THE INFLUENCE OF THERMODYNAMIC STATE OF MINERAL HYDRAULIC OIL ON FLOW RATE THROUGH RADIAL CLEARANCE AT ZERO OVERLAP INSIDE THE HYDRAULIC COMPONENTS

by

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Original scientific paper
DOI: 10.2298/TSCI16S5461K

In control hydraulic components (servo valves, LS regulators, etc.) there is a need for precise mathematical description of fluid flow through radial clearances between the control piston and body of component at zero overlap, small valve opening and small lengths of overlap. Such a mathematical description would allow for a better dynamic analysis and stability analysis of hydraulic systems. The existing formulas in the literature do not take into account the change of the physical properties of the fluid with a change of thermodynamic state of the fluid to determine the flow rate through radial clearances in hydraulic components at zero overlap, a small opening, and a small overlap lengths, which leads to the formation of insufficiently precise mathematical models. In this paper model description of fluid flow through radial clearances at zero overlap is developed, taking into account the changes of physical properties of hydraulic fluid as a function of pressure and temperature. In addition, the experimental verification of the mathematical model is performed.

Key words: *radial clearances, discharge coefficient, density, viscosity, thermodynamic change of state of the fluid*

Introductory remarks on fluid flow through small orifices

The equation for flow rate through the orifices (fig. 1) is derived from the Bernoulli's equation and continuity equation, assuming that the fluid is incompressible and turbulent flow [1]:

$$Q = C_\mathrm{d} A \sqrt{\frac{2(p_D - p_m)}{\rho}} \qquad (1)$$

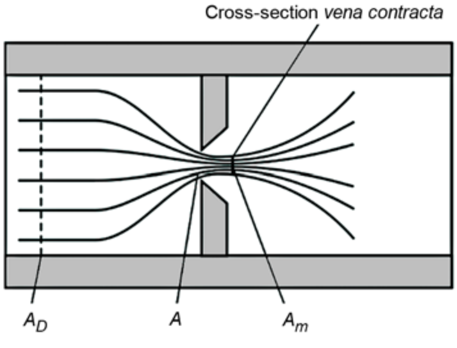

Figure 1. Fluid flow through a sharp-edged orifice

where C_d is the discharge coefficient, A – the orifice cross-sectional area, p_D – the pressure at upstream, p_m – the pressure at *vena contracta*, and ρ – the density of fluid.

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The coefficient C_d in eq. (1) is defined by:

$$C_\mathrm{d} = \frac{C_\mathrm{v} C_\mathrm{c}}{\sqrt{1-\left(\dfrac{A_m}{A_D}\right)^2}} = \frac{C_\mathrm{v} C_\mathrm{c}}{\sqrt{1-C_\mathrm{c}^2\left(\dfrac{A}{A_D}\right)^2}} \tag{2}$$

where $C_\mathrm{v} = v/[2(p_D - p_m)/\rho]^{1/2}$ is the flow velocity coefficient, v – the velocity of fluid in the cross-section *vena contracta*, $C_\mathrm{c} = A_m/A$ – the area contraction coefficient, A_D – the cross--sectional area at upstream, and A_m – the cross-sectional area at *vena contracta*.

As A_D is much higher than A, it can be said that the discharge coefficient C_d in eq. 2 is almost equal to the product of velocity coefficient C_v and contraction coefficient C_c, eq. (3)

$$C_\mathrm{d} = C_\mathrm{v}\, C_\mathrm{c} \tag{3}$$

The discharge coefficient curve in the function of square root at fluid flow through a sharp-edged orifice is often graphically presented in [1]. At fluid flow through radial clearances models for numerical calculation of the discharge coefficient curve are developed that do not associate the discharge coefficient and the Reynolds number [2].

Wu *et al.* [3] determined the discharge coefficient at fluid flow through radial clearances, C_d, with respect to square root of Reynolds number. They developed a calculation method for the flow rate. When calculating discharge, the thermodynamic change of the state of a hydraulic fluid was not taken into account. Such an approach leads to an error in the region of laminar flow (this being the most common case of flow considering the sizes of radial clearances in hydraulic components), because under common pressure and temperature in hydraulic systems the value of kinematic viscosity coefficient significantly changes, and so does the Reynolds number.

