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COMPUTATIONAL FLUID DYNAMICS MODELING PATTERNS AND FORCE CHARACTERISTICS OF FLOW OVER IN-LINE FOUR SQUARE CYLINDERS

by

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The flow over four square cylinders in an in-line, square arrangement was numerically investigated by using the finite volume method with CFD techniques. The working fluid is an incompressible ideal gas. The length of the sides of the array, L, is equal. The analysis is carried out for a Reynolds number of 300, with center-to-center distance ratios, L/D, ranging from 1.5 to 8.0. To fully understand the flow mechanism, details in terms of lift and drag coefficients and Strouhal numbers of the unsteady wake frequencies are analyzed, and the vortex shedding patterns around the four square cylinders are described. It is concluded that L/D has important effects on the drag and lift coefficients, vortex shedding frequencies, and flow field characteristics.

Key words: *four square cylinders, flow patterns, drag coefficients,
lift coefficients, Strouhal numbers, CFD*

Introduction

Flow past multiple bluff bodies has become an active research topic in recent years. Examples can be found in numerous engineering applications, such as offshore platforms, power generators, cooling towers, heat exchanger tubes, bridge piers, chimney stacks, and so on. Investigation of the flow past a group of cylinders can provide better understanding of fluid characteristics, such as vortex patterns, pressure distributions and body forces, which are important for engineering design practice.

To date, there are a number of experimental and numerical studies published for describing fluid flow over single or two cylinders. Sumner [1] reviewed recent studies with two circular cylinders of equal diameter in different arrangements immersed in a steady cross-flow. He primarily focused on the near-wake flow patterns, Reynolds number effects, intermediate wake structure, and the general trends in the measurements of the aerodynamic force coefficients and Strouhal numbers. Han *et al.* [2] investigated the flow past two circular cylinders in a tandem arrangement utilizing spectral element methods in which the Reynolds number was chosen as 200. The results show three wake flow patterns, which are determined by spacing ratios. Forces are highly depended on flow patterns. One paper of Mittal and Kumar [3] researched incompressible flow past two cylinders in tandem and staggered arrangements at

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Reynolds numbers of 100 and 1000 with a stabilized finite element formulation. The spacing ratio, L/D, ranged from 2.5 to 5.5 in a tandem arrangement. The results show that when the flow is unsteady, the downstream cylinder experiences very large unsteady forces that may lead to wake-induced flutter. Other papers of flow characteristics about flow over two cylinders in different configurations were published in [4-6], and in other studies, that relate to heat transfer and fluid flow over two cylinders can be found in the contemporary literature [7-10].

In contrast to many studies about flow past one or two cylinders, there are fewer in the open literature that relate to research on flow over more than two cylinders. Recent studies in the literature that expand knowledge of flow over multiple cylinders are next discussed. Flow past multiple cylinders generates a complex flow structure as a consequence of wake interactions of shed vortices. The resultant combined wake structures behind multiple cylinders are considerably different from those behind single or two cylinders. Sayers [11] experimentally studied flow over four circular cylinders in an in-line, square arrangement. Drag and lift coefficients were measured by using an open-jet wind tunnel with Re = 30000 and four equally-spaced cylinders. An experimental study of turbulent flow past four cylinders in a square arrangement with equal space ratios ($L/D = 2.88$) was performed by Ladjedel *et al.* [12]. Four smooth cylinders and four grooved cylinders were investigated. The experimental results show that a bi-stable flow pattern exists behind the downstream cylinders. With increasing Reynolds numbers, the absolute value of the pressure coefficient increased. The effects of Reynolds number on flow past four square cylinders in an in-line, square configuration for different spacing distances were computed by Waqas *et al.* [13]. The results show that the jets between the gaps strongly influence the wake interactions for certain gap spacing and Reynolds number combinations. Lam and Zou [14] investigated turbulent cross-flow past four cylinders under an in-line configuration with different spacing ratios at a subcritical Reynolds number. The numerical results accurately predict certain wake flow characteristics over the four cylinders, agreeing well with experimental data. A 3-D numerical investigation of cross-flow around four circular cylinders in a diamond array at a Reynolds number of 200 was carried out by Zou *et al.* [15]. It was concluded that the spacing ratio has important effects on development of the free shear layers about the cylinders and has significant effects on forces and pressure characteristics of the four cylinders as well. Beyond that, there were several experimental and numerical papers examining four cylinders in an in-line, square configuration with high and low Reynolds numbers [16-19].

