

## THERMODYNAMIC ANALYSIS OF VISCOELASTIC FLUID IN A POROUS MEDIUM WITH PRESCRIBED WALL HEAT FLUX OVER STRETCHING SHEET SUBJECTED TO A TRANSITIVE MAGNETIC FIELD

by

**Zineb CHAICH<sup>a\*</sup>, Salah SAOULI<sup>b</sup>, and Issam REZAYGUIA<sup>c\*</sup>**

<sup>a</sup> Laboratory of Process Engineering, Faculty of Applied Science, University of Ouargla, Ouargla, Algeria

<sup>b</sup> Institute for Science and Technology, Mila University Center, Mila, Algeria

<sup>c</sup> Faculty of Natural Science, Life Science, Earth and the Universe, University of Ouargla, Ouargla, Algeria

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*An analysis is performed for entropy generation in a steady laminar boundary-layer flow of an electrically conducting second grade fluid in a porous medium prescribed wall heat flux subject to a transverse uniform magnetic field past a semi-infinite stretching sheet. The effects of viscous dissipation, internal heat generation of absorption due to deformation are considered in the energy equation. Kummer's functions are used to obtain temperature field. The velocity, temperature are used to compute the entropy generation number with a change in various dimensionless parameters.*

Key words: *entropy generation, prescribed wall heat flux case, magnetic field, porous medium, second grade fluid, stretching sheet*

### Introduction

The design of thermal systems can be achieved by optimization of entropy generated in the systems. Due to the number of applications in the industrial manufacturing process, the problem of the boundary-layer flow past a stretching plate has attracted considerable attention of researchers during the past few decades for examples heat exchangers, cooling of nuclear reactors, MHD power generators, geophysical fluid dynamics, energy storage systems, cooling of electronic devices, *etc.* A series of studies on heat transfer effects on viscoelastic fluid have been made by many authors under different physical situations, including [1-8] have derived similarity solution of viscoelastic boundary-layer flow and heat transfer over an exponential stretching surface. Cortell [9] have studied flow and heat transfer of a viscoelastic fluid over stretching surface considering both constant sheet temperature and prescribed sheet temperature. Abel *et al.* [10] carried out a study of viscoelastic boundary-layer flow and heat transfer over a stretching surface in the presence of non-uniform heat source and viscous dissipation considering prescribed surface temperature and prescribed surface heat flux. Liu [11] analysis the flow and the heat transfer of a steady laminar boundary-layer flow of an electrically conducting fluid of second grade in a porous medium subject to a transverse uniform magnetic field past a semi-infinite stretching sheet with power-law surface temperature or power-law

\* Corresponding authors, e-mail: chaichrabab@yahoo.com; issam.rezaiguia@yahoo.fr

surface heat flux. Khan [12] studied the case of the boundary-layer problem on heat transfer in a viscoelastic boundary-layer fluid flow over a non-isothermal porous sheet, taking into account the effect a continuous suction/blowing of the fluid, through the porous boundary. The effects of a transverse magnetic field and electric field on momentum and heat transfer characteristics in a viscoelastic fluid over a stretching sheet taking into account viscous dissipation and ohmic dissipation are presented by Abel *et al.* [13]. Hsiao [14] studied the conjugate heat transfer of mixed convection in the presence of radiative viscous dissipation in viscoelastic fluid past a stretching sheet. The case of unsteady MHD was carried out by Abbas *et al.* [15]. Using Kummer's functions, Singh [16] carried out the study of heat source and radiation effects on MHD flow of a viscoelastic fluid past a stretching sheet with prescribed power law surface heat flux. The effects of non-uniform heat source, viscous dissipation and thermal radiation on the flow and heat transfer in a viscoelastic fluid over a stretching surface was considered in Prasad *et al.* [17]. The case of the heat transfer in MHD flow of viscoelastic fluids over stretching sheet in the case of variable thermal conductivity and in the presence of non-uniform heat source and radiation is reported in Abel and Mahesha [18]. Using the homotopy analysis, Hayat *et al.* [19] looked at the hydrodynamic of 3-D flow of viscoelastic fluid over a stretching surface. The investigation of biomagnetic flow of a non-Newtonian viscoelastic fluid over a stretching sheet under the influence of an applied magnetic field is done by Misra and Shit [20]. Subhas *et al.* [21] analyzed the momentum and heat transfer characteristics in a hydromagnetic flow of viscoelastic liquid over a stretching sheet with non-uniform heat source. Nandeppanavar *et al.* [22] analyzed the flow and heat transfer characteristics in a viscoelastic fluid flow in porous medium over a stretching surface with surface prescribed temperature and surface prescribed heat flux and including the effects of viscous dissipation. Chen [23] studied the MHD flow and heat transfer characteristics viscoelastic fluid past a stretching surface, taking into account the effects of Joule and viscous dissipation, internal heat generation/absorption, work done due to deformation and thermal radiation. Nandeppanavar *et al.* [24] considered the heat transfer in viscoelastic boundary-layer flow over a stretching sheet with thermal radiation and non-uniform heat source/sink in the presence of a magnetic field. Cortell [25] also reported the flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation. Free convective heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of the radiation and buoyancy effects has been investigated by Rashidi *et al.* [26, 27]. Although the foregoing research works have covered a wide range of problems involving the flow and heat transfer of viscoelastic fluid over a stretching surface, they have been restricted, from a thermodynamic point of view, to only the first law analysis. The contemporary trend in the field of heat transfer and thermal design is the Second law of thermodynamics analysis and its related concept of entropy generation minimization. Entropy generation is closely associated with thermodynamic irreversibility, which is encountered in all heat transfer processes. Different sources are responsible for the generation of entropy such as heat transfer and viscous dissipation Bejan [28, 29] The analysis of entropy generation rate in a circular duct with the imposed heat flux at the wall and its extension to determine the optimum Reynolds number as a function of the Prandtl number and the duty parameter were presented by Bejan [30]. Sahin [31] introduced the Second law analysis of a viscous fluid in a circular duct with isothermal boundary conditions. In another paper, Sahin [32] presented the effect of variable viscosity on the entropy generation rate for heated circular duct. A comparative study of entropy generation rate inside the duct of different shapes and the determination of the optimum duct shape subjected to the isothermal boundary condition were done by Sahin [33]. Narusawa [34] gave an analytical and numerical analysis of the Second law for flow and heat