This paper provides an empirical discharge coefficient model of flow rate through radial clearances while considering the change of viscosity of hydraulic oil at fluid flow in the function of the change of temperature and pressure, whereas the change of density in the function of change of temperature and pressure is taken into account when calculating discharge.

Model to determine the discharge coefficient through the radial clearance at zero overlap

From the literature, it is well known that there is a transition in a plot of discharge coefficient in the function of $\mathrm{Re}^{1/2}$ from being proportional to the square root of the Reynolds number (at low Reynolds number), to being constant at high Reynolds number. Although the curve shapes vary as the orifice geometry varies, they can be approximated by an empirical model as an exponential function [3]:

$$C_\mathrm{d} = C_{\mathrm{d}\infty}\left(1 + a\mathrm{e}^{-\frac{\delta_1}{C_{\mathrm{d}\infty}}\sqrt{\mathrm{Re}}} + b\mathrm{e}^{-\frac{\delta_2}{C_{\mathrm{d}\infty}}\sqrt{\mathrm{Re}}}\right) \tag{4}$$

where the parameters $C_{\mathrm{d}\infty}$ (the turbulent discharge coefficient), a, b, δ_1, and δ_2 are specific flow dependent coefficients to be determined. Equation (4) can be applied to most types of orifices.

The method of the experimental determination of the discharge coefficient C_d, and the corresponding Reynolds number, for flow through radial clearances at zero overlap (fig. 2) is based on the following general flow equations:

$$C_\mathrm{d} = \frac{Q}{Q_\mathrm{t\,max}} = \frac{Q}{A\sqrt{\dfrac{2\Delta p}{\rho}}} \tag{5}$$

$$\mathrm{Re} = \frac{\rho \dfrac{Q}{A} D_\mathrm{h}}{\mu} \tag{6}$$

where Q is the actual flow rate through the clearance, $Q_\mathrm{t\,max}$ – the maximal theoretical flow rate, Δp – the pressure drop through the orifice, D_h – the hydraulic diameter, and μ – the dynamic viscosity of fluid.

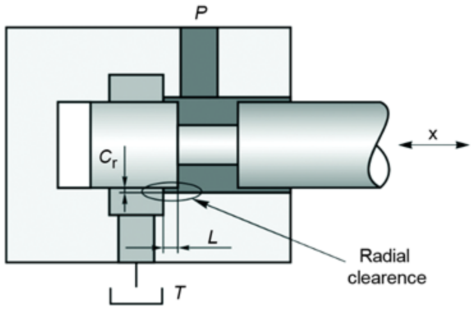

Figure 2. Fluid flow through radial clearance

At zero overlap, the length of the overlap is $L = 0$.

The identification of unknown parameters of the mathematical model

The function describing the change of the discharge coefficient as a function of the root of the Reynolds number is given by eq. (4).

Based on the previous knowledge of problems of fluid flow through the orifices, $C_{\mathrm{d}\infty}$ coefficient can be determined experimentally by performing a larger number of experiments at high values of Reynolds numbers (since the discharge coefficient is constant in turbulent flow). With $C_{\mathrm{d}\infty}$ known, it remains to determine the remaining four unknown parameters of the mathematical model a, b, δ_1, and δ_2, using the method of least squares.

The root of the Reynolds number is the independent variable.

The difference between model predictions and values obtained by measurements can be described by the function:

$$\frac{1}{2}\sum_{j=1}^{m}\left[C_{\mathrm{d}j} - C_{\mathrm{d}\infty}\left(1 + a\mathrm{e}^{-\frac{\delta_1}{C_{\mathrm{d}\infty}}\sqrt{\mathrm{Re}_j}} + b\mathrm{e}^{-\frac{\delta_2}{C_{\mathrm{d}\infty}}\sqrt{\mathrm{Re}_j}}\right)\right]^2 \tag{7}$$

where $C_{\mathrm{d}j}$ is the experimental discharge coefficient at point Reynolds number, Re_j.

