NUMERICAL INVESTIGATION OF MAGNETIC NANOFLUIDS FLOW OVER ROTATING DISK EMBEDDED IN A POROUS MEDIUM

by

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Combined effects of thermal radiation and variable viscosity on a time-dependent boundary-layer flow of magnetic nanofluids over a rotating disk in the presence of the porous medium have been numerically investigated. To carry out the study, hydrocarbon based magnetic nanofluid containing magnetite Fe_3O_4 particles of 10 nm with magnetic phase concentration of 10% has been taken. For numerical solutions of the modelled system containing the governing equation of the flow, a MATLAB tool ODE45 is employed with shooting technique for the initial guess of the unknown boundary conditions. The flow phenomenon and heat transfer on the plate surface are characterised by various flow parameters such as viscosity variations, unsteady rotation parameter, Prandtl number, and radiation parameter. Also, a comparative thermal analysis has been carried out for magnetic nanofluids having three different bases viz. hydrocarbon, fluorocarbon, and water. Results reveal that heat transfer rate of hydrocarbon base magnetic nanofluids is 73.4511% faster than water base magnetic nanofluids, and 239.7458% faster than fluorocarbon base magnetic nanofluids. This enhanced heat transfer capacity of hydrocarbon base magnetic nanofluids will help in improving the performance of oil and ore extraction drilling systems used in mining industry and other geothermal applications.

Key words: *depth and temperature dependent viscosity, magnetic nanofluids, thermal radiation, boundary-layer flow*

Introduction

Magnetic nanofluids (MNF), often referred as ferro fluids, are suspension of nanosized particles having an average size of range about 3-15 nm of, Fe_3O_4, γ-Fe_2O_3 or $C_0Fe_2O_4$ in a carrier liquid (*e. g.* water, kerosene, hydrocarbon, fluorocarbon, toluene, glycol and lubricants, *etc.*) having surfactant (antimony) coating to prevent from agglomeration even when a strong magnetic field gradient is applied. Since last few past decades, wide investigations supported by experiments have been made to understand the behaviour of MNF so that they can be utilized in the fields where enhancement and depression of heat transfer of the thermal devices are paramount to improve the process performance. The study of rheological properties of MNF has a number of wide applications for various industrial thermal devices, computer storage devices, magnetic sealing, damping and bearing of machines, energy conversion system, a thermal pow-

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er generating system, nuclear reactors, transportation, electronics as well as biomedicine (drug delivery in disease curing). In all these processes, fluid under use is made to retain its original flow behaviour by keeping the viscosity changes due to temperature or changes due to externally applied field or changes due to the geophysical positioning (depth dependence changes) under control.

The problem of flow over a rotating disk is one of the classical problems of fluid mechanics that has both theoretical and practical values. The infinite rotating disk problem was first discussed by Karman [1] for an ordinary viscous fluid-flow using a similarity transformation for solving the system of non-linear coupled PDE governing the flow. After his pioneering study, the rotating disk problem again came into focus since disk-shaped bodies are often encountered in many real world engineering and industrial applications which require the study of rotating flows. Some of such applications are rotating heat exchangers, rotating disk reactors for bio-fuels production, gas or marine turbine, chemical and automobile industries, *etc*. The steady laminar MHD flow of an electrically conducting fluid over a rotating disk with slip boundary condition is investigated taking into account the variable rheological properties of a fluid by Frusteri and Osalusi [2]. Ram and Sharma [3] deal with the effects of rotation on the ferro fluid due to a rotating disk by applying Neuringer-Rosensweig model.

Disk driven flows with heat transfer through porous media have constituted a major field of study in fluid mechanics. The enhancement of heat transfer from a rotating body has a number of wide applications in case of various types of machinery for example, gas turbine rotors [4] and computer disk drives [5]. Attia [6] studied the effects of the porous medium and temperature-dependent viscosity on the unsteady flow and heat transfer for a viscous laminar incompressible fluid due to an impulsively started rotating the infinite disc. Attia [7] further extended his work with Darcy model assumption and investigated the steady flow of an incompressible viscous fluid above an infinite rotating disk in the porous medium. Ram and Kumar [8] discussed the effects of field dependent viscosity on ferro fluid flow due to a rotating disk embedded in the porous medium. Rashidi *et al*. [9] described the approximate analytical solutions of the steady flow over a rotating disk in the porous medium with heat transfer through the homotopy analysis method (HAM). Ellahi [10] examined the effect of the temperature dependent viscosity on boundary-layer flow (BLF) of a nanofluid in a pipe using the HAM. The effects of variable fluid properties like density, viscosity, thermal conductivity, *etc*. on the steady laminar conducting fluid-flow due to a porous rotating disk were investigated by Osalusi and Sibanda [11]. Ellahi *et al*. [12] analysed the thermal behaviour of mixed convection flow of power law fluid in the presence of copper nanoparticles**.** Akbar *et al*. [13] discussed the flow behaviour of a temperature dependent viscous nanofluid flow under the influence of gravitational force effect using Buonjiornio model. Ram and Kumar [14] studied the effects of magnetic field dependent viscosity and viscous dissipation on heat transfer in steady axisymmetric ferrohydrodynamic (FHD) BLF of an electrically non-conducting incompressible ferro fluid in the porous medium. They analysed 3-D rotationally symmetric BLF of field dependent viscous ferro fluid saturating porous medium due to the rotation of an infinite disk maintained at a uniform temperature.