Additionally, there are many reported works on flow over square cylinders with various arrangements. Here, one must note that the flow patterns behind a group of square cylinders are considerably different from those over a row of circular cylinders. For example, the separation points on square cylinders are fixed, causing different transitions to critical regimes compared to those of circular cylinders. Moreover, the shedding frequencies, depending on the separation mechanisms, and the aerodynamic forces of these two geometries are different. For more details, readers are referred to Alam *et al.* [20]. The effect of gap spacing of 0.7 and 2.5 with Re = 73 for flow past two side-by-side square cylinders was numerically investigated by Agrawal *et al.* [5] using the lattice Boltzmann method. They found that the wake patterns strongly depend on the jets between the cylinders and their interactions with the wakes and the strength of this interaction heavily depend on gap spacing. Yen *et al.* [21] studied the effects of Reynolds number, spacing ratio, and rotation angle of a downstream cylinder of two, in-line, square cylinders using particle image velocimetry.

The survey indicates that, although there are results for flow around four circular cylinders arranged in a staggered configuration, the square-cylinder counterparts are surprisingly

unavailable. The present work investigates flow behavior around four square cylinders in an in-line configuration with an air cross-flow. The simulation is conducted at a Reynolds number of 300 and center-to-center distance ratios varying from 1.5 to 8.0. The side dimension of each square cylinder is fixed at 25 mm. The numerical simulations were carried out by CFD for constant-property air. The main objective of this paper is to document the effect of spacing ratios between cylinders on vortex dynamics and force characteristics.

The computational model and equations

The computational domain and the configuration of the problem are schematically illustrated in fig. 1. A $50D \times 30D$ rectangular domain is provided for locating four square cylinders in an in-line, square arrangement. The parameter D is the side dimension of these square cylinders, which is fixed as 25 mm. The downstream side of the downstream square cylinder is located $15D$ from the inlet flow plane and $35D$ from the outlet plane. The computational domain is symmetric and the cylinder locations are uniquely determined by the center-to-center distance, L/D, between two adjacent square cylinders. The distance between the upper wall and the lower wall of the computational zone is $30D$. The flow enters at the upstream side of the computational domain with a uniform velocity,

Figure 1. Computational domain for four square cylinders in an in-line configuration; S1-S4 identify the cylinder numbers for reference

u_∞. The outlet boundary condition is an outflow and the surfaces of the cylinders in the domain are smooth, with no-slip velocity boundary conditions. The upper and lower boundaries of the computational field were defined by a moving surface with velocity u_∞.
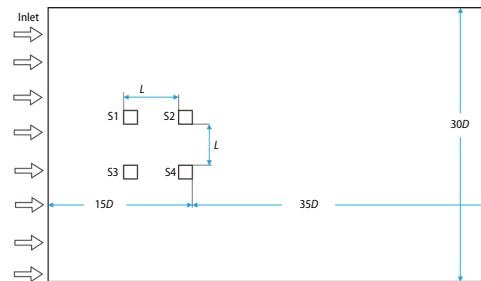
The equations governing the unsteady, incompressible, viscous, 2-D flow of this problem are the continuity and momentum equations:

– for continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

– for momentum

$$\frac{\partial u}{\partial t} + \frac{\partial (uu)}{\partial x} + \frac{\partial (uv)}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial v}{\partial t} + \frac{\partial (uv)}{\partial x} + \frac{\partial (vv)}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{2}$$

where u and v are the dimensionless velocity components along the x- and y-directions of a Cartesian co-ordinate system and p is the non-dimensional static pressure. The Reynolds number is $\rho u_\infty D/\mu$, with u_∞ being the oncoming free-stream velocity, μ – the fluid viscosity, and D – the side dimension of these square cylinders.