Graphic, every member in the eq. (7), represents the square of the vertical distance between the curve C_d (a, b, δ_1, δ_2; $\mathrm{Re}^{1/2}$) (plotted as a function of $\mathrm{Re}^{1/2}$), and point ($\mathrm{Re}_j^{1/2}$, $C_{\mathrm{d}j}$), as can be seen in fig. 3.

The parameters a, b, δ_1, and δ_2 are chosen such that the eq. (7) has a minimum value.

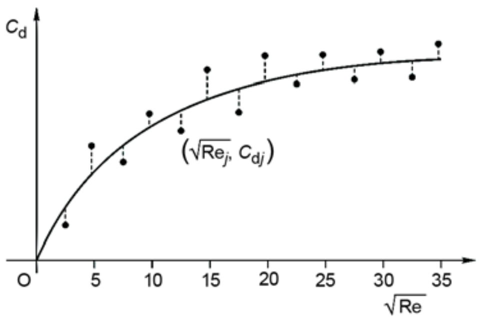

Figure 3. Deviations between models (smooth curve) and measured values (indicated by vertical dashed lines)

The influence of thermodynamic state on the value of density and viscosity of hydraulic oil

The most important physical properties of hydraulic oil, which have fundamental importance for the study of efficiency and dynamic behavior of hydraulic components and systems, are density and viscosity.

Mineral base oils for hydraulic fluids are normally composed of complex hydrocarbon molecules. According to dominant presence of specific hydrocarbons in crude oil, mineral oils are divided as follows: paraffinic, naphthenic and mixed oils. Hydraulic oils, are almost entirely based on highly refined paraffinic oils. Naphthenic oils are very rarely used as hydraulic oils, due to reduced availability [4].

As paraffinic mineral oils are the most widely spread hydraulic oils (about 90% used hydraulic fluids), the following analysis of influence of temperature and pressure on density and dynamic viscosity, is applied on this type of oil.

Density of mineral hydraulic oils

Thermal equation of state for liquids:

$$\rho = \rho(p, T) \tag{8}$$

has no direct mathematical derivation from physical laws (as opposed to gases):

$$\mathrm{d}\rho = \left(\frac{\partial \rho}{\partial T}\right)_p \mathrm{d}T + \left(\frac{\partial \rho}{\partial p}\right)_T \mathrm{d}p \tag{9}$$

Experimental data for the value of density with changing pressure and temperature of a mineral hydraulic oil HM 46 is given in tab. 1.

Table 1. Density of mineral hidraulic oil HM 46 – experimental data

Absolute pressure p [bar]	Temperature [°C]									
	15	20	30	40	50	60	70	80	90	100
1	877.6	874.5	868.3	862.1	856	849.8	843.6	837.4	831.2	825
101	882.6	879.7	873.7	867.8	861.8	855.8	849.9	843.9	838	832
201	887.5	884.6	878.8	873.1	867.3	861.6	855.9	850.1	844.4	838.6
301	892.1	889.3	883.7	878.2	872.6	867.1	861.5	856	850.4	844.9
401	896.5	893.8	888.4	883	877.7	872.3	866.9	861.5	856.1	850.8
501	900.7	898.1	892.9	887.7	882.5	877.2	872.0	866.8	861.6	856.4

The experimental data in tab. 1 and fig. 4 shows that, for practical use, change of the density of mineral oils as a function of pressure and temperature (for common values operating pressure and temperature in hydraulic systems) can be described by a linear function.

Density of mineral oil, at specific temperature and atmospheric pressure, can be calculated by using experimentally measured value of density at the temperature of 15 °C and volume-temperature expansion coefficient, α_p, for the same temperature, using following equation [4]

$$\rho = \rho_0 - \rho_0 \alpha_{p0}(T - 15) = \rho_0[1 - 0.0007(T - 15)] \tag{10}$$

Temperature dependence of viscosity

The viscosity of hydraulic and lubricating oil is extremely sensitive to the operating temperature. With increasing temperature the viscosity of oils falls rapidly.

The oil viscosity at a specific temperature can be either calculated from the viscosity-temperature equations or obtained from the viscosity-temperature ASTM chart.

There are several viscosity-temperature equations. Some of them are purely empirical whereas others are derived from theoretical models. The most commonly used equations are given in tab. 2 [5].