The thermal radiation and heat transfer characteristics of a fluid over a rotating surface has been a subject of great interest from the industrial and energy saving perspectives. These characteristics have been widely studied in the recent years due to its vast applications in engineering, nuclear reactors, thermal power plants, process industries, solar collectors, drying processes, heat exchangers, geothermal and oil recovery, building construction, *etc*. Mukhopadhyay [15] has analysed the effects of thermal radiation on heat and mass transfer on unsteady boundary-layer mixed convection flow over a vertical stretching surface in a porous medium with suction. Uddin *et al*. [16] studied the non-linear Rosseland radiation effect on BLF of a viscous nanofluid embedded in the porous medium. Khidir [17] investigated the effects of viscous dissipation and Ohmic heating on steady MHD convective flow due to a porous rotating disk taking into account the variable fluid properties (density, viscosity, and thermal conductivity) in the presence of Hall current and thermal radiation. These properties are taken to be dependent on temperature. For better accuracy, successive linearization method has been successfully applied to different fluid flow problems. The time-dependent flow and the heat transfer of a nanofluid caused by the linear motion of a horizontal flat plate have been analysed by Ahmadi *et al*. [18]. The heat transfer behaviour of the micropolar fluid-flow in a channel subject to a chemical reaction has been discussed by Fakour *et al*. [19] and governing equations have been analytically solved using least square method. The combined hydromagnetic and slip flow of a steady, laminar conducting viscous fluid in the presence of thermal radiation due to an impulsively started rotating porous disk has been discussed by Osalusi [20]. Babu and Sandeep [21] compared oblique and free stream flow cases of a nanofluid in the presence of non-linear thermal radiation and variable viscosity. Shit and Majee [22] investigated the effects of thermal radiation on the MHD flow and heat transfer over an inclined non-linear stretching sheet. The effects of thermal radiation and heat transfer on the ferromagnetic fluid-flow in the presence of a stretching sheet have been investigated by Zeeshan *et al*. [23]. Water based nanofluids containing nanoparticles volume fraction of Cu, Ag, CuO, Al_2O_3, and TiO_2 are taken into account in the work of Turkyilmazoglu [24]. He described the flow and heat transfer influenced by the existence of such nanoparticles due to a rotating disk. In this direction, many applications of nanofluids in different types of transport phenomena have recently been reported in [25-29].

Many researchers in MHD and few in FHD have been studied flow behaviour due to the geophysical depth and thermal variation of viscosity, taken one at a time. However, the combined effects of depth and temperature on viscosity have not been popularly analysed on rotating disk problem yet. The modelled rotating disk problem of our manuscript focuses on the heat transfer and thermal radiation of time-dependent BLF of an incompressible MNF embedded in the porous medium. The problem is designed for flow under the combined influence of depth and temperature dependent viscosity. The plate is subjected to a magnetic field with components and the plate is maintained at a uniform temperature. The equations governing the time-dependent BLF in component form are non-dimensionalized using the similarity transformations. The resultant non-linear coupled differential equations are then solved numerically by using Runge-Kutta method in MATLAB with a systematic guess of missing boundary conditions. In this model, the investigations are performed for hydrocarbon based MNF (C1-20B) with an average magnetite particle size of 10 nm and a magnetite phase concentration of 10%. Also, a comparative study has been carried out for MNF of three different bases: hydrocarbon based magnetic nanofluid (C1-20B), water based magnetic nanofluid (Taiho W-40) and fluorocarbon based magnetic nanofluid (FC-72). The physical properties of all three types of MNF [30-32] are tabulated in tab. 1.

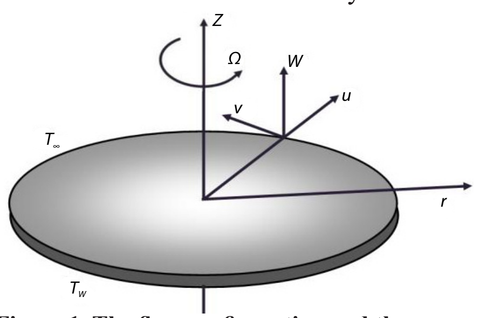

Figure 1. The flow configuration and the co-ordinate system

Mathematical formulation of the problem

The flow configuration and the co-ordinate system is presented on fig. 1. The viscosity of MNF is considered to be both depth and temperature dependent [33] given:

Table 1. Physical properties of the MNF: Taiho W-40, FC-72, and C1-20B

Properties	FC-72 Rini *et al.* [30]	Taiho W-40 Snyder *et al.* [31]	C1-20B Hong *et al.* [32]
Base fluids	Fluorocarbon (perfluorohexane C_6F_{14})	Water	Hydrocarbon (kerosene)
Density [kgm^{-3}]	$1.68 \cdot 10^3$	$1.4 \cdot 10^3$	$1.25 \cdot 10^3$
Coefficient of thermal expansion [K^{-1}]	$1.6 \cdot 10^{-3}$	$0.026 \cdot 10^{-4}$	$0.86 \cdot 10^{-3}$
Pyromagnetic coefficient [Am^{-1}K^{-1}]	–	240	80
Dynamical viscosity in zero magnetic field [kgm^{-1}s^{-1}]	$6.4 \cdot 10^{-4}$	$3.99 \cdot 10^{-2}$	$6 \cdot 10^{-3}$
Thermal diffusivity [m^{2}s^{-1}]	$3.084 \cdot 10^{-8}$	$64.3 \cdot 10^{-8}$	$5 \cdot 10^{-8}$
Prandtl number	12.3	44.3	128

$$\mu(z,T) = \frac{\mu_\infty (1-\alpha z)}{1+\alpha (T-T_\infty)} \tag{1}$$

where μ_∞ is the uniform viscosity of a fluid and $\alpha \geq 0$ is a constant.

The modelled differential equations (equation of continuity, equations of momentum, energy equation) governing the axi-symmetric unsteady FHD BLF in component form are given [34]:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{2}$$

$$\begin{gathered}\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z}\right) + \frac{\partial p}{\partial r} = \frac{\partial}{\partial r}\left[\mu(z,T)\frac{\partial u}{\partial r}\right] + \\ + \frac{\partial}{\partial r}\left[\mu(z,T)\frac{u}{r}\right] + \frac{\partial}{\partial z}\left[\mu(z,T)\frac{\partial u}{\partial z}\right] + \mu_0 \left|\vec{M}\right| \frac{\partial}{\partial r}\left|\vec{H}\right| - \frac{\mu}{k_0} u\end{gathered} \tag{3}$$

$$\begin{gathered}\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{uv}{r} + w\frac{\partial v}{\partial z}\right) = \frac{\partial}{\partial r}\left[\mu(z,T)\frac{\partial v}{\partial r}\right] + \\ + \frac{\partial}{\partial r}\left[\mu(z,T)\frac{v}{r}\right] + \frac{\partial}{\partial z}\left[\mu(z,T)\frac{\partial v}{\partial z}\right] + \mu_0 \left|\vec{M}\right| \frac{\partial}{\partial \phi}\left|\vec{H}\right| - \frac{\mu}{k_0} v\end{gathered} \tag{4}$$

$$\begin{gathered}\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) + \frac{\partial p}{\partial z} = \\ = \frac{\partial}{\partial r}\left[\mu(z,T)\frac{\partial w}{\partial r}\right] + \frac{1}{r}\frac{\partial}{\partial r}[\mu(z,T)w] + \frac{\partial}{\partial z}\left[\mu(z,T)\frac{\partial w}{\partial z}\right] + \mu_0 \left|\vec{M}\right| \frac{\partial}{\partial z}\left|\vec{H}\right| - \frac{\mu}{k_0} w\end{gathered} \tag{5}$$

$$\rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) - \frac{\partial q_r}{\partial z} \tag{6}$$

where ρ is the fluid density, μ_0 – the magnetic permeability in free space, k_0 – the Darcy permeability parameter, k – the thermal conductivity of heat, C_p – the specific heat with constant pressure, and q_r – the radiative heat flux.

The boundary conditions for the flow [35]:

$$\begin{cases} u = 0, \quad v = r\Omega, \quad w = 0, \quad T = T_w \quad \text{at} \quad z = 0, \\ u \to 0, \; v \to 0, \quad T \to T_\infty \quad \text{as} \quad z \to \infty, \\ w \to \text{some finite negative value} \quad \text{as} \quad z \to \infty \end{cases} \tag{7}$$

where Ω is the angular velocity of the disk and $z \to \infty$ is not exactly infinity but a large distance beyond which the boundary-layer vanishes.

Modelling and solution of the problem

The flow of magnetic nanofluid is affected by the magnetic field due to the magnetic dipole, whose magnetic scalar potential:

$$\psi_m = \frac{m}{2\pi r}\cos\phi$$

and the corresponding magnetic field H, considering negligible variation along z-axis, has the components:

$$H_r = -\frac{\partial \psi_m}{\partial r} = \frac{1}{2\pi}\frac{m\cos\phi}{r^2}, \quad H_\phi = -\frac{1}{r}\frac{\partial \psi_m}{\partial \phi} = \frac{1}{2\pi}\frac{m\sin\phi}{r^2}, \quad H_z = 0$$

Hence, the resultant applied magnetic field is:

$$H = \sqrt{H_r^2 + H_\phi^2 + H_z^2} = \frac{m}{2\pi r^2} \tag{8}$$

Assuming that the applied magnetic field, H, is sufficiently strong to saturate the magnetic nanofluid and the variation of magnetization, M, with temperature can be approximated by a linear equation of state:

$$M = K\left(T_c - T\right) \tag{9}$$

where T_c is curie temperature and K is pyromagnetic coefficient.