In the present study, a structured grid is employed to solve the 2-D Navier-Stokes equations and the continuity equation using the commercial CFD software – FLUENT. This code uses the finite volume method to solve the governing equations of fluid flow with appropriate boundary conditions. The well-known semi-implicit pressure linked equations (SIMPLE)

scheme is utilized in the advection calculations, and the second-order discrete format is used for the pressure simulation. A second-order accurate upwind difference scheme is used because of its higher stability and accuracy than the first-order upwind scheme offers.

The mean drag coefficients, maximum amplitude of lift coefficients, and frequency of vortex shedding are discussed in this paper. Some definitions of these parameters are computed from:

$$C_D = 2\frac{F_D}{\rho u_\infty^2 D} \tag{3}$$

$$C_L = 2\frac{F_L}{\rho u_\infty^2 D} \tag{4}$$

$$\mathrm{St} = \frac{fD}{u_\infty} \tag{5}$$

where C_D and C_L are the drag and lift coefficients, respectively, St – the Strouhal number, F_D – the force in the stream wise direction, and F_L – the force in the transverse direction (lift). The velocity of the flow is u_∞ and f is the vortex shedding frequency. The characteristic length scale is D, the side dimension of the square cylinder. It should be emphasized that the fluid is assumed to be incompressible and the dynamic viscosity is $1.5E^{-5}$ kg/ms in the current research.

The model validation

The grid independence test and the validation of numerical models have significant meaning for numerical simulations. The whole domain was discretized by the structured mesh illustrated in fig. 2. To ensure proper boundary-layer growth, flow separation and vortex growth, layers were constructed with $y^+ \approx 1$ in the region near the cylinder walls for all the cases tested. The mesh size, Δy, around the cylinder could be obtained by the following relationship:

$$y^+ = 0.172\frac{\Delta y}{L'}\mathrm{Re}^{0.9} \tag{6}$$

where the L' is the feature length of the cylinder.

In view of the accuracy and cost of computation, four mesh sizes given the total number of meshes of (116400, 124100, 147300, and 158700) were constructed for the grid independence test run at Re = 300. Table 1 shows the mean drag coefficients and maximum lift coefficients of these four mesh number cases. The percentage change in the values of time average coefficients and maximum lift coefficients for the coarsest grid (116400) with respect to the finest grid (158700) are found to be 1.85% and 3.57%, respectively. While the same for the
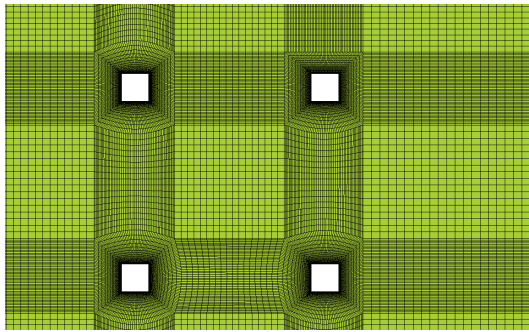

Figure 2. The structured mesh near the four square cylinders surfaces

Table 1. The drag coefficients and maximum lift coefficients under different mesh sizes

Mesh	C_D	$C_{L\max}$	ΔC_D	$\Delta C_{L\max}$
116400	2.12	2.16	1.85%	3.57%
124100	2.10	2.19	2.78%	2.23%
147300	2.13	2.22	1.39%	0.89%
158700	2.16	2.24	baseline	baseline

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grids of 124100 are 2.78% and 2.23%, and the values for mesh 147300 are 1.39% and 0.89%. Accordingly, the 147300 mesh size is preferred, keeping in view the accuracy of the results and computational convenience in the simulations.

In order to validate the computational model, a case with uniform cross-flow along the x-direction over a single square cylinder with a diameter, D, for Reynolds number 250 was computed. Table 2 shows the comparison with other works of computed mean drag coefficients and Strouhal numbers, at Re = 250 [22-24]. The calculated values of the C_D and St match well the values in the literature. The validation results show that the model and methods used for the results to follow are well verified for the single cylinder and it is reasonable to assume that they could be employed for simulating flow past four square cylinders in an in-line, square arrangement.

