide:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(t)}{\rho} u + g\beta_T (T - T_\infty)$$
(7)

$$\rho c_{p} \left[\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \frac{\partial^{2} T}{\partial y^{2}} - \frac{\partial q_{r}}{\partial y}$$
(8)

Figure 1. Physical flow diagram and co-ordinate system

where u and v denote the velocity components in the x, and y-directions, respectively, ρ – the fluid density, v – the kinematic viscosity, σ – the electrical conductivity, T is the temperature, c_p –

the specific heat, k – the thermal conductivity of the fluid, and q_r – the radiative heat flux. In view of Rosseland approximation [17, 18], we can write:

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \tag{9}$$

where σ^* shows the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. Employing Taylor expansion and neglecting higher order terms we get:

$$T^4 \approx 4T^3_\infty T - 3T^4_\infty \tag{10}$$

Hence eqs. (8)-(10) give:

$$\rho c_{p} \left[\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = \frac{\partial}{\partial y} \left[\left(\frac{16\sigma^{*}T_{\infty}^{3}}{3k^{*}} + k \right) \frac{\partial T}{\partial y} \right]$$
(11)

The appropriate boundary conditions are:

$$u = U_w + N\mu \frac{\partial u}{\partial y}, v = V_w, T = T_w + K \frac{\partial T}{\partial y} \text{ at } y = 0$$
(12)

$$u \to 0, T \to T_{\infty} \text{ as } y \to \infty$$
 (13)

$$V_{w} = -\sqrt{\frac{vU_{w}}{x}}f(0) \tag{14}$$

Here indicates the mass transfer at surface with $V_w > 0$ for injection and $V_w < 0$ for suction, $N = N_0 (1-ct)^{1/2}$ is slip factor, $K = K_0 (1-ct)^{1/2}$ is thermal slip factor and for N = 0 = K, the no-slip conditions are recovered. The stretching velocity $U_w(x,t)$ and surface temperature $T_w(x,t)$ are taken:

$$U_{w}(x,t) = \frac{ax}{1-ct}, T_{w}(x,t) = T_{\infty} + \frac{bx}{1-ct}$$
(15)



where *a*, *b*, and *c* denote the constants with a > 0, $b \ge 0$, and $c \ge 0$ and (with ct < 1). Note that *a* and *c* have dimension of time ⁻¹. Consider choosing $B(t) = B_0(1-ct)^{-1/2}$ with B_0 as the uniform magnetic field and introducing:

$$\eta = \sqrt{\frac{U_w}{vx}} y, \psi = \sqrt{vxU_w} f(\eta), \ \theta(\eta) = \frac{T - T_w}{T_w - T_w}$$
(16)

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$$
(17)

The continuity eq. (6) identically satisfied and the resulting problems for f and θ are:

$$f''' + ff'' - f'^{2} - A^{*} \left(f' + \frac{1}{2} \eta f'' \right) - M^{2} f'' + \lambda \theta = 0$$
(18)

$$\frac{1}{\Pr}\left(1+\frac{4}{3N_r}\right)\theta'' + f\theta' - f'\theta - A'\left(\theta + \frac{1}{2}\eta\theta\right) = 0$$
(19)

$$f(0) = S, f'(0) = 1 + S_f f''(0), f'(\eta) \to 0 \text{ as } \eta \to \infty$$

$$\theta(0) = 1 + S_T \theta'(0), \theta(\eta) \to 0 \text{ as } \eta \to \infty$$
(20)

where ψ is the stream function, f(0) = S (with S < 0 corresponds to suction and S < 0 shows injection), $A^* = \frac{C}{a}$ is unsteadiness parameter and for $A^* = 0$ the problems reduce to the steady-state situation. The Hartman number, Prandtl number, the radiation parameter, N_r , mixed convection parameter, λ , the local Grashof number, the non-dimensional slip factor, S_r , and the nondimensional thermal slip parameter, S_r , are defined by:

$$M^{2} = \frac{\sigma B_{0}^{2}}{a\rho}, Pr = \frac{\mu c_{p}}{k}, N_{r} = \frac{4\sigma^{*}T_{x}^{*}}{3k^{*}k}, \lambda = \frac{Gr_{x}}{Re_{x}^{*}}, Re_{x} = \frac{U_{w}x}{v}$$

$$Gr_{x} = \frac{g\beta(T_{w} - T_{w})x^{3}/v^{2}}{U_{w}^{2}x^{2}/v^{2}}, S_{f} = N_{0}\rho\sqrt{av}, S_{T} = K_{0}\sqrt{\frac{a}{v}}$$
(21)

where prime is used for the differentiation with respect to η .