Among them the most accurate is the Vogel equation. Three viscosity measurements at different temperatures for specific oil are needed in order to determine the three constants in this equation, tab. 3.

Apart from being very accurate the Vogel equation is useful in numerical analysis [5].

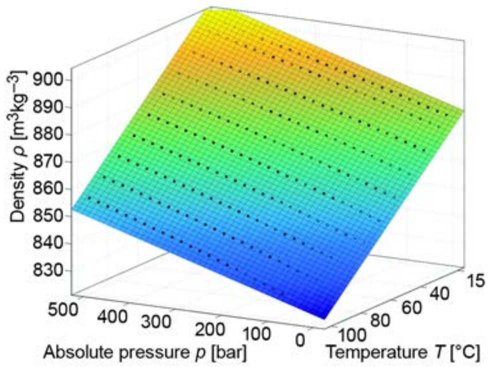

Figure 4. Change of density of mineral hydraulic oil HM 46 as a function of temperature and pressure *(for color image see journal web-site)*

Table 2. Viscosity-temperature equations (*a*, *b*, *c*, *d* – are constants)

Name	Equation	Comments
Reynolds	$\mu_0 = be^{-aT_\mathrm{A}}$	Early equation; accurate only for a very limited temperature range
Slotte	$\mu_0 = \dfrac{a}{(b+T_\mathrm{A})^c}$	Reasonable; useful in numerical analysis
Walther	$(\nu_0 + a) = bd^{1/T_\mathrm{A}^c}$	Forms the basis of the ASTM viscosity-temperature chart
Vogel	$\mu_0 = ae^{b/(T_\mathrm{A}-c)}$	Most accurate; very useful in engineering calculations

Table 3. Kinematic viscosity of testing hydraulic oil, at the three different temperatures and atmospheric pressure, and density of oil at the temperature of 15 °C [4]

	HM 32	HM 46	HM 68	HVL 46
ρ(15 °C) [gcm^{-3}]	0.879	0.883	0.887	0.879
ν_0(20.5 °C) [mm^{2}s^{-1}]	80.13	146.92	217.37	123.88
ν_0(40 °C) [mm^{2}s^{-1}]	30.32	48.5	71.53	47.26
ν_0(100 °C) [mm^{2}s^{-1}]	5.24	6.89	8.8	8.24
IV	101	96	94	149

For known values of kinematic viscosity and density, at specific temperature, value of dynamic viscosity is given by the equation:

$$\mu_0 = \nu_0 \rho \tag{11}$$

Based on data from tab. 4, constants *a*, *b*, and *c*, of testing hydraulic oil, from Vogel equation, are calculated and given in tab. 5.

Table 4. Dynamic viscosity of testing hydraulic oil, at the three different temperatures and atmospheric pressure, calculated by using eq. (11) and tab. 3

	HM 32	HM 46	HM 68	HVL 46
$\mu_{0(20.5\,°\mathrm{C})}\cdot 10^2$ [Pa·s]	7.016	12.57	19.28	10.8469
$\mu_{0(40\,°\mathrm{C})}\cdot 10^2$ [Pa·s]	2.62	4.21	6.234	4.0814
$\mu_{0(100\,°\mathrm{C})}\cdot 10^2$ [Pa·s]	0.433	0.572	0.734	0.6812

Table 5. Constants from Vogel equation for testing hydraulic oil [4]

	HM 32	HM 46	HM 68	HVL 46
a	0.0000736317	0.0000633361	0.0000389689	0.000116198
b	797.7122	879.7742	1083.913	799.7249
c	177.3562	177.7865	166.2304	176.7128

Pressure dependence of viscosity

Viscosity of oil increases along pressure. Chemical composition greatly influences the viscosity-pressure characteristic of a hydraulic fluid.