Table 2. Comparisons of the Strouhal numbers and drag coefficients

Re	Work name	C_D	St
Re = 250	Saha *et al.* [22]	1.77	0.142
	Davis and Moore [23]	1.77	0.165
	Franke *et al.* [24]	1.67	0.141
	Present work	1.72	0.149

Results and discussion

The flow around four square cylinders in an in-line arrangement is expected to lead to different flow fields than those around a single or two cylinders. After validation of mesh independence and numerical methods, a numerical simulation made by varying the spacing ratio for flow over four square cylinders at a Reynolds number of 300 was systematically carried out. In this section, the relationship between center-to-center distances, vortex shedding force parameters and other flow characteristics, are documented.

Flow patterns analysis

Figure 3 depicts the computed instantaneous vorticity patterns at a Reynolds number of 300 and spacing ratios, L/D, varying from 1.5 to 8.0. When L/D =1.5, the flow pattern behind the four square cylinders is similar to that of an isolated cylinder because the four cylinders are sufficiently close to each other. The vortices are immediately downstream of the cylinders, this can be attributed to the fact that the flow interferences between cylinders are quite strong with this small spacing ratio and, consequently, oscillating regions exist. In this case, the downstream cylinders are totally engulfed by the shear layers of the upstream cylinders. The shed vortex wake behind the downstream cylinders (S2 and S4) is narrower than the vortex patterns of cases with the other spacing ratios. This is due to the inner shear layers of the upstream cylinders (S1 and S3) being biased toward one of the downstream cylinders. With this spacing, we can see a bi-stable state of wide and narrow wakes attached behind the downstream cylinders, which was also observed by Lam and Lo [25] with L/D =1.7. The flow pattern is almost steady during the whole process, called a stable shielding flow pattern (I) [18, 26]. The effect of jet flow between cylinders is obvious, and a larger flow acceleration results in strong jet spreading laterally behind the downstream cylinders.

Figures 3(b)-3(d) show that the inner shear layers of the upstream cylinders reattach immediately onto the downstream cylinder. However, there is no reattachment of the outer shear layers from the upstream cylinders on the downstream cylinders, and the outside shear layers are alternately wiggling near the downstream cylinders. Also, the vortex patterns merge behind the downstream cylinders at a small distance, and for those spacing ratios it is hard to show significant wiggling for both inner and outer shear layers. No vortex shedding occurs after the upstream cylinders, and the shear layers of the upper cylinders grow into mature vortices
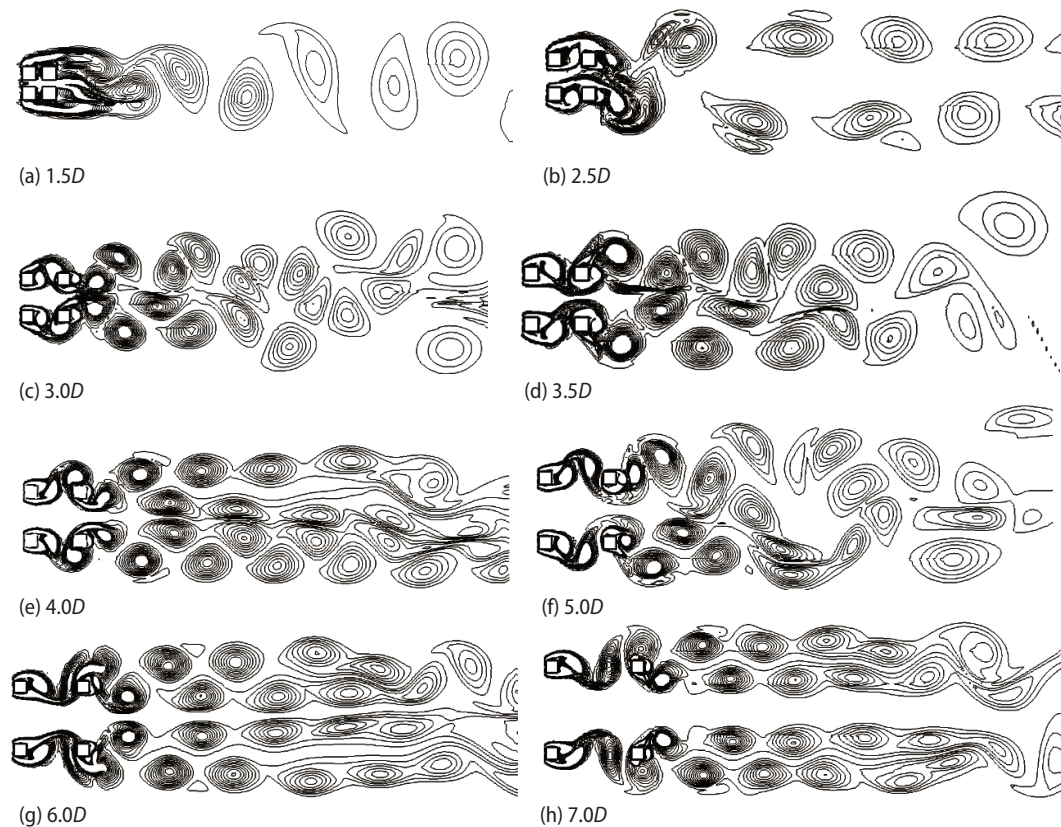
(a) 1.5*D*   (b) 2.5*D*

