MAGNETOHYDRODYNAMIC MIXED THERMO-BIOCONVECTION IN POROUS CAVITY FILLED BY OXYTACTIC MICROORGANISMS

by

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This article describes the steady magnetohydrodynamic mixed thermo-bioconvection in a square enclosure filled with a homogeneous and isotropic porous medium in the presences of oxytactic microorganisms. The model used for the oxytactic microorganisms is based on a continuum model of a suspension of oxytactic microorganisms. The mixed convection is resulting for the interaction between the buoyancy force and the moving of the top wall of the cavity with constant speed. The horizontal walls of the cavity are considered to be adiabatic while the vertical walls are differentially heated. The governing equations are solved using the finite volume method with SIMPLE technique. Comparisons with previously published works are performed and found to be in excellent agreements. It is found that the increase in both Richardson and Hartmann numbers reduces the average Nusselt and Sherwood numbers.

Key words: *bioconvection, magnetohydrodynamic, porous medium, oxytactic microorganisms*

Introduction

As well-known bioconvection occurs due to by upwardly swimming microorganisms whose average density is slightly larger than that of water [1-3]. However, thermo-bioconvection is formed by the up swimming of microorganisms and the temperature gradient across the fluid.

Kuznetsov [4] presented a paper on a continuum model for thermo-bioconvection of oxytactic bacteria in a porous medium. They investigated the combined effects of microorganisms' up swimming and heating from below on the stability of bioconvection in a horizontal layer filled with a porous medium. Biopolymer characteristics is investigated by Beg *et al*. [5]. They performed simulation of free convection magneto-micropolar biopolymer flow over a horizontal circular cylinder by applying Eringen's micropolar model. They showed that their produced code gives good agreement with earlier works. Alloui *et al*. [6] investigated the effect of heating or cooling from below on the stability of a suspension of motile gravitactic microorganisms in a shallow fluid layer. They used the linear perturbation theory and observed that the thermo-effects may either stabilize or destabilize the suspension, and decrease or increase

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the wave length of the bioconvective pattern. In their another work [7] they tested the effect of heating or cooling from below on the development of gravitactic bioconvection in a square enclosure with stress free side walls. They applied the control volume method to solve numerically the complete set of governing equations and obtained that the suspension is destabilized by heating from below and stabilized by cooling from below.

Kuznetsov [8] developed a theory for bio-thermal convection in a suspension that contains two species of microorganisms exhibiting different taxes, gyrotactic and oxytactic microorganisms. He obtained eigenvalue problem and solved by the Galerkin method and presented in the form of a diagram showing that boundary in the (Ra, Rag) plane for different values of Rayleigh number. Taheri and Bilgen [9] investigated the natural convection for constant temperature and constant heat flux on the development of gravitactic bioconvection in vertical cylinders with stress free side walls by applying control volume method. They observed that subcritical bifurcations of bioconvection became supercritical bifurcations when the thermal Rayleigh number, RaT, is different than zero. A theoretical analysis has been performed for a falling bioconvection plume in a deep chamber filled with a fluid saturated porous medium by Kuznetsov *et al*. [10]. They applied similarity solution of full governing equations (without utilizing the boundary-layer approximation). They observed that the utilization of the boundary-layer approximation (parabolization) results in neglecting the effect of cell swimming and cell diffusion on the total cellflux in theplume. Raees *et al*. [11] describes the unsteady flow of liquid containing nanoparticles and motile gyrotactic microorganisms between two parallel plates while keeping one moving and other fixed. They used the similarity transformations to solve governing PDE are transformed to ODE. They used also MATHEMATICA package based on homotopy analysis method is used to solve this problem. They found that the nanoparticle volume fraction increases as the value of squeeze parameter decreases. Kuznetsov [12] investigated the effect of heating from below on the stability of a suspension of motile gyrotactic microorganisms in a fluid layer of finite depth. He observed that a suspension of gyrotactic microorganisms in a horizontal fluid layer heated from below is less stable than the same suspension under isothermal conditions. Raees *et al*. [13] solved 3-D stagnation flow of a Newtonian fluid on a moving plate with anisotropic slip to the case of nanofluid in suspension of both the nanoparticles and microorganisms by using finite difference method. They obtained that addition of nanoparticles into base fluids with suspension of microorganism, the mass transfer enhancement can be improved to a large extent. Other studies on thermos-bioconvection can be found in [4, 14].

There are many researchers were interested on heat transfer by mixed convection due to its importance in many applications. These are nuclear reactors, solar ponds, lakes and reservoirs solar collectors and crystal growth [15]. Mansour and Ahmed [16] discussed the mixed convection cooling of heat source embedded on the bottom wall of an enclosure filled nanofluids. They found that the increase in heat source length leads to not only increasing the flow intensity but also increasing the fluid temperature however it decreases the Nusselt number. The heat transfer enhancement in two-sided lid-driven cavities with the heat source embedded in the left side wall using Al_2O_3-water nanofluid was performed by Mansour and Ahmed [17]. Their results indicated that the fluid-flow and heat transfer characteristics depend strongly on the direction of the horizontal walls. Numerical investigation is presented by Elshehabey and Ahmed [18] to study Buongiorno's model for MHD mixed convection of a lid-driven cavity with nanofluid. Their results demonstrate that the presence of an inclined magnetic field in the flow region leads to lose the fluid movement.