The skin friction coefficient, C_{f} , and local Nusselt number are defined:

$$C_{f} = \frac{\tau_{w}}{\rho U_{w}^{2}/2}, \operatorname{Nu}_{x} = \frac{xq_{w}}{k(T_{w} - T_{\infty})}$$
(22)

where the skin friction, τ_w , and the heat transfer from the plate, q_w , are:

$$\tau_{w} = \mu \left(\frac{\partial u}{\partial y} \right) \Big|_{y=0}, q_{w} = -k \left(\frac{\partial T}{\partial y} \right) \Big|_{y=0}$$
(23)

Dimensionless forms of eq. (23) are:

$$\frac{1}{2}C_{f}\operatorname{Re}_{x}^{1/2} = f''(0), \operatorname{Nu}_{x}/\operatorname{Re}_{x}^{\frac{1}{2}} = -\theta'(0)$$
(24)

Computational design

Keller box method is used to solve the non-linear ODE (18) and (19) subject to boundary conditions eq. (20). This scheme is unconditionally stable and achieves exceptional accuracy. The complete flow chart of this implicit finite difference scheme is:

We start with introducing new independent variables (x, η) , $v(x,\eta)$, and $t(x, \eta)$ with f'=u, u'=v, and $\theta'=t$, so that eqs. (18) and (19) reduces to first order form *i.e.*:

$$v' + fv - u^{2} - A^{*}\left(u + \frac{\eta}{2}v\right) - M^{2}u + \lambda\theta = 0$$
(25)

$$\left(1+\frac{4}{3N_{\nu}}\right)t^{\prime}+\Pr ft-\Pr u\theta-A^{\ast}\left(\theta+\frac{\eta}{2}t\right)=0$$
(26)

The rectangular grid in x- η plane is shown in fig. 2 and net points are:

$$x^{0} = 0, x^{i} = x^{i-1} + k_{i}, i = 1, 2, ..., J_{n}$$

$$\eta_0 = 0, \ \eta_j = \eta_{j-1} + h_j$$
 , $j = 1, 2, ..., J,$

where k_i and k_j are the Δx and $\Delta \eta$ - spacing. Here *i* and *j* are just sequences of numbers that indicate the co-ordinate location.

Using central difference formulation at midpoint $(x^{i}, \eta_{j}^{-1/2})$ as:

$$f_{j}^{i} - f_{j-1}^{i} - \frac{h_{j}}{2} (u_{j}^{i} + u_{j-1}^{i}) = 0$$
(27)
$$u_{j}^{i} - u_{j-1}^{i} - \frac{h_{j}}{2} (v_{j}^{i} + v_{j-1}^{i}) = 0$$
(28)
$$\theta_{j}^{i} - \theta_{j-1}^{i} - \frac{h_{j}}{2} (t_{j}^{i} + t_{j-1}^{i}) = 0$$
(29)





Now applying central differencing, eqs. (25) and (26) at point $(x^{i-1/2}, \eta_i^{-1/2})$:

Keller box scheme flow chart

$$(v_{j}^{i} - v_{j-1}^{i}) + hf_{j-1/2}^{i} v_{j-1/2}^{i} - h(u_{j-1/2}^{i})^{2} - hA^{*}(u_{j-1/2}^{i} + \frac{\eta_{j-1/2}}{3} v_{j-1/2}^{i}) - hM^{2}u_{j-1/2}^{i} + h\lambda\theta_{j-1/2}^{i} = R_{j-1/2}$$
(30)

$$(1 + \frac{4}{3N_{r}})(t_{j}^{i} + t_{j-1}^{i}) + h \Pr f_{j-1/2}^{i} t_{j-1/2}^{i} - h \Pr u_{j-1/2}^{i} \theta_{j-1/2}^{i} - h A^{*}(\theta_{j-1/2}^{i} + \frac{\eta_{j-1/2}}{3} t_{j-1/2}^{i}) = T_{j-1/2}$$
(31)

where $R_{j-1/2}$ and $T_{j-1/2}$ are known quantities *i.e.*:

$$R_{j-1/2} = -(v_{j}^{i-1} - v_{j-1}^{i-1}) - hf_{j-1/2}^{i-1}v_{j-1/2}^{i-1} + h(u_{j-1/2}^{i-1})^{2} + hA^{*}(u_{j-1/2}^{i-1} + \frac{\eta_{j-1/2}}{2}v_{j-1/2}^{i-1}) + hM^{2}u_{j-1/2}^{i-1} + h\lambda\theta_{j-1/2}^{i-1}$$
(32)

$$T_{j-1/2} = -\left(1 + \frac{4}{3N_r}\right) \left(t_j^{i-1} - t_{j-1}^{i-1}\right) - h\Pr f_{j-1/2}^{i-1} t_{j-1/2}^{i-1} + h\Pr u_{j-1/2}^{i-1} \theta_{j-1/2}^{i-1} + hA^*(\theta_{j-1/2}^{i-1} + \frac{\eta_{j-1/2}}{2} t_{j-1/2}^{i-1})$$
(33)

and boundary condition reduce to:

$$f_0^i = S, \, u_0^i = 1 + S_f \, v_0^i, \, u_J^i = 0, \, \theta_0^i = 1 + S_T \, t_0^i, \theta_J^i = 0 \tag{34}$$

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Newton's method to linearize eqs. (30) and (31), we introduce following iterates:

$$f_{j}^{(i+1)} = f_{j}^{(i)} + \delta f_{j}^{(i)}, u_{j}^{(i+1)} = u_{j}^{(i)} + \delta u_{j}^{(i)}, v_{j}^{(i+1)} = v_{j}^{(i)} + \delta v_{j}^{(i)}$$

$$\theta_{j}^{(i+1)} = \theta_{j}^{(i)} + \delta \theta_{j}^{(i)}, t_{j}^{(i+1)} = t_{j}^{(i)} + \delta t_{j}^{(i)}$$
(35)

Substituting previous expressions in eqs. (30) and (31) and then drop quadratic and higher order terms in $\delta f_j^{(i)}$, $\delta u_j^{(i)}$, $\delta v_j^{(i)}$, $\delta \theta_j^{(i)}$, $\delta t_j^{(i)}$ this procedure yields the following tridiagonal system:

$$\delta f_j - \delta f_{j-1} - \frac{h_j}{2} (\delta u_j - \delta u_{j-1}) = (r_1)_j$$
(36)

$$\delta u_{j} - \delta u_{j-1} - \frac{h_{j}}{2} (\delta v_{j} - \delta v_{j-1}) = (r_{2})_{j}$$
(37)

$$\delta\theta_{j} - \delta\theta_{j-1} - \frac{h_{j}}{2} (\delta t_{j} - \delta t_{j-1}) = (r_{2})_{j}$$

$$(a_{1})_{j-1/2} \delta v_{j} + (a_{2})_{j-1/2} \delta u_{j} + (a_{3})_{j-1/2} \delta u_{j} + (a_{4})_{j-1/2} \delta u_{j-1} + (a_{4})_{j-1/2} \delta u_{j-1/2} + (a_{4})_{j-1/2} \delta u_{j-1} + (a_{4})_{j-1/2} \delta$$

$$+ (a_{5})_{j-1/2} \delta f_{j} + (a_{6})_{j-1/2} \delta f_{j-1} + (a_{7})_{j-1/2} \delta \theta_{j} + (a_{8})_{j-1/2} \delta \theta_{j-1} = (r_{4})_{j-1/2}$$
(39)