transfer inside a rectangular duct. In a more recent paper, Mahmud and Fraser [35-37] applied the Second law analysis to fundamental convective heat transfer problems and to non-Newtonian fluid flow through channel made of two parallel plates. The study of entropy generation in a falling liquid film along an inclined heated plate was carried out by Saouli and Aiboud-Saouli [38]. The effects of magnetic field and viscous dissipation on entropy generation in a falling film and channel were studied by Aiboud-Saouli *et al.* [39, 40]. The application of the Second law analysis of thermodynamics to viscoelastic MHD flow over a stretching surface was carried out by Aiboud and Saouli [41, 42]. Irreversibility analysis in a couple stress film flow along an inclined heated plate with adiabatic free surface has been studied by Adesanya and Makinde [43]. Entropy generation and energy conversion rate for the peristaltic flow in a tube with magnetic field has also been investigated by Akbar [44]. Makinde [45] has investigated entropy analysis of MHD boundary-layer flow and heat transfer over a flat plate with a convective surface boundary condition. Entropy analysis of an unsteady MHD flow past a stretching permeable surface in nanofluid has been studied by Abolbashari *et al.* [46]. Chemical reaction effect on MHD free convective surface over a moving vertical plane through porous medium has been studied by Tripathy *et al.* [47].

**Formulation of the problem**

In 2-D Cartesian co-ordinate system  $x, y$  we consider magneto-convection, steady, laminar, electrically conductor, boundary-layer flow of a second grade fluid caused by a stretching surface through porous medium to prescribed wall heat flux in the presence of a uniform transverse magnetic field. As shown in fig. 1 the  $x$ -axis is taken in the direction of the main flow along the plate and the  $y$ -axis is normal to the plate with velocity components  $u, v$  in these directions under the usual boundary-layer approximations, the governing equations are [11]:

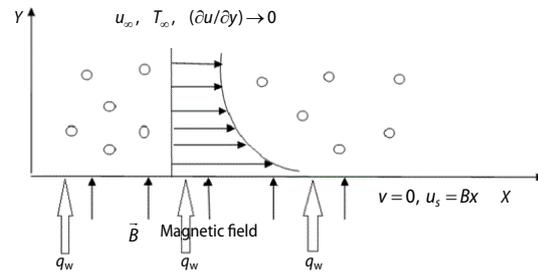


Figure 1. Physical model for the flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho} \left[ \frac{\partial}{\partial x} \left( u \frac{\partial^2 u}{\partial y^2} \right) + \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} + v \frac{\partial^3 u}{\partial y^3} \right] - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{k_1} u \tag{2}$$

The boundary conditions are given:

$$y = 0, \quad B > 0, \quad u = Bx, \quad v = 0 \tag{3a}$$

$$y \rightarrow \infty, \quad u \rightarrow 0, \quad \frac{\partial u}{\partial y} \rightarrow 0 \tag{3b}$$

The energy equation [11], corresponding to the boundary-layer analysis, with viscous dissipation, done by deformation and internal heat generation or absorption is given:

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2 + \alpha_1 \frac{\partial u}{\partial y} \frac{\partial}{\partial y} \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + Q(T - T_\infty) \tag{4}$$

The relevant boundary conditions are:

$$\frac{\partial T}{\partial y} = q_w = D \left( \frac{x}{l} \right)^2, \quad y = 0 \quad (5a)$$

$$T \rightarrow T_\infty, \quad \text{as } y \rightarrow \infty \quad (5b)$$

### Analytical solution

A similarity solution exists if we introduce a transformation:

$$u = Bx f'(\eta), \quad v = -\sqrt{\frac{B}{\nu}} f(\eta), \quad \eta = \sqrt{\frac{B}{\nu}} y, \quad (6)$$

where a prime denotes the differentiation with respect to  $\eta$ . Apparently eq. (8) has already satisfied the continuity equation. From eqs. (2) and (10), we have:

$$f'^2 - ff'' = f''' + k(2f' f''' - f'^2 - ff^{IV}) - (k_0 + Mn) f' \quad (7)$$

Now let us seek a solution of eq. (7):

$$f(\eta) = \frac{1}{m} (1 - e^{-m\eta}) \quad (8)$$

where

$$m = \sqrt{\frac{1 + Mn + k_0}{1 + k_1}}$$

is the parameter associated with the viscoelasticity of the second grade fluid, permeability of the porous medium and the contribution of the magnetic field.