The main aim of this manuscript is to investigate the mixed thermo-bioconvection under magnetic field by using finite volume method. Based on survey and authors' knowledge this study is the first attempt to investigate the mixed convection problem of porous cavity filled by oxytactic microorganism for mixed convection. The applications of such kind of problems can be found in a highly efficient fuel cells where thermophilic microorganisms, such as Bacillus licheniformis and Bacillus thermoglucosidasius are used. Also, it is relevant in applications such as motile thermophilic microorganisms that live in hot springs**.**

Mathematical formulations

The 2-D a square porous enclosure filled with oxytactic microorganisms as shown in fig. 1. It is assumed that the length of the cavity wall is equal to H and there are no-slip boundary conditions at the cavity walls. The top wall moves from left to right with constant speed, U_0. Constant temperature distributions, T_h and T_c, are imposed to the left and right walls, respectively, where $T_\mathrm{h} > T_\mathrm{c}$. However, the top and bottom walls are assumed to be thermally insulated. The cavity is permeated by uniform magnetic field, $\vec{B}$, constant $B_0 = (B_x^2 + B_y^2)$ and it direction makes angle, Φ, with x-axis. An isotropic and homogenous porous medium is considered with constant properties. The Boussinesq approximation is utilized and the suspension is assumed dilute. The inertia terms are neglected due to the small flow velocity associated with bioconvection. The gravity vector, $\vec{\mathrm{g}}$, acts in the opposite direction to the y-axis. The model presented here is based on a continuum model of a suspension of oxytactic microorganisms developed by Hillesdon and Pedley [19].

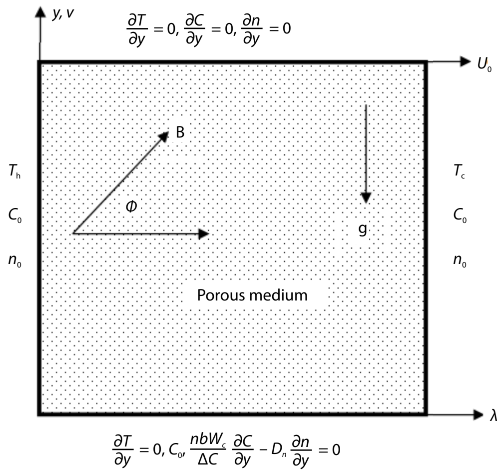

Figure 1. Physical model

Under these assumptions, the vector form of the governing equations can be expressed:
– continuity

$$\nabla\vec{\mathrm{q}} = 0 \tag{1}$$

– momentum equation

$$\rho_\mathrm{f}\left[(\vec{\mathrm{q}}\nabla)\vec{\mathrm{q}}\right] = -\nabla p + \mu_\mathrm{f}\nabla^2\vec{\mathrm{q}} - \frac{\mu_f}{K}\vec{\mathrm{q}} - \left[\gamma\Delta\rho n - \beta\rho_\mathrm{f}(T - T_\mathrm{c})\vec{\mathrm{g}} + \vec{\mathrm{I}}\times\vec{B}\right] \tag{2}$$

– Ohm's law

$$\vec{\mathrm{I}} = \sigma\left[-\nabla\varphi + \vec{\mathrm{q}}\times\vec{B}\right] \tag{3}$$

– the electric current density $\vec{\mathrm{I}}$

$$\nabla\vec{\mathrm{I}} = 0 \tag{4}$$

– energy equation

$$\vec{\mathrm{q}}\nabla T = \alpha_m\nabla^2 T \tag{5}$$

– oxygen conservation equation

$$\vec{\mathrm{q}}\nabla C = D_c\nabla^2 C - \delta n \tag{6}$$

– cell conservation equation

$$\nabla(n\vec{\mathrm{q}} + n\tilde{\vec{\mathrm{q}}} - D_n\nabla n) = 0 \tag{7}$$