$$(b_{1})_{j-1/2} \,\delta u_{j} + (b_{2})_{j-1/2} \,\delta u_{j-1} + (b_{3})_{j-1/2} \,\delta f_{j} + (b_{4})_{j-1/2} \,\delta f_{j-1} + + (b_{5})_{j-1/2} \,\delta t_{j} + (b_{6})_{j-1/2} \,\delta t_{j-1} + (b_{7})_{j-1/2} \,\delta \theta_{j} + (b_{8})_{j-1/2} \,\delta \theta_{j-1} = (r_{5})_{j-1/2}$$

$$(40)$$

In which

$$(a_{1})_{j-1/2} = 1 + \frac{h_{j}}{4} f_{j-1/2} - \frac{h_{j} A^{*} \eta_{j-1/2}}{4} = (a_{2})_{j-1/2} + 2$$
(41)

$$(a_{3})_{j-1/2} = -\frac{h_{j}}{2}u_{j-1/2} - \frac{h_{j}}{2}(A^{*} + M^{2}) = (a_{4})_{j-\frac{1}{2}}$$

$$(42)$$

$$(a_5)_{j-1/2} = \frac{h_j}{4} v_{j-1/2} = (a_6)_{j-1/2}, (a_7)_{j-1/2} = \frac{h_j}{2} \lambda = (a_8)_{j-1/2}$$
(43)

$$(b_{1})_{j-1/2} = -\frac{h_{j} \operatorname{Pr}}{2} \theta_{j-1/2} = (b_{2})_{j-1/2}, (b_{3})_{j-1/2} = \frac{h_{j} \operatorname{Pr}}{4} t_{j-1/2} = (b_{4})_{j-1/2}$$
(44)

$$(b_{5})_{j-1/2} = \left(1 + \frac{4}{3N_{r}}\right) + \frac{h_{j} \Pr}{4} f_{j-1/2} - \frac{h_{j} \Pr A \eta_{j-1/2}}{4}$$

$$(45)$$

$$(b_{6})_{j-1/2} = -\left(1 + \frac{4}{3N_{\rm r}}\right) + \frac{n_{j}\,{\rm PT}}{4} f_{j-1/2} - \frac{n_{j}\,{\rm PT}\,A}{4} \frac{\eta_{j-1/2}}{4} \tag{46}$$

$$(b_{7})_{j-1/2} = -\frac{h_{j} \Pr}{4} u_{j-1/2} - \frac{h_{j} \Pr A}{2} = (b_{8})_{j-1/2}$$
(47)

and

$$(r_{4})_{j-1/2} = -(v_{j} - v_{j-1}) - hf_{j-1/2}v_{j-1/2} + h(u_{j-1/2})^{2} + hA^{*}(u_{j-1/2} + \frac{\eta_{j-1/2}}{2}v_{j-1/2}) + hM^{2}u_{j-1/2} + h\lambda\theta_{j-1/2} + R_{j-1/2}$$
(48)

$$(r_{5})_{j-1/2} = -\left(1 + \frac{4}{3N_{r}}\right)(t_{j} - t_{j-1}) - h\Pr f_{j-1/2} t_{j-1/2} + h\Pr u_{j-1/2}\theta_{j-1/2} + hA^{*}(\theta_{j-1/2} + \frac{\eta_{j-1/2}}{2} t_{j-1/2}) + T_{j-1/2}$$

$$(49)$$

Boundary condition becomes

$$\delta f_0 = 0, \, \delta u_0 = 0, \, \delta \theta_0 = 0, \, \delta u_J = 0, \, \delta \theta_J = 0 \tag{50}$$