This is satisfied by the following boundary conditions:

$$\eta = 0, \quad f = 0 \quad \text{at} \quad f' = 1 \quad (9a)$$

$$\eta \rightarrow \infty, \quad f' \rightarrow 0, \quad f'' \rightarrow 0 \quad (9b)$$

On substituting eq. (8) into eq. (6) and using boundary conditions eqs. (9a) and (9b) the velocity components take the form:

$$u = Bx e^{-m\eta} \quad (10a)$$

$$v = -\frac{\sqrt{B\nu}(1 - e^{-m\eta})}{m} \quad (10b)$$

Defining the dimensionless temperature:

$$\theta(\eta) = \frac{T - T_\infty}{T_s - T_\infty} \quad (11)$$

Using eqs. (8), (10a), (10b), and (11) and the boundary conditions eqs. (5a) and (5b) can be written:

$$\theta'' + \text{Pr} f \theta' + \text{Pr}(\beta - 2f')\theta = -\text{Pr} \text{Ec}[(f'')^2 + kf''(ff'' - ff''')] \quad (12)$$

Introducing the variable:

$$\xi = -re^{-m\eta} \quad \text{with} \quad r = \frac{\text{Pr}}{m^2} \quad (13)$$

Substituting eq. (13) into eq. (12) we find:

$$\xi \frac{d^2\theta}{d\xi^2} + (1-r-\xi) \frac{d\theta}{d\xi} + \left(2 + \frac{\beta r}{\xi}\right) \theta = -\frac{\text{Pr Ec}(1+k)\xi}{r^2} \quad (14)$$

The corresponding boundary conditions become:

$$\theta'(-r) = \frac{-1}{rm}, \quad \theta(0) = 0 \quad (15)$$

The eq. (14) can be transformed into the standard confluent hypergeometric equation or the Kummer's equation [48].

The solution satisfies eq. (14) and eq. (15) is given:

$$\theta(\xi) = \left(\frac{1}{m} + 2H\right) \left[ \left(\frac{\xi}{-r}\right)^{\frac{r+s}{2}} \right] \cdot \frac{M\left(\frac{r+s-4}{2}, s+1, \xi\right)}{\frac{r+s}{2} M\left(\frac{r+s-4}{2}, s+1, -r\right) - \frac{r(r+s-4)}{2(s+1)} M\left(\frac{r+s-4}{2}, s+1, -r\right)} - H \left(\frac{\xi}{-r}\right)^2 \quad (16)$$

where

$$s = r \sqrt{1 - \frac{4\beta}{r}} \quad \text{at} \quad H = \frac{\text{EcPr}(1+k)}{4-2r+\beta r} \quad \text{and} \quad M(a, b, \xi) = 1 + \sum_{n=1}^{\infty} \frac{(a)_n}{(b)_n n!} \xi^n$$

is the Kummer's function.

$$\begin{aligned} (a)_n &= a(a+1)(a+2)\dots(a+n-1) \\ (b)_n &= b(b+1)(b+2)\dots(b+n-1) \end{aligned} \quad (17)$$

The solution (16) can be rewritten, in terms of  $\eta$  as:

$$\theta(\eta) = \left(\frac{1}{m} + 2H\right) e^{\frac{-(r+s)m\eta}{2}} \cdot \frac{M\left(\frac{r+s-4}{2}, s+1, -re^{-m\eta}\right)}{\frac{r+s}{2} M\left(\frac{r+s-4}{2}, s+1, -r\right) - \frac{r(r+s-4)}{2(s+1)} M\left(\frac{r+s-2}{2}, s+2, -r\right)} - He^{-2m\eta} \quad (18)$$

## Second law analysis

According to Wood [49] the local volumetric rate of entropy generation in the presence of a magnetic field and porous medium is given:

$$S_G = \frac{k}{T_0} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{T_0} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{T_0} u^2 + \frac{\mu}{k_1 T_0} (V)^2 \quad (19)$$

The eq. (19) clearly shows contributions of four sources of entropy generation. The first term on the right-hand side of eq. (19) is the entropy generation due to heat transfer; the second term is the local entropy generation due to viscous dissipation, whereas the third term is the local entropy generation due to the effect of the magnetic field and the fourth term is the entropy generation due to the Porous medium.

It is appropriate to define dimensionless number for entropy generation rate  $N_S$ . This number is defined by dividing the local volumetric entropy generation rate  $S_{G_0}$  to a characteristic entropy generation rate  $S_G$  [49].

For prescribed boundary condition, the characteristic entropy generation rate is:

$$S_{G_0} = \frac{k(\Delta T)^2}{l^2 T_0^2} \quad (20)$$

Therefore, the entropy generation number is:

$$N_S = \frac{S_G}{S_{G_0}} \quad (21)$$

Using eqs. (10a), (10b), (18), and (19), the entropy generation number is given:

$$N_S = \frac{4}{\text{Re}_l X^2} \theta^2 + \theta'^2 + \frac{\text{Br Re}_l}{\Omega} f'^2 + \frac{\text{Br}}{\Omega} (\text{Ha}^2 + k_0 \text{Re}_l) f'^2 + \frac{\text{Br } k_0}{\Omega X^2} f^2 \quad (22)$$

## Results and discussion

The flow and heat transfer of an electrically conducting fluid of second grade in a porous medium with prescribed wall heat flux over a stretching sheet subject to a transverse magnetic field has been solved analytically using Kummer's functions after that the velocity and temperature have been used to compute the entropy generation.

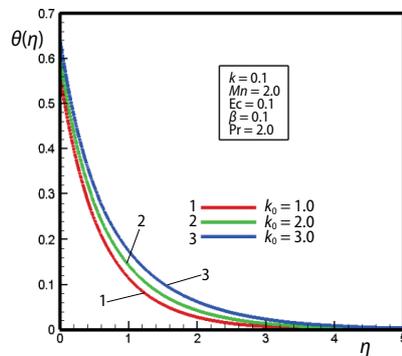
In all the figures of dimensionless temperature profiles plotted we notice that temperature is maximum at the wall where both heat flux and magnetic field are imposed and minimum at the free surface whatever the values of the parameters studied so we conclude that transfer happened in surface extensible. In order to validate our study, the results were compared with those of previous studies as well as the series solution for several values of parameters. It can be seen from tab. 1 that our results agree well with those reported in the literature.

It is clear from figs. 2 and 3 that dimensionless temperature increases slightly with an increase of both medium porous and magnetic parameters this can be explained by Lorentz force which creates by the influence of the vertical magnetic fields of electrically conducting fluid. The decrease of permeability (augmentation of medium porous parameter) increases the friction between the molecule fluid this causes an increase of heat in the boundary-layer.