(c) 3.0*D*   (d) 3.5*D*

(e) 4.0*D*   (f) 5.0*D*

(g) 6.0*D*   (h) 7.0*D*

Figure 3. Instantaneous flow patterns of four-cylinder array with different spacing ratios

after they have reattached to the downstream cylinders. This type of vortex pattern is referred to as a wiggling shielding flow pattern (II) [18, 26].

In figs. 3(e)-3(h), distinct vortex shedding from the upstream cylinders occur and a synchronized shedding pattern is observed. This type of vortex shedding is defined as the vortex shedding flow pattern (III) [18, 26]. These pictures clearly show that between the upstream cylinders (S1 to S2) there is a large scale re-circulation region and the vortex shedding pattern becomes mature because of free shear layers observed on the upstream cylinders which then impinge onto the downstream cylinders. The flow patterns remain distinct and move forward without any lateral spreading or distortion and the wake interactions are weak compared with those with smaller spacing ratios, and the vortices keep the same width and size. This is because of the relatively large distance between the upstream and downstream cylinders, which results in a weaker interaction between the upstream and downstream cylinders. With a larger spacing, the vortex pattern downstream of the four cylinders becomes steadier. A comparison of these four flow patterns shows that the overall flow patterns for *L/D* =6.0 and 7.0 are similar to those for the flow past two cylinders in a tandem arrangement.

Force characteristics and Strouhal numbers

Figure 4 illustrates the variations of time averaged drag coefficients, C_D, with different spacing ratios, *L/D,* (1.5 to 8.0) with Re =300. To compute the time averaged value, the

instantaneous values are averaged over approximately 15 shedding cycles after the periodic behavior condition (where the starting point for processing is unimportant) is met. This figure shows that the values of mean drag coefficients of upstream cylinders (S1 and S3) are close to each other, as are the values for the downstream cylinders (S2 and S4). The mean drag coefficients of upstream cylinders (S1 and S3) are much larger than those of the downstream cylinders (S2 and S4), which is a result of the downstream cylinders being mostly located in the wakes of the upstream cylinders; thus, the drag mainly acts on the upstream cylinders. Furthermore, all of the values are smaller than that for a
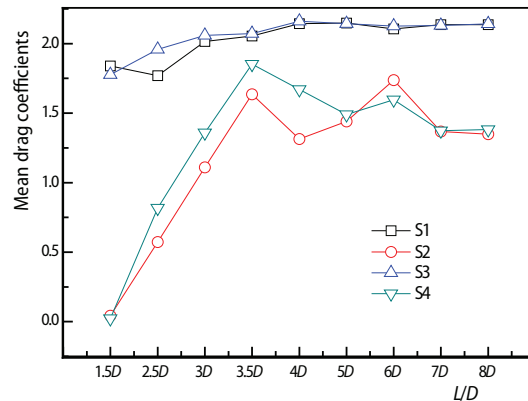