The best known equation, which describes viscosity-pressure behavior of hydraulic fluids, is Barus equation [6]:

$$\mu = \mu_0 \mathrm{e}^{\alpha p} \tag{12}$$

To adopt experimental data by a mathematical model, the so-called *modulus equation* was used. Modulus equation is based on eq. (12). The model comprises the pressure, p [bar], and temperature, T [°C], dependence on the dynamic viscosity [3] and is given by:

$$\mu(p, T) = \mu_0 \mathrm{e}^{\left[\frac{p}{a_1 + a_2 T + (b_1 + b_2 T)p}\right]} \tag{13}$$

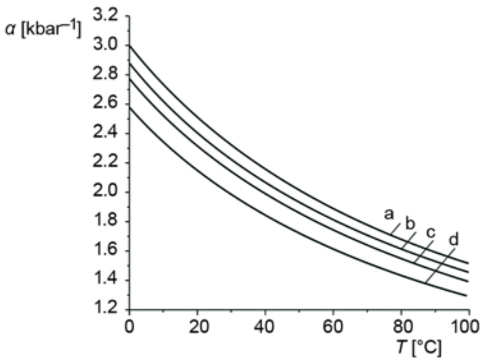

Figure 5. Pressure-viscosity coefficient of hydraulic oil of paraffinic base structure; (a) $p = p_a$, (b) p = 500 bar, (c) p = 1000 bar, and (d) p = 2000 bar

Dependence pressure-viscosity coefficient, α, of pressure and temperature is given by following equation and fig. 5 [4]:

$$\alpha(p, T) = \frac{\ln \mu - \ln \mu_0}{p - p_a} = \frac{1}{a_1 + a_2 T + (b_1 + b_2 T)p} \tag{14}$$

The parameters a_1, a_2, b_1, and b_2 represent the oil behavior and have to be calculated from experimental data. In accordance with the data given by the hydraulic oil producers [3], constants from eq. (14) are calculated by using the method for identifying unknown parameters of the mathematical model.

Dynamic viscosity as a function of temperature and pressure for mineral hydraulic oils

After replacing μ_0 in eq. (12), by Vogel equation, eq. (15), to describe temperature dependence at atmospheric pressure:

$$\mu(T) = \mu_0 = a\mathrm{e}^{\left[\frac{b}{(T+273.15)-c}\right]} \tag{15}$$

an equation is obtained with seven unknown parameters a, b, c, a_1, a_2, b_1, and b_2:

$$\mu(p, T) = a\mathrm{e}^{\left[\frac{b}{(T+273.15)-c}\right]}\mathrm{e}^{\left[\frac{p}{a_1+a_2T+(b_1+b_2T)p}\right]}. \tag{16}$$

The pressures, in hydraulic systems, are usually smaller than 400 [bar], and it results with $a_1 + a_2T \gg (b_1 + b_2T)p$. Thus, eq. (16) can be simplified and the number of unknown parameters can be reduced to five:

$$\mu(p, T) = a\mathrm{e}^{\left[\frac{b}{(T+273.15)-c}\right]}\mathrm{e}^{\left[\frac{p}{a_1+a_2T}\right]} \tag{17}$$

Therefore, for mineral hydraulic oils of paraffinic base structure, pressure viscosity coefficient, α, can be calculated by using the equation:

$$\alpha(T) = \frac{1}{a_1 + a_2T} = \frac{1}{334 + 3.2557\,T} \quad [\mathrm{bar}^{-1}] \tag{18}$$

Values of dynamic viscosity are calculated by using the data from tabs. 5 and 6, and the chart is given for mineral oil HM 46 (fig. 6).

Table 6. Parameter values for pressure-viscosity coefficient, α [4]

	a_1 [bar]	a_2 [bar°C^{-1}]	b_1 [–]	b_2 [°C^{-1}]
Hydraulic oil of paraffinic base structure	334	3.2557	0.026266	0.000315

The demand for highly efficient hydraulic systems is permanently increasing. Because of that, the need for accurate mathematical modeling of fluid behavior is increasing as well.

In this paper, a mathematical model for calculating dynamic viscosity is given for mineral hydraulic oils, as a function of temperature and pressure. It is shown that neglecting the influence of working pressure can lead to significant errors in calculation of the dynamic viscosity of hydraulic oils value. The value of mistake is increasing with the growth of pressure and the decrease of temperature.