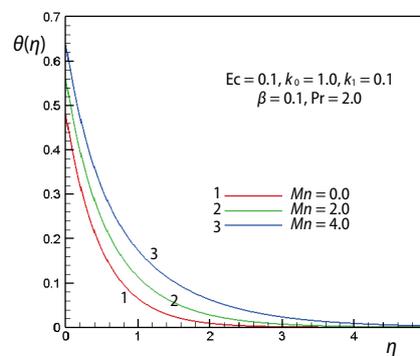
Figure 4 shows the increase of dimensionless temperature profiles as a function of  $\eta$  for different values of the viscoelastic parameter. As it can be noticed, an increase of visco-

**Table 1. Comparison with previously published data for the values of surface temperature**

Pr	M	[50]	[51]	Present study
0.72	0	1.2253	/	1.2367
1	0	1	/	1
10	0	0.2688	/	0.2688
6.7	0	/	0.333303	0.333303
	0.5	/	0.339715	0.339715
	1	/	0.345377	0.345377
	5	/	0.380930	0.380930



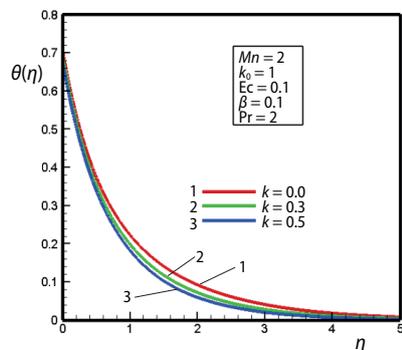
**Figure 2. Effect of the porous medium parameter on the temperature**



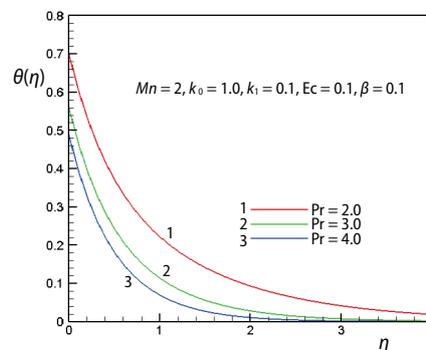
**Figure 3. Effect of the magnetic parameter on the temperature**

elastic parameter produces a reduction in the wall temperature this implies that the increase of viscoelastic parameter increases the flow velocity the contact time fluid-surface decreases this will cause reduction of heat transfer surface-fluid. We notice also from the fig. 5 that an augmentation of the Prandtl number reduces the dimensionless temperature which implies the thermal boundary-layer thickness decreases when the Prandtl number increases it physically means that the flow with large Prandtl presents spreading of heat in the fluid.

Figure 6 demonstrates the effect of the heat source/sink parameter. For a fixed value of  $\eta$ , the dimensionless temperature  $\theta(\eta)$  increases with an increase in heat source/sink. This is due



**Figure 4. Effect of the viscoelastic parameter on the temperature**



**Figure 5. Effect of the Prandtl number on the temperature**

to the increase of the heat generation inside the boundary-layer leading to higher temperature profile.

The fig. 7 presents a variation of dimensionless temperature with Eckert number, we notice that an increase of Eckert number augments the dimensionless temperature in the flow region it confirms that the energy is stocked in the fluid region dissipation due to the viscosity and elastic deformation.

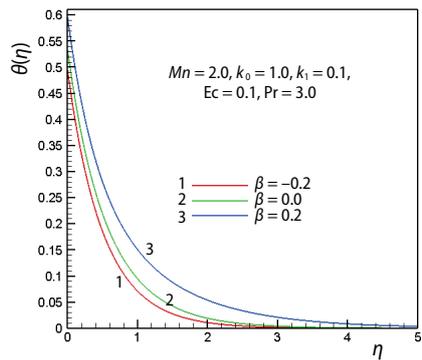


Figure 6. Effect of the heat source/sink parameter on the temperature

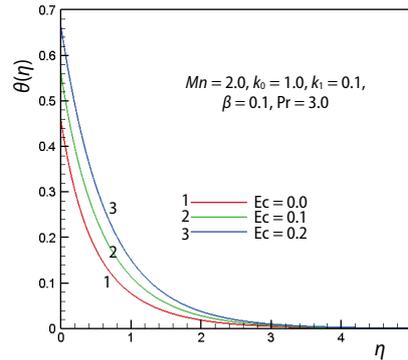


Figure 7. Effect of the Eckert number on the temperature

The results of the entropy generation are presented in function of many parameters such as the viscoelastic, magnetic, heat source/heat sink parameters and dimensionless group, Hartman, Reynolds, Prandtl, Eckert numbers

All the results depict that the entropy decreases with a decrease of mentioned parameters moreover the entropy is higher near the surface where the velocity and magnetic field are in their maximum values also where the heat flux is imposed this means that the surface acts a strong source of irreversibility.