Figure 4. Time averaged drag coefficients of four cylinders at different spacing ratios

single cylinder case. The mean drag forces, C_D, of cylinders (S1 and S3) seem to keep constant with increasing spacing ratios when $L/D > 4.0$. Moreover, the variation of C_D with L/D has a slight increase with the change of L/D from 1.5 to 4.0. The values of the mean drag coefficients for the downstream cylinders increase sharply with increasing spacing ratios when $L/D < 3.5$, this is because the outside free shear layers from the upper cylinders reattach to the downstream cylinders and parts of the drag are assigned to the downstream cylinders. However, at a higher spacing ratio, such as $L/D > 3.5$, the mean drag coefficients of the downstream cylinders decrease gradually with increasing L/D. Lam *et al.* [17] and Lam and Fang [27] report that, with $L/D < 2.0$, C_{D4} is negative, while the present results show that C_{D4} values are small but positive.

Figures 5(a)-5(i) represent the time histories of drag coefficients for different spacing ratios. It is observed that the time signals for $L/D = 1.5$ show steady behavior compared with signals from cases of other spacing ratios. Upstream cylinders (S1 and S3) have similar fluctuating trends, especially for $L/D > 4.0$, where the drag coefficient curves of (S1 and S3) almost coincide with each other. It is found that the spacing ratio, L/D, has a significant influence on amplitudes of signals for downstream cylinders (S2 and S4). It can also be seen that the periodicity of values of drag coefficients for cylinders (S2 and S4) become somewhat lost and the signals become more disordered when $3.0 < L/D < 6.0$.

Time averaged lift coefficients for four cylinders with different spacing ratios can be seen in fig. 6. The mean lift coefficients of upstream cylinders (S1 and S3) are generally equal in magnitude and opposite in direction and the value of lift force asymptotically approaches zero with increasing spacing ratio, L/D, on the upstream cylinders. This shows weakening of flow interference between the two rows of cylinders. The mean lift coefficient is not exactly symmetric about the x-axis because of bi-stable flow pattern that has been observed for this four-square-cylinders array. When $L/D = 1.5-3.0$, the mean lift of cylinder S1 gradually decreases. However, when $L/D > 3.0$, the value slightly increases but still is negative, which results from a transformation of the flow pattern. For $L/D > 3.0$, the upstream (S1 and S3) and downstream (S2 and S4) cylinder pairs each have lift coefficients of opposite sign. The mean lift coefficients of downstream the cylinders (S2 and S4) are equal in magnitude and opposite in direction, except when $L/D = 2.5$ and 3.0. At the flow transition region ($L/D = 3.0-4.0$), a different trend of cylinder S2 can be seen, which indicates the mean lift forces of downstream cylinders are closely related to the transformation of flow patterns, figs. 3(b) and 3(c).
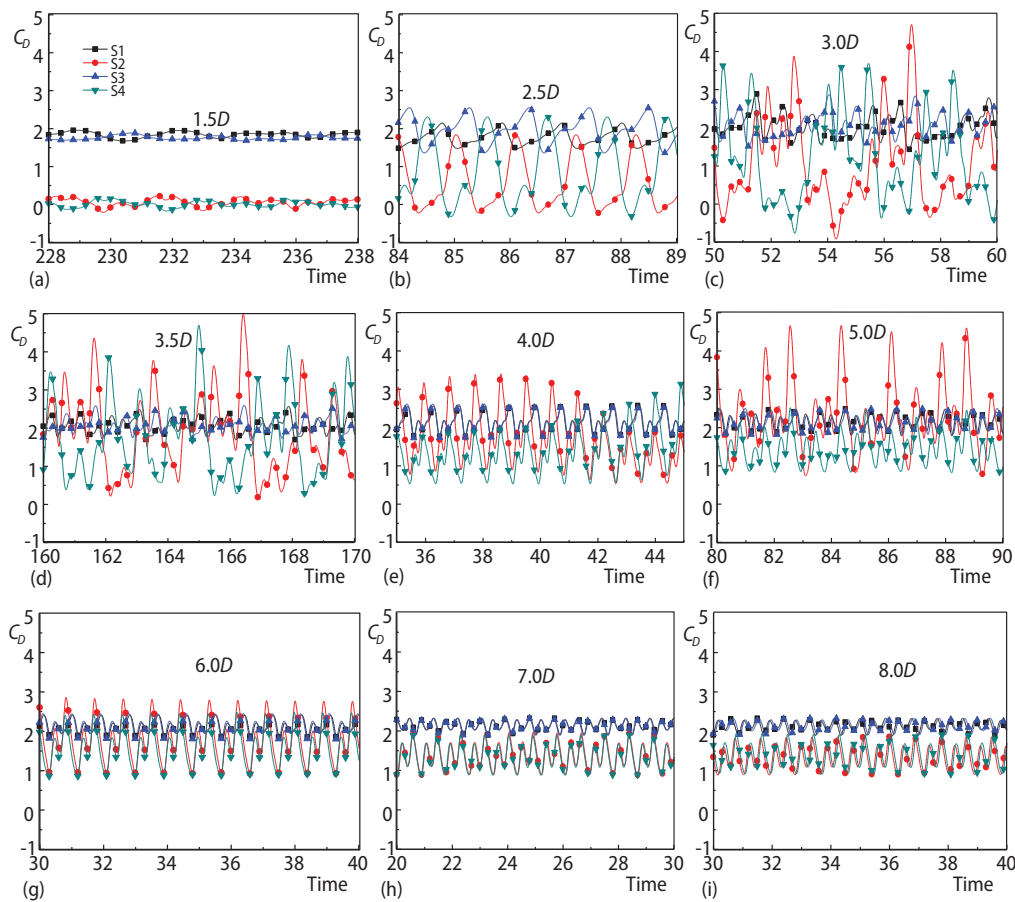
Figure 5. Time history analysis of drag coefficients with different spacing ratios
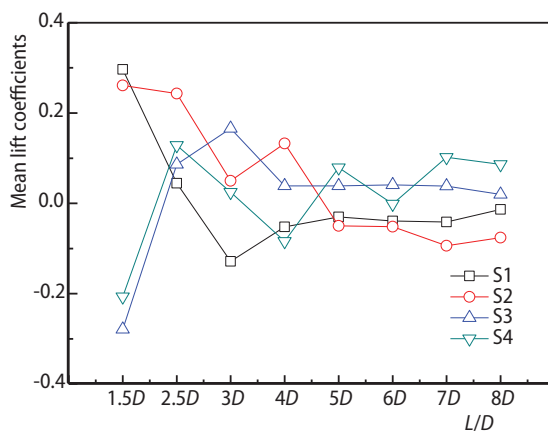