Using Rosseland approximation, the radiative flux, q_r, is modeled:

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial z} \tag{10}$$

where σ is Stefan-Boltzmann constant and k^* – the mean absorption coefficient. Assuming that the differences in temperature within the flow are such that T^4 can be expressed as a linear combination of temperature. Expanding T^4 in Taylor's series about T_∞ and neglecting the higher order terms, we get:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{11}$$

The revolving flow of magnetic nanofluid due to the rotating disk with constant angular velocity, Ω, is in equilibrium under the influence of centrifugal force which is balanced by the radial pressure gradient. So, the boundary-layer approximation for eq. (3):

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = r\Omega^2 \tag{12}$$

Introducing the following similarity transformations to non-dimensionalized the governing equations:

$$u = r\Omega U'(\eta), \quad v = r\Omega \mathrm{V}(\eta), \quad w = \frac{\nu}{\delta} W(\eta), \quad T - T_\infty = \Delta T\,\theta(\eta) \tag{13}$$

where $\eta = z/\delta$, $\Delta T = T_w - T_\infty$, and $\delta(t)$ is a scalar factor responsive for unsteady flow.

Using eq. (13) in equation of continuity (2), we get:

$$W = -2RU \tag{14}$$

Using eqs. (8)-(14) in the set of governing eqs. (3)-(6), we get a system on non-linear coupled ODE:

$$(1-\varepsilon_1\eta)U''' - \varepsilon_1 U'' - (1-\varepsilon_1\eta)(1+\varepsilon\theta)^{-1}\varepsilon\theta' U'' +$$
$$+(1+\varepsilon\theta)\left[\frac{\delta}{\nu}\frac{\mathrm{d}\delta}{\mathrm{d}t}\eta U'' + R(V^2 - U'^2) + 2RUU'' - R - \frac{2B}{R} - R\beta U'\right] = 0 \tag{15}$$

$$(1-\varepsilon_1\eta)V'' - \varepsilon_1 V' - (1-\varepsilon_1\eta)(1+\varepsilon\theta)^{-1}\varepsilon\theta' V' +$$
$$+(1+\varepsilon\theta)\left(\frac{\delta}{\nu}\frac{\mathrm{d}\delta}{\mathrm{d}t}\eta V' + 2R(UV' - VU') - R\beta V\right) = 0 \tag{16}$$

$$(3Q_r + 4)\theta'' + 3\,\mathrm{Pr}\,Q_r\left(\frac{\delta}{\nu}\frac{\mathrm{d}\delta}{\mathrm{d}t}\eta + 2RU\right)\theta' = 0 \tag{17}$$

where $\varepsilon = \alpha\Delta T$, $\varepsilon_1 = \alpha\delta(t)$ are viscosity variation parameter and modified viscosity variation parameter, respectively, unsteady rotation parameter $R = \Omega\delta^2/\nu$, FHD interaction parameter $B = m\mu_0 K(T_c - T)\rho/2\pi\mu^2$, permeability parameter $\beta = \mu/k_0\rho\Omega$, Prandtl number $\mathrm{Pr} = \mu C_p/k$, and radiation parameter $Q_r = kk^*/4\sigma T_\infty^3$.

For unsteady flow problem, the term $(\delta/\nu)(\mathrm{d}\delta/\mathrm{d}t)$ should not be dropped from the eqs. (15)-(17). Considering the usual scaling factor for various unsteady BLF [35]:

$$\delta = 2\sqrt{\frac{\nu}{t}} + L \tag{18}$$

where L represents the length scale of steady flow.

Introducing (18) in eqs. (15)-(17), respectively, we have the following dimensionless non-linear ODE:

$$(1-\varepsilon_1\eta)U''' - \varepsilon_1 U'' - (1-\varepsilon_1\eta)(1+\varepsilon\theta)^{-1}\varepsilon\theta' U'' +$$
$$+(1+\varepsilon\theta)\left[2\eta U'' + R(V^2 - U'^2) + 2RUU'' - R - \frac{2B}{R} - R\beta U'\right] = 0 \tag{19}$$

$$(1-\varepsilon_1\eta)V'' - \varepsilon_1 V' - (1-\varepsilon_1\eta)(1+\varepsilon\theta)^{-1}\varepsilon\theta' V' + (1+\varepsilon\theta)\left[2\eta V' + 2R(UV' - VU') - R\beta V\right] = 0 \tag{20}$$

$$(3Q_r + 4)\theta'' + 6\,\mathrm{Pr}\,Q_r(\eta + RU)\theta' = 0 \tag{21}$$

Also, the boundary conditions (7) reduces:

$$\left.\begin{array}{c} U'(0) = 0, \quad V(0) = 1, \quad W(0) = 0, \quad \theta(0) = 1 \\ U'(\infty) \to 0, \quad V(\infty) \to 0, \quad \theta(\infty) \to 0 \\ W(\infty)\ \text{tends to some negative value} \end{array}\right\} \tag{22}$$

A fourth order Runge-Kutta method is employed to obtain the numerical solution followed by shooting technique with a systematic guessing of missing boundary conditions $U'(0), V'(0)$, and $\theta'(0)$ as employed by Ram *et al*. [36].