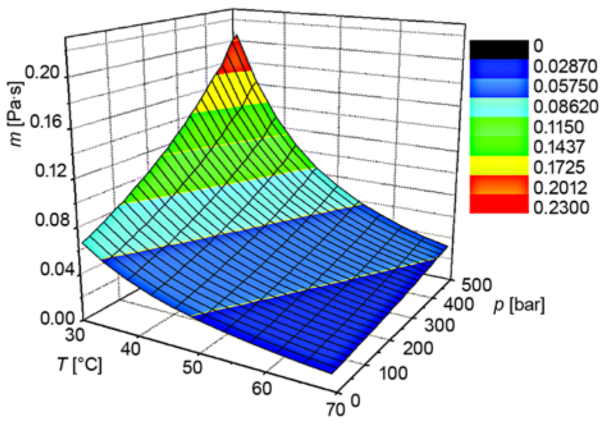

Figure 6. Change of dynamic viscosity of mineral hydraulic oil HM 46 as a function of temperature and pressure *(for color image see journal web-site)*

Equations (16), or (17), are very useful for the analysis of flow of hydraulic oil through clearances into hydraulic components, especially in hydraulic components of automation control (servo valve, LS compensator, *etc.*).

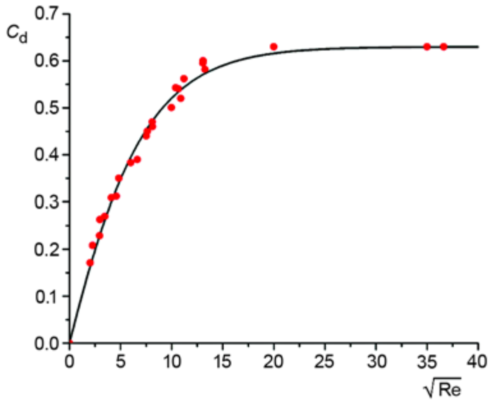

Figure 7. Discharge coefficient for the radial clearance at zero overlap

The parameters in eqs. (16), or (17), represent fluid behavior and have to be calculated from experimental data for each one of the oils used separately. However, for engineering applications and ISO viscosity gradations and types (HM and HV), previously calculated values of parameters can be used with very satisfactory accuracy.

Zero overlap – experimental determination of the discharge coefficient

For different types of hydraulic oil and different operating conditions (pressure, temperature), discharge coefficient and the corresponding Reynolds number are experimentally determined. Each point in fig. 7 is obtained as the mean value of 2 to 4 measurements under the same operating conditions.

The experimental data, shown in fig. 7, are given in the tab. 7.

Table 7. Experimentally determined values of discharge coefficient at zero overlap

$\sqrt{Re}$	0	2.036	2.2757	2.941	2.981	3.462	4.113	4.579	4.834
C_d	0	0.171	0.2082	0.229	0.263	0.27	0.3087	0.312	0.35
$\sqrt{Re}$	6	6.63	7.51	7.621	8.11	8.12	9.98	10.4	10.67
C_d	0.383	0.39	0.44	0.45	0.47	0.46	0.5	0.5426	0.5404
$\sqrt{Re}$	10.886	11.2	13.259	13.0513	13.0772	20	35	36.93	
C_d	0.52	0.5619	0.582	0.5951	0.6	0.63	0.63	0.63	

Zero overlap

Section *Model to determine the discharge…* presents a method for identification of unknown parameters of the mathematical model, which determines the discharge coefficient, C_d, through the radial clearance at zero overlap.

The function describing the change of discharge coefficient as a function of the root Reynolds number is given by eq. (4).

Parameter $C_{d\infty}$ represents the value of discharge coefficient in a turbulent area of flow. Its value has been experimentally determined (by performing experiments with water, at high values of Reynolds number), and it is 0.63 (tab. 7). With the known value $C_{d\infty}$, eq. (4) can be simplified and written in the form:

$$C_d = 0.63\left(1 + ae^{-\frac{\delta_1}{C_{d\infty}}\sqrt{Re}} + be^{-\frac{\delta_2}{C_{d\infty}}\sqrt{Re}}\right) \tag{19}$$

It is still necessary to determine the remaining four unknown parameters in eq. (19) a, b, δ_1, and δ_2.

Table 7 provides experimental data to determine the discharge coefficient as a function of the Reynolds number.

Each set of experimental data is obtained as the average value of 2 to 4 measurements.

By applying the method of Gauss-Newton the parameters of the mathematical model are obtained, as presented in tab. 8.