Linearized differ	rence eqs. (36) - (40) can be written in the block tridiagonal form, <i>i.e.</i>
ments are:	$\begin{bmatrix} [A_1][C_2] \\ [B_2][A_2][C_2] \\ \cdots & \cdots \\ [B_{J-1}][A_{J-1}][C_{J-1}] \\ & [B_J] & [A_J] \end{bmatrix} \begin{bmatrix} [\delta_1] \\ [\delta_2] \\ \cdots \\ [\delta_{J-1}] \end{bmatrix} = \begin{bmatrix} [r_1] \\ [r_2] \\ \cdots \\ [r_{J-1}] \\ [r_J] \end{bmatrix},$
anents are.	F 0 0 1 1 0 T
	(h/2) 0 0 $(h/2)$ 0
	$\begin{bmatrix} -(n/2) & 0 & 0 & -(n/2) & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
	$[A_1] = \begin{bmatrix} 0 & -(h/2) & 0 & 0 & -(h/2) \end{bmatrix}$
	$(a_2)_1 0 (a_5)_1 \ (a_1)_1 0$
	$L = 0 = (b_6)_1 = (b_3)_1 = 0 = (b_5)_1 = J$
	$[A_{j}] = \begin{bmatrix} -(h_{j}/2) & 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & -(h_{j}/2) & 0 \\ 0 & -1 & 0 & 0 & -(h_{j}/2) \\ (a_{4})_{j} & 0 & (a_{5})_{1} & (a_{1})_{1} & 0 \\ (b_{2})_{j} & (b_{8})_{1} & (b_{3})_{j} & 0 & (b_{5})_{j} \end{bmatrix}, 2 \leq j \leq J$
	$[B_{j}] = \begin{bmatrix} 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -(h/2) & 0 \\ 0 & 0 & 0 & 0 & -(h_{j}/2) \\ 0 & 0 & (a_{6})_{j} & (a_{2})_{1} & 0 \\ 0 & 0 & (b_{4})_{j} & 0 & (b_{6})_{j} \end{bmatrix}, 2 \le j \le J$
	$\begin{bmatrix} C_j \end{bmatrix} = \begin{bmatrix} -(h/2) & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \ 2 \le j \le J$
	$(a_3)_i$ $(a_7)_i$ 0 0 0
	$\begin{bmatrix} & & & & & \\ & & & & \\ & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & & \\ & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & $

where elements are:

By using LU method the block tridiagonal matrix is solved. Value of δ is calculated again and again until it satisfies the following condition:

$$\delta v_0^{(i)} \leq \epsilon_1,$$

where $\epsilon_1 = 10^{-6}$ is small prescribed value.

Theoretical results and discussion

This section is represented physical interpretation of predicted parameter on velocity and temperature profile. Blue^{*} lines are plotted against suction (S < 0) while Red^{*} lines represented the injection case (S > 0). The effects of mixed convection parameter λ on $f'(\eta)$ and $\theta(\eta)$ are shown in figs. 3 and 4, respectively. As λ is directly related to buoyancy force this mean increasing values of mixed convection parameter yields stronger buoyancy force. Due to this velocity of the flow increases, fig. 3. This trend is more prominent for suction as compared to injection. Figure 4 depict that an opposite behavior is observed for temperature profile because when viscous boundary-layer increases as a results thermal boundary-layer decreases. Ratio of

^{*}for colour image see journal web site

electromagnetic force to the viscous force is called Hartman number is a decreasing function of Hartman number due to the reason that increasing Hartman number lead to increase in electromagnetic force therefore magnitude of the velocity decrease, fig. 5. It is observed from fig. 6 that temperature is an increasing function of Hartman number but this increase is more rapid for S < 0. The influence of unsteady parameter A^* is demonstrated from figs. 7 and 8. Both velocity and temperature reduces with rising values of unsteady parameter also momentum and thermal



boundary-layer thickness decreases. The contribution of Prandtl number on $f'(\eta)$ and $\theta(\eta)$ have been displayed in figs. 9 and 10. Increase in Prandtl number tends to enhancing momentum diffusivity. As a result velocity of the fluid and corresponding viscous boundary-layer thickness decays, fig. 9. Whereas thermal diffusivity is inversely related to Prandtl number due to this fluid