Figure 8 shows the variation of the entropy with magnetic parameter as we can see for fixed value of the entropy increases with an increase in magnetic parameter because the magnetic field creates more entropy, the figure also displays that the increase of magnetic parameter the entropy increases up to certain distance from the sheet after that point the behavior mentioned is reverse. The fig. 9 displays the variation of the entropy with medium porous parameter, the entropo-

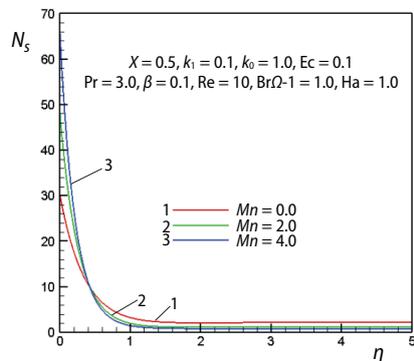


Figure 8. Effect of the magnetic parameter on the entropy generation number

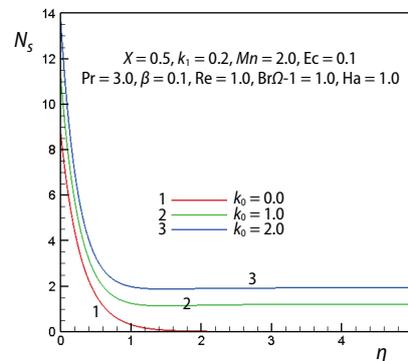
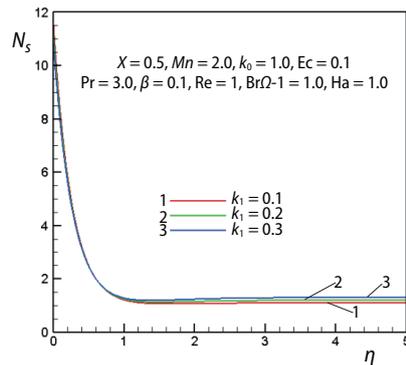


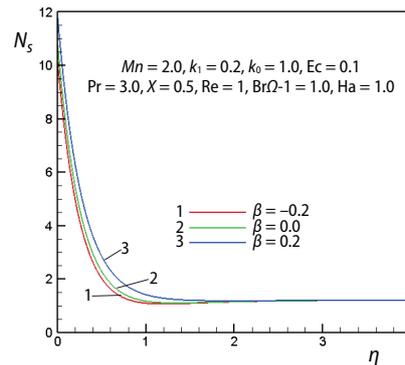
Figure 9. Effect of the porous medium parameter on the entropy generation number

py production number increases with a diminution of permeability of porous medium, whenever the medium is more restrictive the particles of the fluid are more and more messy.

From figs. 10 and 11 we conclude that an augmentation of viscoelastic and heat source/heat sinks parameters has a small effect on the entropy generation.

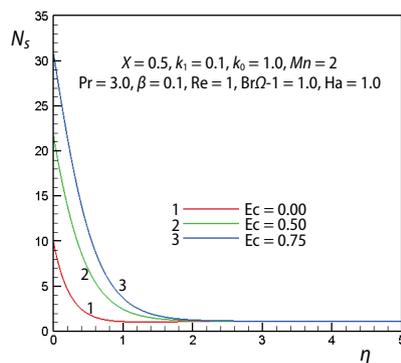


**Figure 10.** Effect of the viscoelastic parameter on the entropy generation number  $k_1$

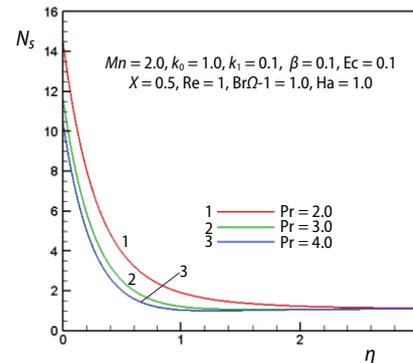


**Figure 11.** Effect of the heat source/heat sink parameter on the entropy generation number

The fig. 12 depicts that the production of the entropy increases with an increase of Eckert number this behavior is attributes to the increase of the viscous dissipation that is a source of entropy generation. The variations in the number of entropy production depending on  $\eta$  for different values of Prandtl number is shown in fig. 13. For a given thickness, the entropy production decreases with the increase of the Prandtl number. This is due to the fact that the temperature decreases with increasing Prandtl number.



**Figure 12.** Effect of the Eckert number on the temperature



**Figure 13.** Effect of the Prandtl number on the entropy generation number

The influence of the Reynolds number on the entropy generation number is plotted in fig. 14. For a given  $\eta$ , the entropy generation number increases as Reynolds number. The augmentation of the Reynolds number increases the contribution of the entropy generation number due to fluid friction. The variation of the entropy generation with different values of characteristic length is shown on the fig. 15, for fixed value an augmentation of characteristic length decreases the entropy generation this behavior can be interpreted by the energy lost inside the fluid flow.

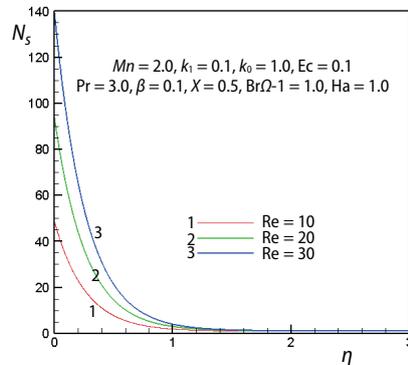


Figure 14. Effect of the  $Re_1$  on the entropy generation number

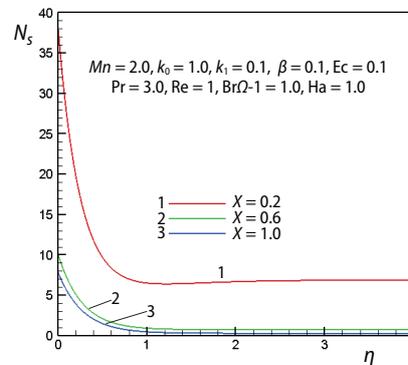


Figure 15. Effect of the characteristic length on the entropy generation number

The effect of the dimensionless group parameter  $Br\Omega-1$  on the entropy generation number is depicted in fig. 16 for a given  $\eta$ , the entropy generation number is higher for higher dimensionless group. This is due to the fact that for higher dimensionless group, the entropy generation numbers due to the fluid friction. The effect of the Hartman number on the entropy generation number is plotted in fig. 17. For a given  $\eta$ , as the Hartman number increases, the entropy generation number increases. The entropy generation number is proportional to the Hartman number which is proportional to the magnetic field. The presence of the magnetic field creates additional entropy.