Table 8. The parameters of the mathematical model for discharge coefficient through the radial clearances at zero overlap

a	b	δ_1	δ_2
–2	1	0,1388	0,1917

By using these parameters, an empirical model for the discharge coefficient eq. (19) takes the form:

$$C_\mathrm{d} = 0.63\left(1-2\mathrm{e}^{-0.22\sqrt{\mathrm{Re}}}+\mathrm{e}^{-0.304\sqrt{\mathrm{Re}}}\right) \tag{20}$$

The curve of the discharge coefficient, graph of eq. (20) is shown in fig. 7.

Determination of flow rate through the radial clearance at zero overlap

Using eq. (20), or curve of the discharge coefficient (fig. 7) for calculation of flow rate of hydraulic fluid through radial clearances at zero overlap, an iterative process (application of the method of approximation) is required. The reason for this is that the flow rate Q depends on the discharge coefficient, C_d (which is a function of the Reynolds number), and the Reynolds number depends on the flow rate.

By using eq. (20), for the flow of hydraulic fluid through radial clearances at zero overlap, the equation for flow rate through orifices eq. (5), may be written in the form:

$$Q = 0.63\left(1-2\mathrm{e}^{-0.22\sqrt{\mathrm{Re}}}+\mathrm{e}^{-0.304\sqrt{\mathrm{Re}}}\right) d\pi c_\mathrm{r}\sqrt{\frac{2\Delta p}{\rho}} \tag{21}$$

If the pressure at the outlet of the clearance is atmospheric, then Δp can replace p_0 in eq. (21).

Reynolds number as a function of flow is:

$$\mathrm{Re} = \frac{vD_h}{\nu} = \frac{2vc_r}{\nu} = \frac{2\rho Q_0}{d\pi\mu} \tag{22}$$

The dynamic viscosity, μ, which exists in the eq. (22) is determined by eq. (17) and can be written:

$$\mu = a\mathrm{e}^{\left[\frac{b}{(T+273.15)-c}\right]}\mathrm{e}^{\left[\frac{p_0}{a_1+a_2T}\right]} = \mu_0^*\mathrm{e}^{\alpha p_0} \tag{23}$$

Determination of flow rate through the radial clearance at zero overlap, by using the method of approximation, can be done in the following steps:

– Assume the turbulent flow and discharge coefficient $C_\mathrm{d} = C_{\mathrm{d}\infty} = 0.63$.
– With this value of discharge coefficient C_d, $Q_0^{(0)}$ is calculated using eq. (1), and based on it $\mathrm{Re}^{(0)}$ using eq. (22).

– Calculated $\mathrm{Re}^{(0)}$ is to be included into the equation for the flow rate, eq. (21), and receives the flow rate $Q_0^{(1)}$, on the basis of which $\mathrm{Re}^{(1)}$ will be recalculated.
– The process is repeated until the difference between two last values of the flow rate is acceptably small $(Q_{0,n} - Q_{0,n-1} \approx 0)$.

Example of determination of flow rate through the radial clearance at zero overlap

Experimental data: Hydraulic fluid HM46, the mean diameter: d = 9.995 mm = 0.009995 m, radial clearance: c_r = 17 μm, temperature of oil: T_0 = 42.2 °C, pressure drop through orifice: $\Delta p = p_0$ = 38 bar, density at a temperature of 15 °C: ρ_{15} = 883 kg/m^3, density at working temperature: ρ = 883 [1 – 0.0007 (42.2 – 15)] = 866.2 kg/m^3 (eq. 10), dynamic viscosity at the atmospheric pressure: μ_0 = 0.037944 Pa·s (Vogel equation), pressure-viscosity coefficient: $\alpha = 1/(a_1 + a_2T)$ = 0.0021214 bar^{-1} (eq. 18).