has a thinner thermal boundary-layer which increases gradient of temperature and lowering $\theta(\eta)$ as shown in fig. 10. The N_r being a radiation parameter is portrayed through fig. 11 for velocity and fig. 12 for temperature. Thermal conductivity is inversely proportional to N_r which contributes in up surging $f'(\eta)$ as represented in fig. 11. Radiation parameter amplifies thermal capability of fluid-flow. Hence temperature rises with thermal radiation parameter N_r . This information is demonstrated through fig. 12 influence of thermal slip parameter, S_r , and velocity slip parameter, S_r , are depicted through figs. 13-16. From figs. 13 and 14 confirms that velocity and temperature of the fluid decreases when thermal slip parameter rises. As by the definition of S_T kinematic viscosity of fluid decrease when thermal slip parameter increases due to which enhances viscous forces and lowering thermal forces.



Figure 13. Variation of S_{τ} on velocity



Figures 15 and 16 are portrayed against S_f for $f'(\eta)$ and $\theta(\eta)$. It is observed that increases values of velocity slip parameter results a lowering fluid velocity and up surging temperature profile.



Check the accuracy and reliability of proposed method a comparative analysis of skin friction coefficient and local Nusselt number is presented through tables. Table 1 shows relative investigation of - f''(0) corresponding to S_f when $A^* = 0 = S = M$, and compared with the exact analytical solution [26]. The variation of heat transfer characteristic at the wall $-\theta'(0)$ when $M = S_T = S_f = 0 = N_r$ for different values of A^* , S, Pr is given in tab. 2 and comparison with existing results also shown. We predict from this table that proposed technique having an excellent agreement with exact solution [27]. Table 3 presents values of $-\theta'(0)$ for some values of A^* , M, N_r when Pr = 0.5 = S, $S_f = 0.0 = S_T$. The magnitude of $-\theta'(0)$ decreases when M and N_r increase. However it increases for larger values of A^* . Table 4 consists of the values of skin friction coefficient $1/2 C_f \operatorname{Re}_s^{1/2}$ and $-\theta'(0)$ for the parameters A^* , M, S_r and S_f when Pr = 1.0, $N_r = 0.2$, and $S_T = 0.1$. Numerical values of this table shows that skin friction coefficient enhances for rising values of M, S, and A^* whereas it decays corresponding to S_f . While local heat flux is decreasing function of M, S_f , and opposite trend is witness for S, A^* . Table 5 presents the values of $-\theta'(0)$ for some values of $-\theta'(0)$ for

Table 1. Comparison of values of C_f with those of [26] for various values of S_f when $A = S = M = \lambda = 0$

S_{f}	[26]	Finite difference approximation
0.0	1.0000	1.000000
0.1	0.8721	0.872082
0.2	0.7764	0.776377
0.5	0.5912	0.591195
1.0	0.4302	0.430162
2.0	0.2840	0.283981
5.0	0.1448	0.144841
10.0	0.0812	0.081249
20.0	0.0438	0.043782
50.0	0.0186	0.01834
100.0	0.0095	0.009581

Table 2. Comparison of values of $-\theta'(0)$ with those of [27] for various values of A, S, and Pr when $M = N_r = S_r = S_T = 0$

A^{*}	S	Pr	[27]	Finite difference approximation
0	-1.5	0.72	0.4570268	0.4570268
		1	0.5000000	0.5000000
		10	0.6541612	0.6541612
	0	0.72	0.8086313	0.8086313
		1	1.0000000	1.0000000
		3	1.9236825	1.9236825
	1.5	10	3.7206739	3.7206739
		0.72	1.4943684	1.4943684
1	-1.5	1	2.0000000	2.0000000
	0	10	16.084218	16.084218
	2	1	0.8095484	0.8095484
			1.3205534	1.3205534
			2.2223486	2.2223486

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magnitude of $-\theta'(0)$ increases for large values of Pr and decreases for rising values of S_{τ} and N_{r} . Moreover, flow patterns are displayed in fig. 17 for different values of A. It is evident that symmetry about vertical axis exists for flow pattern.