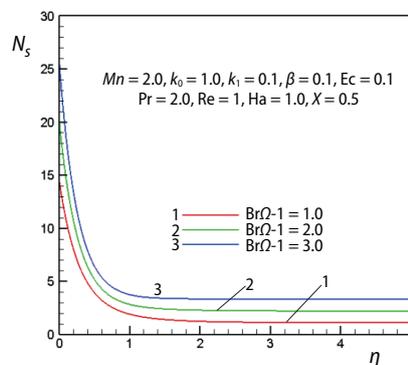


Figure 16. Effect of the dimensionless group on the entropy generation number

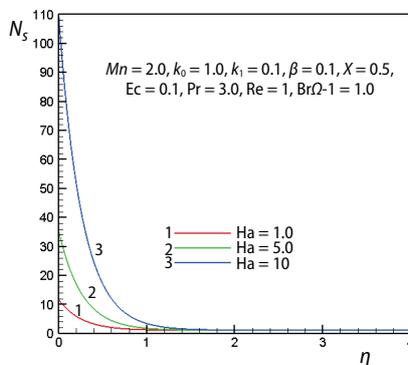


Figure 17. Effect of the Hartmann number on the entropy generation number

## Conclusions

The velocity and temperature profiles are obtained analytically and used to compute the entropy generation number in a viscoelastic fluid over a stretching sheet prescribed wall heat flux subject to a transverse magnetic field. The influences of the Prandtl number, the magnetic parameter and the heat source/sink parameter on the temperature profiles are presented. As far as the entropy generation number is concerned, it is dependent on the magnetic parameter, the Prandtl and Reynolds numbers, the dimensionless group, the Hartman number. From the results the following conclusions could be drawn:

- The temperature increases as the porous medium, the magnetic and heat source sink parameters increase, but it decreases as the viscoelastic parameter and Prandtl number increase.
- The entropy generation number increases as Hartman number, dimensionless group parameter and Reynolds number increase.
- The entropy generation number is slightly influenced by Prandtl number, magnetic parameter.
- The surface acts as a strong source of irreversibility.

### Nomenclature

$A$	– constant, [K]	$T$	– temperature, [K]
$B$	– proportional, constant	$u, v$	– axial and transverse velocity, [ $\text{ms}^{-1}$ ]
$B_0$	– uniform magnetic field strength, [ $\text{Wbm}^{-2}$ ]	$u_l$	– plate velocity based on the characteristic length, [ $\text{ms}^{-1}$ ]
$Br$	– Brinkman number, [ $= (\mu u_l^2)/k\Delta T$ ], [–]	$x, y$	– axial and transverse distance, [m]
$C_p$	– specific heat of the fluid, [ $\text{Jkg}^{-1}\text{K}^{-1}$ ]	$X$	– dimensionless axial distance, ( $= x/l$ )
$D$	– constant, [K]	<i>Greek symbols</i>	
$Ec$	– Eckert number, ( $= B^2 l^2 / AC_p$ ), [–]	$\alpha$	– positive constant
$f$	– dimensionless function, [–]	$\beta$	– heat source/sink parameter, ( $= Q/B\rho C_p$ )
$Ha$	– Hartman number, [ $= B_0 l (\sigma/\mu)^{1/2}$ ], [–]	$\theta$	– dimensionless temperature, [ $= (T - T_\infty)/(T_p - T_\infty)$ ], [–]
$k$	– thermal conductivity of the fluid, [ $\text{Wm}^{-1}\text{K}^{-1}$ ]	$\Omega$	– dimensionless temperature difference, ( $= \Delta T/T_\infty$ ), [–]
$k_0$	– medium porous parameter, ( $= \nu/k_1 B$ )	$\eta$	– dimensionless variable, [–]
$k_1$	– viscoelastic parameter, ( $= \alpha_1 B/\mu$ )	$\mu$	– dynamic viscosity of the fluid, [ $\text{kgm}^{-1}\text{s}^{-1}$ ]
$l$	– characteristic length, [m]	$\nu$	– kinematic viscosity of the fluid, [ $\text{m}^2\text{s}^{-1}$ ]
$M$	– Kummer's function	$\rho$	– density of the fluid, [ $\text{kgm}^{-3}$ ]
$Mn$	– magnetic parameter, [ $= (\sigma B_0^2)/(\rho B)$ ]	$\sigma$	– electric conductivity, [ $\Omega^{-1}\text{m}^{-1}$ ]
$N_S$	– entropy generation number,	$\zeta$	– dimensionless variable, ( $= -re^{-m\eta}$ ), [–]
$Pr$	– Prandtl number, ( $= \mu C_p/k$ ), [–]	<i>Subscript</i>	
$Q$	– rate of internal heat generation or absorption, [ $\text{Wm}^{-3}\text{K}^{-1}$ ]	$\infty$	– far from the sheet
$Re_l$	– Reynolds number based on the characteristic length, ( $= u_l l/\nu$ ), [–]		
$S_G$	– local volumetric rate of entropy generation, [ $\text{Wm}^{-3}\text{K}^{-1}$ ]		