In the equations $\vec{q} = (u, v)$ is the velocity vector, p – the pressure, ρ_f – the fluid density, ∇^2 – the Laplacian operator, μ_f – the dynamic viscosity, K – the permeability of the porous medium, γ – the average volume of a microorganisms, $\Delta\rho = \rho_{cell} - \rho_f$ – the density difference between cells and fluid, n – the number density of motile microorganisms, β – the volume expansion coefficient of water at constant pressure, T – the dimensional temperature, $\vec{I}$ – the electric current density vector, σ – the electrical conductivity, φ – the electric potential function and is considered to be constant due to electrically non conducting boundaries, α_m – the effective thermal diffusivity of the porous medium, C – the oxygen concentration, $-\delta n$ – the describes the consumption of the microorganisms, D_c – the diffusivity of oxygen, D_n – the diffusivity of the microorganisms, and $\tilde{\vec{q}}$ the average directional swimming velocity of microorganism and it is assumed as Hillesdon and Pedley [19]:

$$\tilde{\vec{q}} = \frac{bW_c \nabla C}{\Delta C} \tag{8}$$

where b is the chemotaxis constant, W_c – the maximum cell swimming speed, and $\Delta C = C_0 - C_{\min}$ where C_0 is the free-surface oxygen concentration, and $C_{\min}$ – the minimum oxygen concentration that microorganisms need in order to be active.

Introducing the following dimensionless set of variables:

$$X = \frac{x}{H},\ Y = \frac{y}{H},\ U = \frac{u}{U_0},\ V = \frac{v}{U_0},\ P = \frac{p}{\rho_f U_0^2},\ \theta = \frac{T - T_c}{T_h - T_c},\ N = \frac{n}{n_0},\ \phi = \frac{C - C_{\min}}{\Delta C} \tag{9}$$

where n_0 is the average number density of the microorganisms. Substituting eq. (9) into eqs. (1)-(7), the following dimensionless Cartesian co-ordinates X and Y equations are obtained:

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\mathrm{Re}}\left[\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right] - \frac{1}{\mathrm{Da\,Re}}U + \frac{\mathrm{Ha}^2}{\mathrm{Re}}(V\sin\Phi\cos\Phi - U\sin^2\Phi) \tag{10}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\mathrm{Re}}\left[\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right] - \frac{1}{\mathrm{Da\,Re}}V +$$

$$+\frac{\mathrm{Ha}^2}{\mathrm{Re}}(U\sin\Phi\cos\Phi - V\cos^2\Phi) + \mathrm{Ri}(\theta - \mathrm{Rb}N) \tag{11}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{\mathrm{Re\,Pr}}\left[\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right] \tag{12}$$

$$U\frac{\partial \phi}{\partial X} + V\frac{\partial \phi}{\partial Y} = \frac{1}{\mathrm{Re\,Sc}}\left[\frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2}\right] - \frac{\sigma_1}{\mathrm{Re\,Sc}}N \tag{13}$$

$$\mathrm{Sc}\chi\left[U\frac{\partial N}{\partial X} + V\frac{\partial N}{\partial Y}\right] + \frac{\mathrm{Pe}}{\mathrm{Re}}\left(N\nabla 2\phi + \frac{\partial N}{\partial X}\frac{\partial \phi}{\partial X} + \frac{\partial N}{\partial Y}\frac{\partial \phi}{\partial Y}\right) = \frac{1}{\mathrm{Re}}\left[\frac{\partial^2 N}{\partial X^2} + \frac{\partial^2 N}{\partial Y^2}\right] \tag{14}$$

where $\mathrm{Re} = U_0 H/\nu$, $\mathrm{Da} = K/H^2$, $\mathrm{Ha} = (\sigma B_0^2 H^2/\mu_f)^{1/2}$, $\mathrm{Gr} = [g\beta(T_h - T_c)H^3]/\nu^2$, $\mathrm{Ri} = \mathrm{Gr}/\mathrm{Re}^2$, $\mathrm{Rb} = \gamma\Delta\rho n_0/[\rho_f\beta(T_h - T_c)]$, $\mathrm{Pe} = bW_c/D_n$, $\mathrm{Pr} = \nu/\alpha_m$, $\mathrm{Sc} = \nu/D_c$, $\sigma_1 = \delta n_0 H^2/D_c\Delta C$ are the Reynolds, Darcy, Hartman, Grashof, Richardson, bioconvection Rayleigh, Peclet, Prandtl, Schmitt numbers, and ratio of the rate of oxygen consumption to the rate of oxygen diffusion, respectively. Following Sheremet and Pop [20], the appropriate dimensionless boundary conditions are:

$$X = 0:\quad U = V = 0, \quad \theta = 1, \quad \phi = 1, \quad N = 1 \tag{15a}$$

$$X = 1:\ U = V = 0, \quad \theta = 0, \quad \phi = 1, \quad N = 1 \tag{15b}$$

$$Y = 0:\ U = V = 0, \quad \frac{\partial \theta}{\partial Y} = 0, \quad \phi = 1, \quad \mathrm{Pe}N\frac{\partial \phi}{\partial Y} - \frac{\partial N}{\partial Y} = 0 \tag{15c}$$

$$Y = 1:\ U = 1 = 0, \quad V = 0, \quad \frac{\partial \theta}{\partial Y} = 0, \quad \frac{\partial \phi}{\partial Y} = 0, \quad \frac{\partial N}{\partial Y} = 0 \tag{15d}$$