Table 3. Variation of $-\theta'(0)$ for some values of A', M, and N_r , when

$Pr = 0.5 = S, S_f = 0.0 = S_T$				
A^*	М	N _r	<i>- θ</i> '(0)	
0.4	1.0	0.2	0.69411	
0.8			0.80061	
1.5			0.94861	
0.3	1.2		0.64979	
	1.4		0.63665	
	2.5		0.57780	
	1.0	0.1	0.71902	
		0.3	0.61683	
		0.5	0.54402	

Table 5. Values of $-\theta'(0)$ for some values of *Pr*, *N*, and *S*₇ when *A* = 0.2, *S* = 0.5, *S*₆=1.0

	/	, ,	
Pr	N _r	S_{T}	<i>- θ</i> '(0)
5.0	0.2	0.5	0.455178
7.2			0.573501
10.0			0.636947
7.2	0.0	0.5	0.709564
	0.3		0.614107
	0.6		0.621022
1.0	0.2	0.0	0.934592
		1.0	0.483096
		2.0	0.325735



	-				
Μ	S	S_{f}	A^{*}	$-1/2C_f \operatorname{Re}_x^{1/2}$	$-\theta$ '(0)
0.0	1.0	1.0	0.2	0.580873	0.724131
0.5				0.601575	6.715271
1.0				0.645009	0.697184
1.0	0.0	1.0	0.2	0.557754	0.460126
	0.2			0.575633	0.501501
	0.7			0.619717	0.621022
1.0	0.5	0.0	0.2	1.733191	0.695134
		1.0		0.602285	0.571204
1.0		5.0		0.175027	0.491062
		1.0	0.0	0.593809	0.516683
			0.3	0.606342	0.595319
			0.6	0.617821	0.655138



Summary and novelty of article

Current article examined mixed convection flow with velocity and temperature slips, transverse magnetic field and thermal radiation effects over a permeable elongated sheet. Computational analysis is carried out by means of unconditionally stable Keller-Box method. Graphical and tabulated results lead to the following notable findings.

- Transverse magnetic field resists fluid-flow.
- The effects of M, Pr, S_{τ} , S_{f} , and A^{*} on $f'(\eta)$ are similar in a qualitative sense whereas an opposite trend is witness for λ and N_{r} .
- Thermal boundary-layer is a decreasing function of λ , A^* , Pr, and S_T .
- Influence of M, N_r , and S_t on the temperature θ are opposite.
- The M, S, A^* enhance skin friction coefficient.
- Nusselt number is an increasing function of Prandtl number. However, reverse trend is observed for N_r and S_r.
- Present analysis overlaps with existing results in literature in a limiting sense.

• Application of present analysis is useful in plastic films, polymer extrusion, glass fiber, metallurgical processes and metal spinning.

Nomenclature

- A^* unsteadiness parameter, [–]
- a,b,c constants, [s⁻¹]
- B(t) time dependent magnetic field, [T]
- c_p specific heat, [Jkg⁻¹K⁻¹]
- Gr grashof number, [–]
- f,g dimensionless and components
- of velocity, [-]
- k^* mean absorption coefficient, [m⁻¹]
- T, T_{∞} fluid temperature and ambient temperatures respectively [K]
- x,y co-ordinate axes [m]

Greek symbols

- η dimensionless space variable, [–]
- θ density of fluid, [kgm⁻³]
- κ thermal conductivity, [Wm⁻¹K⁻¹]
- λ mixed convection parameter, [–]

M – hartman number, [–]

- N_r radiation parameter, [–]
- Pr prandtl number, [–]
- q_w radiative heat flux, [Wm⁻²]
- S_f velocity slip factor [–]
- S_{T} thermal slip factor [–]
- *S* suction/injection parameter [–]
- u, v velocity components [ms⁻¹]
- v kinematic viscosity, $[m^2 s^{-1}]$
- ρ stefan-Boltzmann constant, [Wm⁻²K⁻⁴]
- σ^* dimensionless temperature, [–]
- σ electric conductivity, [1/ $\Omega\mu$]
- Subscript
- w wall conditions
- ∞ conditions at infinity

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