### References

- [1] Abel, M. S., et al., Study of Viscoelastic Fluid Flow and Heat Transfer over Stretching Sheet with Variable Viscosity, *Int. J. Non-Linear Mech.*, 37 (2002), 1, pp. 81-88
- [2] Bhattacharya, S., et al., Heat Transfer in the Flow of a Viscoelastic Fluid over a Stretching Surface, *Heat Mass Transfer*, 34 (1998), 1, pp. 41-45
- [3] Datti, P. S., et al., MHD Viscoelastic Fluid Flow over a Non-Isothermal Stretching Sheet, *Int. J. Eng. Sci.*, 42 (2004), 8-9, pp. 935-946
- [4] Idrees, M. K., Abel, M. S., Viscoelastic Flow Past a Stretching Sheet in Porous Media and Heat Transfer with Internal Heat Source, *Indian J. Theory. Phys.*, 44 (1996), pp. 233-244
- [5] Lawrence, P. S., Rao, B. N., Heat Transfer in the Flow of Viscoelastic Fluid over Stretching Sheet, *Acta Mech.*, 93 (1992), 1-4, pp. 53-61
- [6] Prasad, K. V., et al., Momentum and Heat Transfer in Viscoelastic Fluid Flow in a Porous Medium over a Non-Isothermal Stretching Sheet, *Int. J. Numer. Method Heat Flow*, 10 (2000), 8, pp. 786-801
- [7] Prasad, K. V., et al., Non-Darcy Forced Convective Heat Transfer in a Viscoelastic Fluid Flow over a Non-Isothermal Stretching Sheet, *J. Porous Media*, 5 (2002), 1, pp. 41-47
- [8] Khan, S. K., Sanjayanand, E., Viscoelastic Boundary Layer Flow and Heat Transfer over an Exponential Stretching Sheet, *Int. J. Heat Mass Transfer*, 48 (2005), 8, pp. 1534-1542
- [9] Cortell, R., A Note on Flow and Heat Transfer of a Viscoelastic Fluid over a Stretching Sheet, *Int. J. Non-Linear Mech.*, 41 (2006), 1, pp. 78-85
- [10] Abel, M. S., et al., Heat Transfer in a Viscoelastic Boundary Layer Low over a Stretching Sheet with Viscous Dissipation and Non-Uniform Heat Source, *Int. J. Heat Mass Transfer*, 50 (2007), 5-6, pp. 960-966

- [11] Liu, I.-C., Flow and Heat Transfer of an Electrically Conducting Fluid of Second Grade in a Porous Medium over a Stretching Sheet Subject to a Transverse Magnetic Field, *Int. J. Non-Linear Mech.*, 40 (2005), 4, pp. 465-474
- [12] Khan, S. K., Heat Transfer in a Viscoelastic Fluid over a Stretching Surface with Source/Sink, Suction/Blowing and Radiation, *Int. J. Heat Mass Transfer*, 49 (2006), 3-4, pp. 628-639
- [13] Abel, M. S., et al., Viscoelastic MHD Flow and Heat Transfer over a Stretching Sheet with Viscous and Ohmic Dissipation, *Comm. Nonlinear. Sci. and Num. Simulation*, 13 (2008), 9, pp. 1808-1821
- [14] Hsiao, K. L., Conjugate Heat Transfer of Magnetic Mixed Convection with Viscous Dissipation Effects for Second-Grade Viscoelastic Fluid Past a Stretching Sheet, *Appl. Therm. Eng.*, 27 (2007), 11, pp. 1895-1903
- [15] Abbas, Z., et al., Hydromagnetic Flow of a Viscoelastic Fluid Due to the Oscillatory Stretching Surface, *Int. J. Non-Linear Mech.*, 43 (2008), 8, pp. 783-793
- [16] Singh, A. K., Heat Source and Radiation Effects on Magneto-Convection Flow of a Viscoelastic Fluid Past a Stretching Sheet: Analysis with Kummer's Functions, *Int. Comm. Heat Mass Transfer*, 35 (2008), 5, pp. 637-642
- [17] Prasad, K.V., et al., The Effect of Variable Viscosity on MHD Viscoelastic Fluid Flow and Heat Transfer over a Stretching Sheet, *Comm. Nonlinear Sci. and Num. Simulation*, 15 (2010), 2, pp. 331-344
- [18] Abel, M. S., Mahesha, N., Heat Transfer in MHD Viscoelastic Fluid over a Stretching Sheet with Variable Thermal Conductivity, Non-Uniform Heat Source and Radiation, *Appl. Math. Modelling*, 32 (2008), 10, pp. 1965-1983
- [19] Hayat, T., et al., Three-Dimensional Flow over a Stretching Sheet in a Viscoelastic Fluid, *Nonlinear. Ana. Real World Appl.*, 9 (2008), 4, pp. 1811-1822
- [20] Misra, J. C., Shit, G. C., Biomagnetic Viscoelastic Fluid Flow over a Stretching Sheet, *Appl. Math. And Compu.*, 210 (2009), 2, pp. 350-361
- [21] Abel, M. S., Nandeppanavar, M. M., Heat Transfer in MHD Viscoelastic Boundary Layer Flow over a Stretching Sheet with Non-Uniform Heat Source/Sink, *Comm. NonLinear Sci. and Num. Simu.*, 14 (2009), 5, pp. 2120-2131
- [22] Nandeppanavar, M. M., et al., Flow and Heat Transfer Characteristics of a Viscoelastic Fluid in a Porous Medium over an Impermeable Stretching Sheet with Viscous Dissipation, *Int. J. Heat Mass Transfer*, 53 (2010), 21-22, pp. 4707-4713
- [23] Chen, C. H., On the Analytic Solution of MHD Flow and Heat Transfer for two Types of Viscoelastic Fluid over a Stretching Sheet with Energy Dissipation Internal Heat Source and Thermal Radiation, *Int. J. Heat Mass Transfer*, 53 (2010), 19-20, pp. 4264-4273
- [24] Nandeppanavar, M. M., et al., Heat Transfer in MHD Viscoelastic Boundary Layer Flow over a Stretching Sheet with Thermal Radiation and Non-Uniform Heat Source/Sink, *Comm. Nonlinear. Sci. and Num. Simulation*, 16 (2011), 9, pp. 3578-3590
- [25] Cortell, R., Flow and Heat Transfer of a Fluid through a Porous Medium over a Stretching Surface with Internal Heat Generation/Absorption and Suction/Blowing, *Fluid Dyn Res*, 37 (2005), 4, pp. 231-245
- [26] Rashidi, M. M., et al., Free Convective Heat and Mass Transfer for MHD Fluid Flow over a Permeable Vertical Stretching Sheet in the Presence of the Radiation and Buoyancy Effects, *Ain Shams Eng. J.*, 5 (2014), 3, pp. 901-912
- [27] Rashidi, M. M., et al., Buoyancy Effect on MHD Flow of Nanofluid over a Stretching Sheet in the Presence of Thermal Radiation, *J. Mol. Liq.*, 198 (2014), Oct., pp. 234-238
- [28] Bejan, A., A Study of Entropy Generation in Fundamental Convective Heat Transfer, *J. Heat Transfer*, 101 (1979), 4, pp. 718-725
- [29] Bejan, A., Second-Law Analysis in Heat Transfer and Thermal Design, *Adv. Heat Transfer*, 15 (1982), Dec., pp.1-58
- [30] Bejan, A., *Entropy Generation Minimization*, CRC Press, Boca Raton, Fla., USA, 1996
- [31] Sahin, A. Z., Second Law Analysis of Laminar Viscous Flow through a Duct Subjected to Constant Wall Temperature, *J. Heat Transfer*, 120 (1998), 1, pp. 76-83
- [32] Sahin, A. Z., The Effect of Variable Viscosity on the Entropy Generation and Pumping Power in a Laminar Fluid Flow through a Duct Subjected to Constant Heat Flux, *Heat Mass Transfer*, 35 (1999), 6, pp. 499-506
- [33] Sahin, A. Z., A Second Law Comparison for Optimum Shape of Duct Subjected to Constant Wall Temperature and Laminar Flow, *Heat Mass Transfer*, 33 (1998), 5-6, pp.425-430
- [34] Narusawa, U., The Second-Law Analysis of Mixed Convection in Rectangular Ducts, *Heat Mass Transfer*, 37 (1998), 2-3, pp. 197-203