The local Nusselt and Sherwood numbers at the heated wall are:

$$\mathrm{Nu} = -\left[\frac{\partial \theta}{\partial X}\right]_{X=0}, \quad \mathrm{Sh} = -\left[\frac{\partial \phi}{\partial X}\right]_{X=0} \tag{16, 17}$$

The average Nusselt and Sherwood numbers are, also, defined:

$$\mathrm{Nu}_m = \int_0^1 \mathrm{Nu}\,\mathrm{d}Y, \quad \mathrm{Sh}_m = \int_0^1 \mathrm{Sh}\,\mathrm{d}Y \tag{18, 19}$$

Numerical method and validation

The control volume method is applied to solve the governing eqs. (10)-(14) with the boundary conditions as given in eq. (15). In order to explain this numerical solution, let us start with writing eqs. (10)-(14) in the following form:

$$\nabla(\vec{U}Q) = \zeta\nabla(\nabla Q) + S_Q \tag{20}$$

where Q refer to U, V, θ, ϕ, and N, $\vec{\mathrm{U}} = (U, V)$ is the dimensionless velocity vector, S_Q – the source terms, $\nabla(\vec{\mathrm{U}}Q)$ is the convective term and ∇, ∇Q – the diffusive term. Integrating (20) over the control volume Ω gives:

$$\int_\Omega \nabla(\vec{\mathrm{U}}Q)\mathrm{d}V = \zeta\int_\Omega \nabla(\nabla Q)\mathrm{d}V + \int_\Omega S_Q\mathrm{d}V \tag{21}$$

Combining the convective and the diffusive terms in eq. (21):

$$\int_\Omega \nabla(\vec{\mathrm{U}}Q - \zeta\nabla Q)\mathrm{d}V = \int_\Omega S_Q\mathrm{d}V \tag{22}$$

Using the divergence theorem to convert the volume integral in the left hand side in eq. (23) to an integral $\delta\Omega$ around the boundary, see fig. 2:

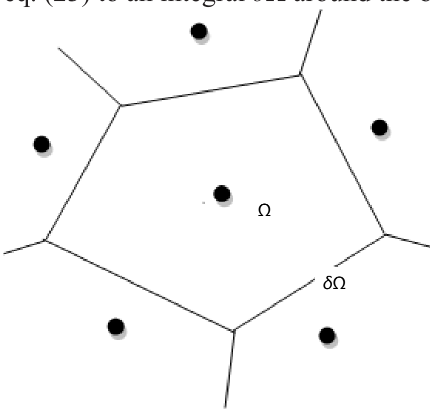

Figure 2. The control volume

$$\int_{\delta\Omega} (\vec{\mathrm{U}}Q - \zeta\nabla Q)\vec{\mathrm{n}}\mathrm{d}S = \int_\Omega S_Q\mathrm{d}V \tag{23}$$

In eq. (24), $\vec{\mathrm{U}}Q\vec{\mathrm{n}}\mathrm{d}S$ and $\zeta\,\nabla Q\vec{\mathrm{n}}\mathrm{d}S$ is the convective and diffusive fluxes of Q across the part of the boundary edge dS, respectively. The surface integral in eq. (23) is generally approximated in a discretized form by:

$$\int_{\delta\Omega} (\vec{\mathrm{U}}Q - \zeta\nabla Q)\vec{\mathrm{n}}\mathrm{d}S \approx \sum\nolimits_k (\vec{\mathrm{U}}Q - \zeta\nabla Q)_k(\vec{\mathrm{n}}\mathrm{d}S)_k \tag{24}$$

In eq. (24), $(\vec{\mathrm{U}}Q - \zeta\,\nabla Q)$ evaluated at the center of edge segment k and ΔS, and $\vec{\mathrm{n}}$ are the area and the unit vector normal to this edge, respectively. The source terms in the volume integral of eq. (25) are approximated:

$$\int_{\Omega} S_Q \mathrm{d}V \approx \overline{S}_Q Vol \approx (S_Q)_P Vol \tag{25}$$

where $\overline{S}_Q$ is the average value of S_Q over the control volume and $(S_Q)_p$ is its value at the cell centre node P. The resulting discretized equations have been solved iteratively, through alternate direction implicit ADI with SIMPLE algorithm [21]. The velocity correction has been made using the Rhie and Chow interpolation. For convergence, under-relaxation technique has been employed. The 10^5 was set as the convergence criterion. A uniform grid resolution of 81×81 is found to be suitable. In order to verify the accuracy of the present numerical study, the present numerical model was validated against the results obtained by Iwatsu *et al.* [22], and Chamkha [23] in special cases of the current investigation. These comparisons are presented in tab. 1. An excellent agreement was found between the present results and the results obtained by Iwatsu *et al.* [22], and Chamkha [23]. Therefore, we are confident that the results presented in this paper are very accurate.