With the previous scheme, the solution of the coupled differential eqs. (19)-(21) along with the boundary conditions (22) provides us the variations in velocity and temperature profiles besides the skin friction coefficients and the rate of heat transfer at the surface of the plate. The Newtonian formulae are used to calculate the radial stress τ_r and tangential shear stress τ_ϕ:

$$\begin{cases} \tau_r = \left[\mu(z,T)\left(\dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial r} \right) \right]_{z=0} = \mu_\infty (1-b)\sqrt{\mathrm{Re}}\Omega U''(0) \\ \tau_\phi = \left[\mu(z,T)\left(\dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial \theta} \right) \right]_{z=0} = \mu_\infty (1-b)\sqrt{\mathrm{Re}}\Omega V'(0) \end{cases} \tag{23}$$

where Re is the local rotational Reynolds number. Therefore, the radial and tangential skin frictions are, respectively, given:

$$(1-b)^{-1}\sqrt{\mathrm{Re}}C_{f_r} = U''(0), \qquad (1-b)^{-1}\sqrt{\mathrm{Re}}C_{f_\phi} = V'(0) \tag{24}$$

where c_{f_R} and c_{f_ϕ} are the coefficients of radial and tangential skin frictions, respectively. Also, the rate of heat transfer from the surface of the plate to the magnetic nanofluid is calculated by using Fourier law given:

$$q = -\left(k\frac{\partial T}{\partial z} \right)_{z=0} = -k\Delta T\sqrt{\frac{\Omega}{\nu_\infty}}\theta'(0) \tag{25}$$

Therefore, the Nusselt number is given by $\sqrt{\mathrm{Re}}\mathrm{Nu} = -\theta'(0)$.

Results and discussion

The system of non-linear coupled ODE (19)-(21) governing the fluid-flow along with boundary conditions (22) have been solved using the numerical procedure in MATLAB as given in *Appendix*. This enables us to carry out the influence of rotation, viscosity variation and thermal radiation on flow behaviour and enhancement of heat transfer of a hydrocarbon based magnetic nanofluid. Also, the thermal behaviour of C1-20B has been compared with Taiho W-40 and FC-72 taking FHD interaction parameter B = 1.0, permeability parameter β = 1.0, viscosity variation parameters ε and ε_1 ranging from 0.0-1.0, unsteady rotation parameter ranging from 0.5-2.0, and radiation parameter ranging from 0.5-5.0.

The effect of increase in viscosity variation parameter ε and ε_1 for the set of values R = 1.0, B = 1.0, β = 1.0, Pr = 128, and Q_r = 1.0 on the dimensionless fluid-flow profiles (velocity and temperature profiles) is depicted in figs. 2-5. An increasing value of ε implies the dependency of viscosity on temperature (heated surface) while viscosity depends on depth with the increase in the value of ε_1. From the figures, we may see how the velocity distribution is affected by variation in viscosity dependency on temperature and geophysical position in comparison to the uniform viscosity *i. e.* when $\varepsilon = \varepsilon_1 = 0$. The numerical results show an increase in dimensionless radial and axial velocity profiles for increasing values of ε and ε_1. Here, η = 0.21 is a critical point as for $0 \le \eta \le 0.21$, the tangential velocity decreases with increasing values of ε and ε_1, however, the trend is reverse for $0.21 < \eta \le 0.96$ *i. e.* as we move away from the centre disk.

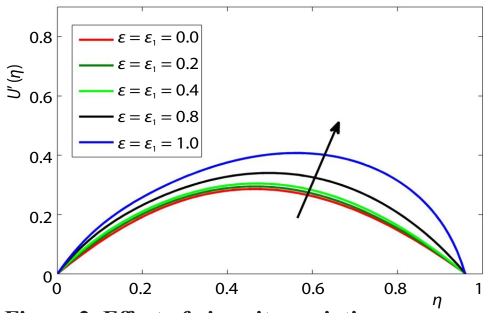

Figure 2. Effect of viscosity variation on radial velocity profile
(*for color image see journal web site*)