Figure 6. Time averaged lift coefficients of four cylinders with different spacing ratios

Figure 7 shows the amplitude values of lift coefficient oscillations for the four cylinders arrangements with different spacing ratios. It can be seen that the amplitude values of lift of the downstream cylinders (S2 and S4) are about 1.5 times those of the upstream cylinders (S1 and S3) when $L/D > 2.5$, which illustrates a stabilizing effect of shielding flow on the upper cylinders and vortex shedding after the downstream cylinders. It is interesting to note that the trends and magnitudes of amplitude values of lift force of upstream cylinders (S1 and S3) are basically the same, as are those of downstream cylinders (S2 and S4). The changes of the mean
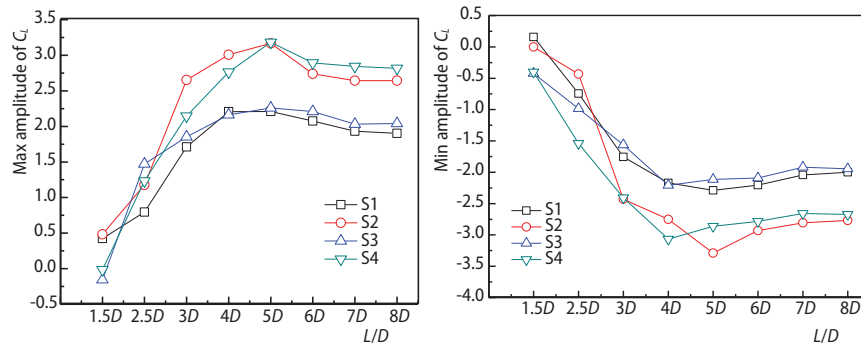
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Figure 7. The amplitudes of lift coefficients for four cylinders with different spacing ratios; (a) the maximum amplitude, (b) the minimum amplitude

lift coefficients on the downstream cylinders are significantly larger than changes on the upstream cylinders for $L/D > 4.0$.