Table 1. Comparisons of the local Nusselt number at the lid wall, for different values of Re at Pr = 0.71, Gr = 10^2, Ha = 0, Rb = ∞

Re	Iwatsu *et al.* [22]	Chamkha [23]	Present
100	1.94	2.01	1.93
400	3.84	3.91	3.91
1000	6.33	6.33	6.31

Results and discussion

In this section, the results of laminar, 2-D mixed convection of incompressible electrically conducting fluid inside a lid-driven enclosure filled with a porous medium in the presences of oxytactic microorganisms are discussed. The results of the problem are represented in terms of contours of streamlines, ψ, isotherms, θ, oxygen isoconcentrations, ϕ, and microorganisms isoconcentrationsas, N, well as profiles of the velocity components, local Nusselt number, Sherwood number, and average Nusselt and Sherwood numbers. This parametric study is computed for wide ranges of the governing parameters. Richardson number varies from 0.001-10, the bioconvection Rayleigh number varies from 0-15, the Hartmann number varies from 0-100, and the inclination angle of the magnetic field varies from 0° to 120°. In all the obtained results, the reference case of other parameters are considered to be Gr = 10^4, Da = 10^{-3}, Ha = 10, Rb = 1, Ri = 1, Pr = 6.2, Sc = σ_1 = 1, Pe = 0.1, Φ = 0°, x = 1. Figure 3 shows the contours of streamlines, isotherms, oxygen isoconcentrations, and microorganisms iso-concentrations for different values of the Richardson number. It is observed that the streamlines affect clearly by the moving of the top wall. A large clockwise circular cell is formed inside the enclosure at the low value of Ri = 0.001.

In addition, a gathering of streamlines is seen near the lid-moving wall and top part of the right wall indicating a strong flow of the fluid in these regions. As increases, the shear force resulting from the lid-driven moving decreases and consequently, the streamlines stretch towards the bottom wall indicating a high buoyancy force at Ri = 10 comparing with low values of Richardson number. Regarding the isotherms contours, at Ri = 0.001, there is a non-isothermal region is seen near the top wall indicating large temperature differences in this area. Also, the temperature contours gather near the heated wall indicating a high rate of heat transfer in this case. The increasing values of Richardson number makes the isotherms distribute inside the entire region of the enclosure indicating a low temperature gradients in these cases. The distributions of oxygen iso-concentrations and microorganisms iso-concen-