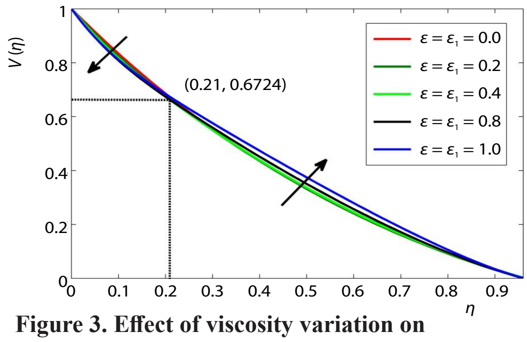

Figure 3. Effect of viscosity variation on tangential velocity profile
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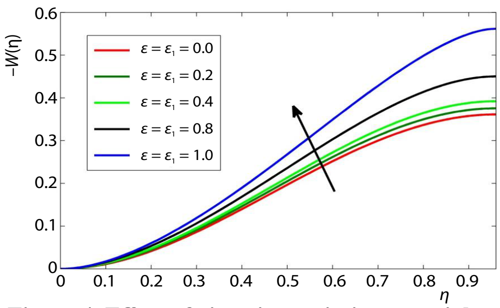

Figure 4. Effect of viscosity variation on axial velocity profile
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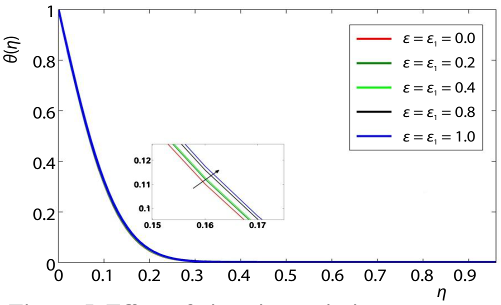

Figure 5. Effect of viscosity variation on temperature profile
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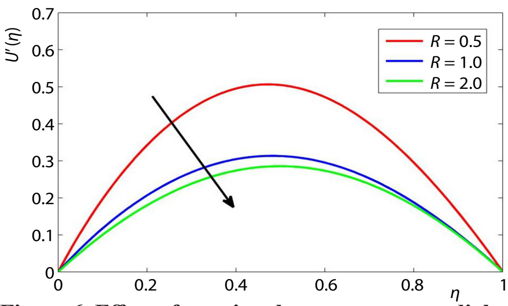

Figure 6. Effect of rotational parameter on radial velocity profile (*for color image see journal web site*)

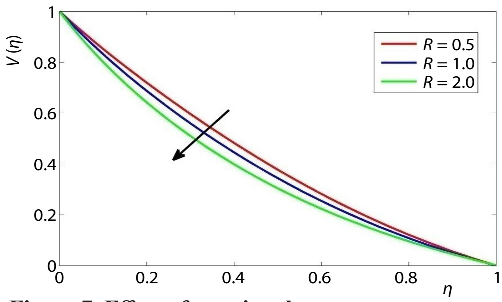

Figure 7. Effect of rotational parameter on tangential velocity profile
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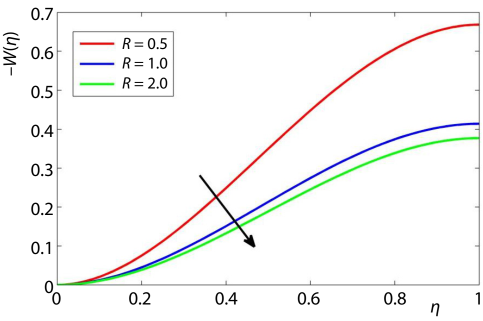

Figure 8. Effect of rotational parameter on axial velocity profile (*for color image see journal web site*)

Figures 6-8 describe the influence of rotation of the plate on flow profiles. From these figures, it has been noted that on increasing the rotation of the disk there is a significant change in velocity profiles (radial, tangential, and axial). An increase in rotation of the disk causes decrease in velocity distribution profiles and attains the steady-state more rapidly. However, for slower rotation of the plate, the radial and axial profiles show large influence while the influence becomes smaller when rotation increases.

We examined the effects of different values of radiation parameter Q_r on the velocity and temperature profiles with a set of values $\varepsilon = \varepsilon_1 = 0.1$, $R = 1.0$, $B = 1.0$, and $\beta = 1.0$ for hydrocarbon based magnetic nanofluid (C1-20B). It is observed that the thermal radiation does not affect significantly on the velocity of the fluid and it is only the temperature distribution, fig. 9, which is affected the most. The increasing radiation parameter, Q_r, decreases the temperature profile due to the faster dissipation of the heat. Also, it is observed that the temperature is maximum at the surface of the disk and asymptotically decreases to its steady-state.