The variation of Strouhal number with center-to-center distance ratio is depicted in fig. 8. The values of Strouhal number are obtained through frequency analysis of the lift force using fast fourier transform function. It is worth mentioning that the frequencies of cylinder S1 are the same as those of cylinder S2, and, similarly, for cylinders (S3 and S4), so the Strouhal number values of cylinders (S1 and S3) are discussed in this paper. From fig. 8, we can see that the trends of cylinders (S1 and S3) are almost the same but there exists a slight difference of magni-

Figure 8. Strouhal numbers of S1 and S3 with different spacings

tudes of Strouhal number. The values of Strouhal number gradually increase and approach the vortex frequency of flow around a single cylinder with increasing of L/D, while the growth rate of Strouhal numbers reduces beyond $L/D = 4.0$.
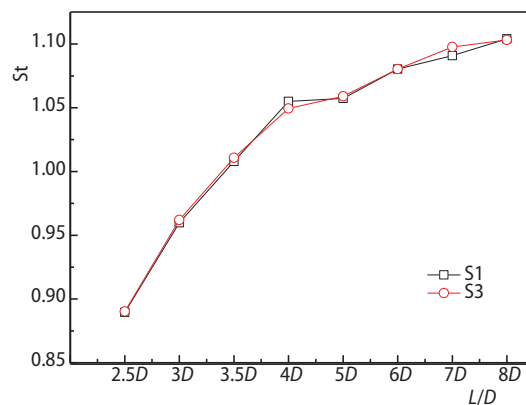
Conclusions

This paper presents a 2-D numerical scheme for documentation of the effects of spacing ratio on laminar flow fields and forces for field around four square cylinders in an in-line square configuration at a Reynolds number of 300. Valuable data, such as the wake structure, drag and lift force characteristics, and Strouhal numbers are obtained.

In general, the wake patterns vary from a bi-stable shielded flow pattern to a vortex shedding flow pattern with increases of spacing ratio. The critical spacing ratio for Re = 300 is around a value of $L/D = 3.5$. For a smaller spacing ratio, the interferences from the side cylinders become more pronounced and forces on the cylinders opposed in direction increase. The interference between two cylinders in tandem is much stronger than seen between side-by-side cylinders, with a larger L/D.

The patterns of flow around four cylinders have an essential influence on the force distributions. Smaller drag forces are exerted on downstream cylinders because they are engulfed

by the outer shear layers of upstream cylinders. The mean lift coefficients of the two upstream cylinders are generally equal in magnitude and opposite in direction to those of the two downstream cylinders.

Acknowledgment

Nomenclature

C_D – mean drag coefficient of the cylinders, [–]
C_{D4} – mean drag coefficient of the four cylinders, [–]
C_L – mean lift coefficient of the cylinders, [–]
D – square cylinders side dimension, [m]
F_D – force in the stream-wise direction, [N]]
F_L – force in the transverse direction, [N]
f – vortex shedding frequency, [–]
L – distance between two cylinders, [m]
L' – feature length of the cylinders, [m]
L/D – spacing ratio of two cylinders, [–]
p – pressure, [Pa]
Re – Reynolds number, [–]
St – Strouhal numbers of downstream cylinder, [–]

S1 – number of cylinder 1
S2 – number of cylinder 2
S3 – number of cylinder 3
S4 – number of cylinder 4
t – temperature [K]
u_∞ – inlet velocity of flow, [ms^{-1}]
u – dimensionless x-velocity, [–]
v – dimensionless y-velocity, [–]
x, y – axial and normal co-ordinates, [m]
Δy – mesh size close the cylinders wall, [–]

Greek symbols

μ – kinematic viscosity of fluid, [m^{2}s^{-1}]
ρ – flow density, [kgm^{-3}]

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