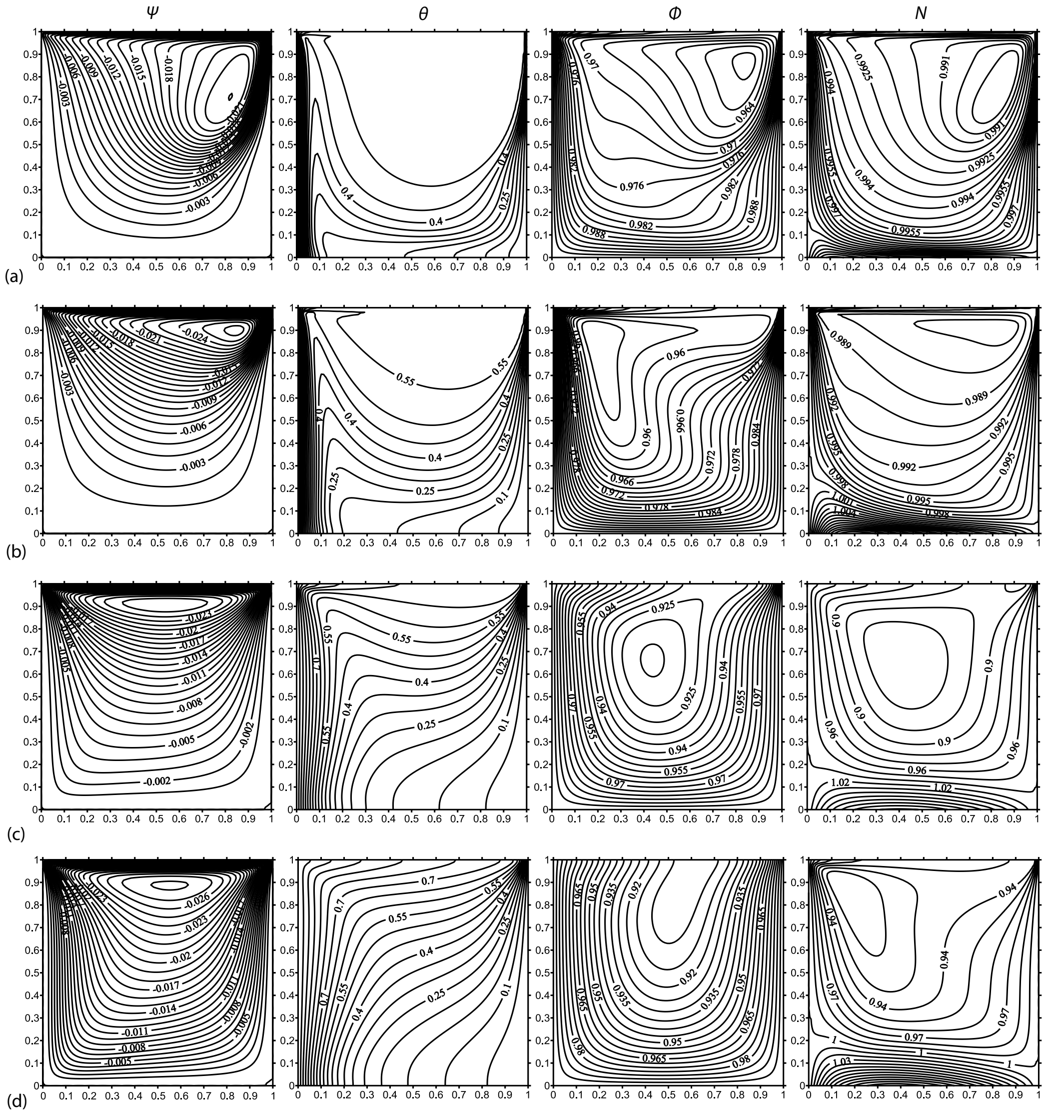

Figure 3. Contours of streamlines, isotherms, oxygen isoconcentrations, and microorganisms isoconcentrations for; (a) Ri = 0.001 , (b) Ri = 0.01, (c) Ri = 1, (d) Ri = 10 at Gr = 10^4, Da = 10^{-3}, Ha = 10, Rb = 1, Pr = 6.2, Sc = σ_1 = 1, Pe = 0.1, Φ = 0°, χ = 1

trations affect clearly by the behavior of the streamlines. As Richardson number increases, a decrease in motile bacteria concentration is seen in the cavity. The effects of Richardson number on the profiles of vertical velocity component at the enclosure mid-section, local Nusselt numbers at the left wall are depicted in figs. 4 and 5, respectively. It should be noted that, in the left half of the cavity, the vertical velocity component increases as increases, whereas, a reverse behavior is observed at the right half. As mentioned above, the increase in Richardson number causes a reduction of the temperature gradients, this in turn reduces the local Nusselt number as it can be noted from fig. 5.

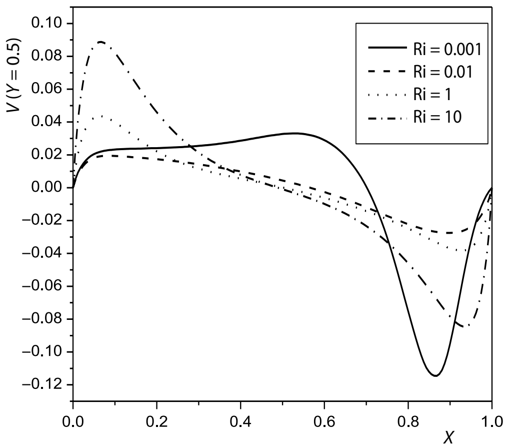

Figure 4. Vertical velocity profiles at the enclosure mid-section for different values of Ri at: Gr = 10^4, Da = 10^{-3}, Ha = 10, Rb = 1, Pr = 6.2, Sc = σ_1 = 1, Pe = 0.1, Φ = 0°, χ = 1

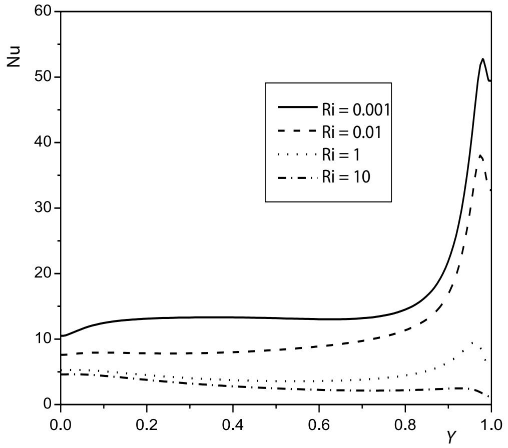

Figure 5. Profiles of the local Nusselt number at the left wall for different values of at: Gr = 10^4, Da = 10^{-3}, Ha = 10, Rb = 1, Pr = 6.2, Sc = σ_1 = 1, Pe = 0.1, Φ = 0°, χ = 1

Figures 6 and 7 present effect of the bioconvection Rayleigh number on the profiles of the local Nusselt number, and the local Sherwood number at the heated wall. The results indicate that both local Nusselt number and Sherwood number increase as increases. Figures 8 and 9 show the effect of the presence of magnetic field represented by the variation of the Hartmann number on the local Nusselt number and local Sherwood number, respectively. It is observed that the increase of causes a significant reduction in the profiles of local Nusselt number and a similar behavior is noted for the local Sherwood number. The physical explanation for these behaviors is due to the magnetic force that slowdown the fluid motion and reduces the thermo bioconvection and consequently decreases the gradients of the temperature and oxygen concentration.