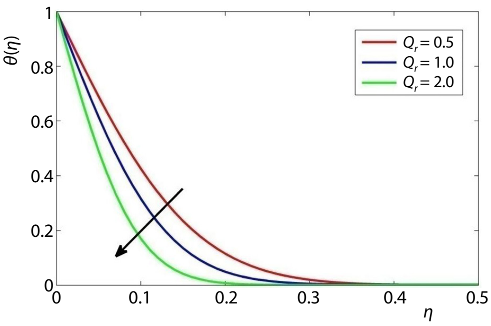

Figure 9. Effect of radiation parameter on temperature profile
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Figure 10 shows a comparative study of the modelled problem for three type of MNF: hydrocarbon based magnetic nanofluid (C1-20B), water based magnetic nanofluid (Taiho W-40), and fluorocarbon based magnetic nanofluid (FC-72) with a specified set of values of various physical parameters, such as $\varepsilon = \varepsilon_1 = 0.1$, $R = 1.0$, $B = 1.0$, $\beta = 1.0$, and $Q_r = 1.0$. As expected, the numerical results show that on increasing the Prandtl number, the temperature profile decreases and reaches to its boundary condition more fastly due to the fact that the thermal diffusion of the fluid decreases with increase in the Prandtl number. It means that hydrocarbon based magnetic nanofluid (C1-20B) dissipates heat faster than water based magnetic nanofluid (Taiho W-40), and much faster than fluorocarbon based magnetic nanofluid (FC-72). Precisely, we can conclude that the maximum cooling of the rotating disk is achieved in C1-20B.

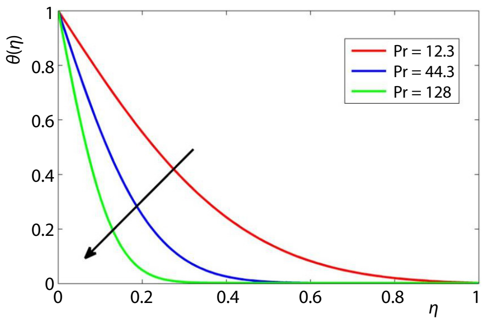

Figure 10. Effect of Prandtl number on temperature profile
(*for color image see journal web site*)

The effects of the rheological parameters: viscosity variation parameters ε and ε_1, unsteady rotation parameter, R, radiation parameter, Q_R, and Prandtl number on the shear stresses $U'(0)$, $V'(0)$, and the rate of heat transfer, $\theta'(0)$, have been illustrated in tab. 2. We noticed that the effect of the increasing ε, ε_1, and R, increases the tangential skin friction. Also the rate of heat transfer increases with increasing value of Q_r and Prandtl number. The negative values of $\theta'(0)$ indicates the heat flow from the disk surface to the ambient fluid. This is in perfect agreement with the physical fact that the thermal boundary-layer thickness decreases with increasing Prandtl number and radiation parameter. Table 2 reveals that heat transfer of Taiho W-40 is 95.8739% and C1-20B is 239.7458% faster than FC-72.

Validation and comparison with the existing results

The present numerical model has been validated with the help of previous research findings. A comparison of rate of heat transfer of the reduced case of steady and ordinary ($\delta = L$, $\beta = 0$) viscous fluid-flow for Pr = 10 and 100 have been presented in tab. 3, and are in good agreement with those of Gregg and Sparrow [37] and Maleque [33]. Also, a compar-

Table 2. Skin friction coefficients and rate of heat transfer for B = 1.0 and β = 1.0

ε	ε_1	R	Q_r	Pr	$U''(0)$	$V'(0)$	$-\theta'(0)$
0.0	0.0	1.0	1.0	128	1.186871532109	1.758431289289	8.099769432637
0.2	0.2	1.0	1.0	128	1.360915853218	1.969879289653	8.069976943264
0.4	0.4	1.0	1.0	128	1.520976297153	2.139450312893	8.053255794326
0.8	0.8	1.0	1.0	128	1.830685321096	2.365974503129	8.002455794327
1.0	1.0	1.0	1.0	128	2.074909132109	2.383589259745	7.968455794326
0.1	0.1	0.5	1.0	128	2.254319279897	1.576839668793	8.128997694326
0.1	0.1	1.0	1.0	128	1.327989678451	1.839668792897	8.069976943264
0.1	0.1	2.0	1.0	128	1.100567932799	2.252839668792	7.876943263659
0.1	0.1	1.0	0.5	128	1.325201419533	1.833966879289	6.399769432637
0.1	0.1	1.0	1.0	128	1.327989678451	1.839668792896	8.069976943264
0.1	0.1	1.0	5.0	128	1.328938896785	1.843969687929	11.059976943263
0.1	0.1	1.0	1.0	12.3	1.319229245229	1.802579668793	2.375298652594
0.1	0.1	1.0	1.0	44.3	1.322924522952	1.825796687928	4.652594216998
0.1	0.1	1.0	1.0	128	1.327989678451	1.839668792896	8.069976943263

Table 3. The rate of heat transfer: comparison and validation

Prandtl number	Present study	Reduced case of the present study	Maleque [33]	Gregg and Sparrow [37]
10	2.1404453579	1.13654449	1.13331	1.13410
100	7.1235709136	2.68691999	2.68682	2.68710