- [35] Mahmud, S., Fraser, R. A., Thermodynamic Analysis of Flow and Heat Transfer Inside Channel with Two Parallel Plates, *Exergy*, 2 (2002), 3, pp. 140-146
- [36] Mahmud, S., Fraser, R. A., Inherent Irreversibility of Channel and Pipe Flows for Non-Newtonian Fluids, *Int. Comm. Heat Mass Transfer*, 29 (2002), 5, pp. 577-587
- [37] Mahmud, S., Fraser, R. A., The Second Law Analysis in Fundamental Convective Heat Transfer Problems, *Int. J. Therm. Sci.*, 42 (2003), 2, pp. 177-186
- [38] Saouli, S., Aiboud-Saouli, S., Second Law Analysis of Laminar Falling Liquid Film Along an Inclined Heated Plate, *Int. Comm. Heat Mass Transfer*, 31 (2004), 6, pp. 879-886
- [39] Aiboud-Saouli, S., et al., Thermodynamic Analysis of Gravity-Driven Liquid Film Along an Inclined Heated Plate with Hydromagnetic and Viscous Dissipation Effects, *Entropy*, 8 (2006), 4, pp. 188-199
- [40] Aiboud-Saouli, S., et al., Second-Law Analysis of Laminar Fluid Flow in a Heated Channel with Hydromagnetic and Viscous Dissipation Effects, *Applied Energy*, 84 (2007), 3, pp. 279-289
- [41] Aiboud, S., Saouli, S., Second Law Analysis of Viscoelastic Fluid over a Stretching Sheet Subject to a Transverse Magnetic Field with Heat and Mass Transfer, *Entropy*, 12 (2010), 8, pp. 1867-1884
- [42] Aiboud, S., Saouli, S., Entropy Analysis for Viscoelastic Magnetohydrodynamics Flow over a Stretching Surface, *Int. J. Non-Linear Mech.*, 45 (2010), 5, pp. 482-489
- [43] Adesanya, S. O., Makinde, O. D., Irreversibility Analysis in a Couple Stress Film Flow Along an Inclined Heated Plate with Adiabatic Free Surface, *Physica A*, 5 (2015), Feb., pp. 115-124
- [44] Akbar, N. S., Entropy Generation and Energy Conversion Rate for the Peristaltic Flow in a Tube with Magnetic Field, *Energy*, 82 (2015), Mar., pp. 23-30
- [45] Makinde, O. D., Entropy Analysis for MHD Boundary Layer Flow and Heat Transfer over a Flat Plate with a Convective Surface Boundary Condition, *Int J Exergy*, 10 (2012), 2, pp. 142-154
- [46] Abolbashari, M. H., et al., Entropy Analysis for an Unsteady MHD Flow Past a Stretching Permeable Surface in Nano-Fluid, *Powder Technol.*, 267 (2014), Nov., pp. 256-267
- [47] Tripathy, R. S., et al., Chemical Reaction Effect on MHD Free Convective Surface over a Moving Vertical Plane through Porous Medium, *Alexandria Eng. J.*, 54 (2015), 3, pp. 673-679
- [48] Chiam, T. C., Magnetohydrodynamic Heat Transfer over a Non-Isothermal Stretching Sheet, *Acta Mechanica*, 122 (1997), 1-4, pp. 169-179
- [49] Woods, L. C., *Thermodynamics of Fluid Systems*, Oxford University Press, Oxford, UK, 1975
- [50] Elbashbeshy, E. M. A., Heat Transfer over a Stretching Surface with Variable Surface Heat Flux, *J. Phys. D Appl. Phys.*, 31 (1998), 16, pp. 1951-1954
- [51] Liu, I.-C., A note on Heat and Mass Transfer for a Hydromagnetic Flow over a Stretching Sheet. *Int. Comm. Heat Mass Transf.*, 32 (2005), 8, pp. 1075-1084