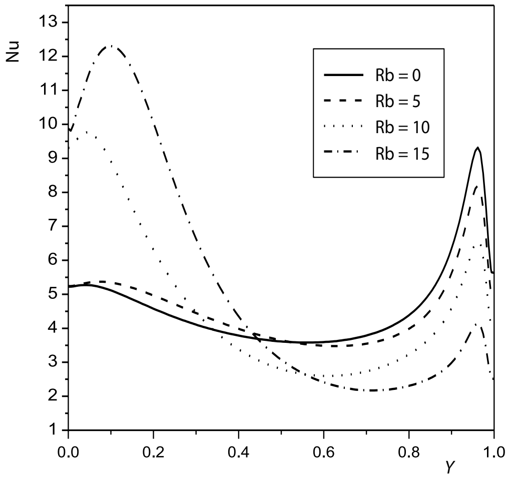

Figure 6. Profiles of the local Nusselt number at the left wall for different values of Rb at: Gr = 10^4, Da = 10^{-3}, Ha = 10, Rb = 1, Pr = 6.2, Sc = σ_1 = 1, Pe = 0.1, Φ = 0°, χ = 1

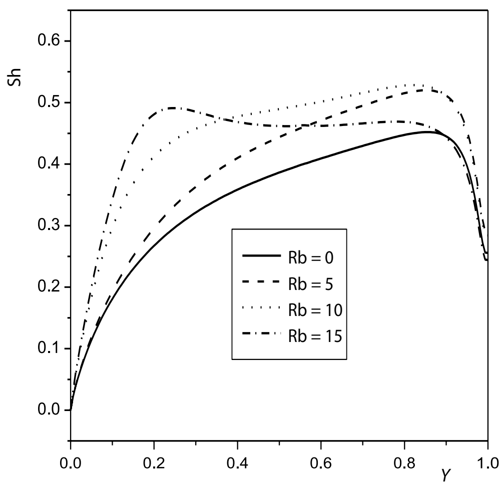

Figure 7. Profiles of the local Sherwood number at the left wall for different values of Rb at: Gr = 10^4, Da = 10^{-3}, Ha = 10, Rb = 1, Pr = 6.2, Sc = σ_1 = 1, Pe = 0.1, Φ = 0°, χ = 1

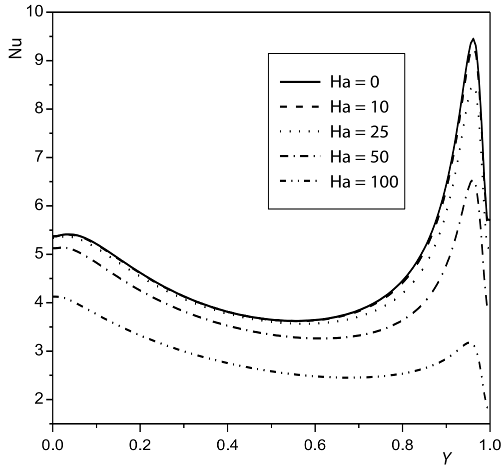

Figure 8. Profiles of the local Nu number at the left wall for different values of Ha at: Gr = 10^4, Da = 10^{-3}, Rb = 1, Ri = 1, Pr = 6.2, Sc = σ_1 = 1, Pe = 0.1, Φ = 60°, χ = 1

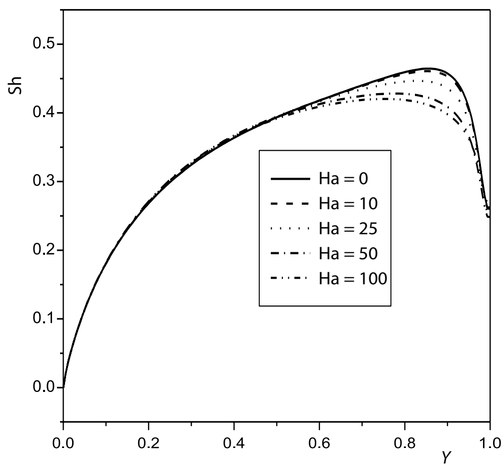

Figure 9. Profiles of the local Sh number at the left wall for different values of Ha at: Gr = 10^4, Da = 10^{-3}, Ri = 1, Rb = 1, , Pr = 6.2, Sc = σ_1 = 1, Pe = 0.1, Φ = 60°, χ = 1