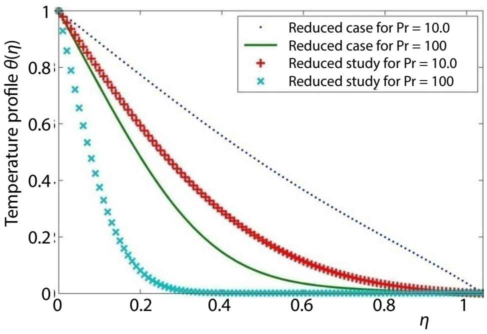

Figure 11. Effect of Prandtl number on radial velocity profile
(*for color image see journal web site*)

ison of the present study with its reduced case is described in fig. 11. A remarkable difference in thermal boundary-layer thickness is observed when we replace ordinary fluids with MNF.

Conclusions

The numerical investigations exhibit many interesting rheological features concerning the effects of thermal radiation and depth and temperature dependent viscosity on hydrocarbon based magnetic nanofluid (C1-20B) flow over a rotating disk embedded in the porous medium. The limiting case of the numerical model has an excellent agreement with the previous works. Also, a comparative study has been done for three types of MNF: hydrocarbon based magnetic nanofluid (C1-20B), water based magnetic nanofluid (Taiho W-40), and fluorocarbon based magnetic nanofluid (FC-72). The main findings of this work are.

- Viscosity variation of the fluid and the rotations of the plate alters the velocity distribution, however, temperature profile is unaltered. The velocity profiles increases and approaches to its steady-state when we move from uniform viscosity to variable viscosity (temperature and depth dependent) while the trends are reverse when we increase the rotation of the plate.

- The achievement of the steady-state in temperature distribution is faster as we move from low thermal radiation to high thermal radiation and increase the rate of heat transfer from the disk surface to the fluid.
- Thermal boundary-layer thickness of C1-20B is thinner than Taiho W-40 and much thinner than FC-72, meaning thereby that hydrocarbon based magnetic nanofluid C1-20B is better coolant than Taiho W-40 and FC-72.
- Thus fast cooling of the plate can be achieved by implementing these effects.

In the present study, the hydrocarbon based magnetic nanofluids has been considered, which is directly applicable to control the heat losses (or in keeping cool the instrument), and avoiding the damage caused due to the heat generation by the motion of its blades (shafts). This study is also applicable in thermal power generating systems, high speed rotating machinery and aerodynamic extrusion of plastic sheets. In the future study, we generalize this model for the CNT nanofluids and non-Newtonian nanofluids with the effects of magnetic fields.

Appendix

Numerical Coding for the Model

The code is written in the MATLAB environment using ODE45. To reduce the equations to first order equations, we set:

$$y_1 = U,\quad y_2 = U',\quad y_3 = U'',\quad y_4 = V,\quad y_5 = V',\quad y_6 = \theta,\quad y_7 = \theta'$$

The initial condition are given by:

$$y_1(0) = 0,\quad y_2(0) = 0,\quad y_4(0) = 1,\quad y_6(0) = 1$$

Viscosity variation parameter due to temperature and depth:

$$\varepsilon = m = 0.1, 0.4, 0.8, 1.0,\quad \varepsilon_1 = n = 0.1, 0.4, 0.8, 1.0$$

Unsteady rotation parameter: $R = 0.5, 1.0, 2.0$

Prandtl number: $\mathrm{Pr} = 44.3, 79.3, 128$

The FHD interaction parameter: $B = 1.0$

Permeability parameter: $\beta = b = 1.0$

Radiation parameter: $Q_r = 0.5, 1.0, 5.0$

Model equations used in the problem:

$$y_3' = \frac{ny_3 + (1-nt)\left\{my_7y_3/(1+my_6) - (1+my_6)[2ty_3 - R\left(y_2^2 - y_4^2\right) + 2Ry_1y_3 - R - 2B/R - Rby_2]\right\}}{1-nt} \tag{26}$$

$$y_3 = s_3$$

$$y_5' = \frac{ny_5 + (1-nt)my_7y_5/(1+my_6) - 2(1+my_6)\left[ty_5 - R(y_2y_4 - y_1y_5) - Rby_4\right]}{1-nt}, \tag{27}$$

$$y_5 = s_5$$

$$y_7' = \frac{-3Qr\Pr(2ty_7 + 2Ry_1y_7)}{3Qr+4}$$
$$y_7 = s_7 \tag{28}$$

where s_3, s_5, and s_7 are constants.

The tool ODE45 reduces the boundary value problem to initial value problem. Shooting technique is used to guess the values of s_3, s_5, and s_7 satisfying the boundary conditions: $y_2(\infty) \to 0$, $y_4(\infty) \to 0$, $y_1(\infty) \to -c$, ($c > 0$, and $y_6(\infty) \to 0$ with the desired degree of accuracy, viz. 10^{-12}. The solution is then compared quantitatively and qualitatively through tab. 2 and figs. 2-10.

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