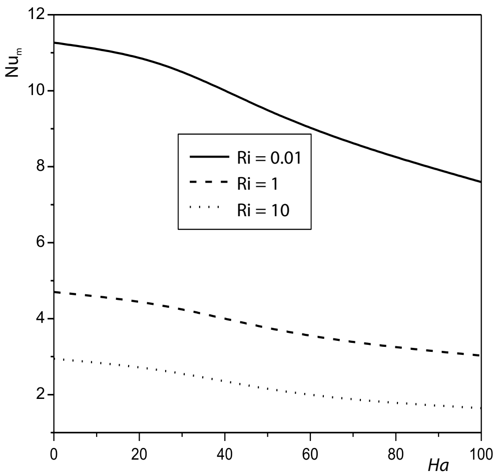

Figure 10. Profiles of the average Nusselt number at the left wall for different values of Ha and Ri at: Gr = 10^4, Da = 10^{-3}, Rb = 1, , Pr = 6.2, Sc = σ_1 = 1, Pe = 0.1, Φ = 0°, χ = 1

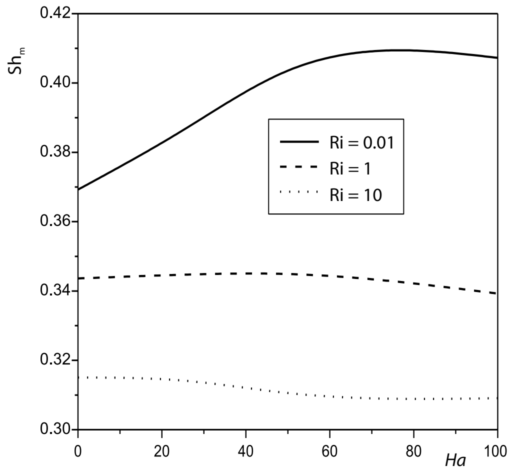

Figure 11. Profiles of the average Sherwood number at the left wall for different values of Ha and Ri at: Gr = 10^4, Da = 10^{-3}, Rb = 1, Pr = 6.2, Sc = σ_1 = 1, Pe = 0.1, Φ = 0°, χ = 1

Figures 10 and 11 depict the profiles of the local Nusselt number and local Sherwood number at the enclosure left wall for different values of the magnetic field inclination angle Φ. The rate of heat transfer takes its minimum at Φ = 0° and it takes its maximum at Φ = 60°, whereas more increase in Φ (Φ = 120°) causes a reduction in Nusselt number. This behavior occurs at the bottom half of the enclosure, however, at the top half, these behaviors are reverse. The same observations are, almost, found for the local Sherwood number but the local Nusselt number is noted to be more affected by the variations of Φ than the oxygen transfer rate.

Conclusion

The problem of MHD thermo-bioconvection of an incompressible, laminar electrically conductive fluid of lid-driven enclosure filled with a homogenous and isotropic porous medium was investigated. The PDE governing the problem were converted to dimensionless form and then solved numerically using the control volume method. Comparison with previously published results was formed and found to be in excellent agreements. From this investigation, it can be concluded that the increase in the Richardson number causes a decrease in the heat transfer rate, whereas the local Sherwood number, almost, increases as the Richardson number increases. Also, an enhancement in the profiles of the local Nusselt and Sherwood numbers at the bottom half of the left wall is obtained by increasing the bioconvection Rayleigh number. Moreover, the increase of Hartman number reduces both of the local and average Nusselt and Sherwood numbers.

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Nomenclatures

$\vec{B}$ – magnetic field
b – chemotaxis constant
C – oxygen concentration
Da – Darcy number
D_n – diffusivity of the microorganisms
D_c – diffusivity of the oxygen
Gr – Grashof number
g – gravity acceleration
H – height of the cavity
Ha – Hartmann number
$\vec{\mathrm{I}}$ – electric current density vector
K – permeability of the porous medium
N – dimensionless number density of motile microorganisms
n – dimensional number density of motile microorganisms
Nu – Nusselt number
P – dimensionless pressure
p – dimensional pressure
Pe – Peclet number
Pr – Prandtl number
$\vec{\mathrm{q}}$ – velocity vector
Rb – bioconvection Rayleigh number
Re – Reynolds number
Ri – Richardson number
Sc – Schmitt number
Sh – Sherwood number
T – dimensional temperature
U, V – dimensionless velocity components
u, v – dimensional velocity component
U_0 – led velocity
W_c – maximum cell swimming speed
X, Y – dimensionless Cartesian co-ordinates
x, y – dimensional Cartesian co-ordinates

Greek symbols

α – thermal diffusivity
β – volume expansion coefficient
γ – average volume of microorganisms
θ – dimensionless temperature
μ – dynamic viscosity
ν – kinematic viscosity, [$\mathrm{m^2s^{-1}}$]
ρ – fluid density
σ – electrical conductivity
σ_1 – ratio of the rate of oxygen consumption to the rate of oxygen diffusion
φ – electric potential function
ϕ – dimensionless oxygen concentration n
Φ – magnetic field inclination angle

Subscripts

c – cold
h – hot
f – fluid
0 – reference state

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