Solution:

(1) For C_d = 0.63, flow rate through clearance is:

$$Q_0^{(0)} = 0.63 d\pi c_\mathrm{r}\sqrt{\frac{2\Delta p}{\rho}} = 31.5\cdot 10^{-6}\ \mathrm{m^3/s} = 1.89\ \mathrm{L\ per\ minute}$$

$$\mu = \mu_0 \mathrm{e}^{\alpha p_0} = 0.037944\ \mathrm{e}^{0.0021214\cdot 38} = 0.04113\ \mathrm{Pa{\cdot}s}, \qquad \mathrm{Re}^{(0)} = \frac{2\rho Q_0^{(0)}}{d\pi\mu} = 42.254$$

(2) $$Q_0^{(1)} = 0.63\left(1 - 2\,\mathrm{e}^{-0.22\sqrt{42.254}} + \mathrm{e}^{-0.304\sqrt{42.254}}\right) 0.00995\,\pi 17\cdot 10^{-6}\sqrt{\frac{2\cdot 38\cdot 10^5}{866.2}}$$

$$Q_0^{(1)} = 20.697\cdot 10^{-6}\ \mathrm{m^3/s} = 1.242\ \mathrm{L\ per\ minute}, \qquad \mathrm{Re}^{(1)} = 27.763$$

(3) $Q_0^{(2)} = 18\cdot 10^{-6}\ \mathrm{m^3/s} = 1.08$ L per minute, $\quad \mathrm{Re}^{(1)} = 24.1173$

(4) $Q_0^{(3)} = 17.1156\cdot 10^{-6}\ \mathrm{m^3/s} = 1.027$ L per minute, $\quad \mathrm{Re}^{(3)} = 22.96$

(5) $Q_0^{(4)} = 16.81\cdot 10^{-6}\ \mathrm{m^3/s} = 1.01$ L per minute

Since $Q_0^{(4)} - Q_0^{(3)} = 0.017$ L per minute, and the solution converge quickly, it can be accepted that the flow rate is: $Q_0 = 1.01$ L per minute, and $\mathrm{Re}^{1/2} = 22.96^{1/2} = 4.79$.

Discharge coefficient for this example is:

$$C_\mathrm{d} = 0.63\left(1 - 2\mathrm{e}^{-0.22\sqrt{22.96}} + \mathrm{e}^{-0.304\sqrt{22.96}}\right) = 0.338$$

Experimentally determined values are: C_d = 0.35 and $\mathrm{Re}^{1/2}$ = 4.834 (tab. 7).

Experimentally measured value of flow rate for the given clearance at zero overlap is Q_0 = 1.4 L per minute.

Conclusions

The clearances (and throttles) in different construction forms are applied in all hydraulic control and operating elements, and that shows that the research, which is the subject of this paper, is very important. This paper:

- Gives a detailed description of changes of density and viscosity of hydraulic fluid with the change of pressure and temperature. It is shown that precise mathematical modeling of fluid properties is necessary for accurate modeling of phenomena within hydraulic components.
- Gives the equation for determining the discharge coefficient at zero overlap between the piston and the body (cylinder), as a function of the Reynolds number. It should be noted that the Reynolds number is calculated by taking into account the change of dynamic viscosity of hydraulic fluid with change of temperature and working pressure. Calculating the Reynolds number, previous studies have ignored the impact of pressure on viscosity, which is why the experimental data in the diagram of the discharge coefficient showed greater dissipation.
- Gives a mathematical model for determining the flow rate through radial clearances inside hydraulic components at zero overlap. Such a model is very important for research into hydraulic control components (hydraulic servo distributors, LS regulators, pilot valves, *etc*).

The results obtained by using the developed mathematical model presented in this paper, have shown excellent agreement with experimental results.

Nomenclature

A – cross-sectional area of the orifice, [m^2]
A_D – cross-sectional area of the tube in front of the orifice, [m^2]
A_m – cross-sectional area of the fluid stream in the section *vena contracta*, [m^2]
C_c – contraction coefficient, [–]
C_d – discharge coefficient, [–]
C_v – velocity coefficient, [–]
c_r – radial clearance, [m]
d – diameter of a spool, [m]
D_h – hydraulic diameter, [m]
p_0 – pressure in the hydraulic fluid at the entrance of the clearance, [bar]
p_D – pressure in front of the orifice, [bar]
p_m – pressure in the section *vena contracta* , [bar]
Δp – pressure drop through the orifice, [bar]
Q – actual flow rate through the clearance (determined experimentally), [m^3s^{-1}]
$Q_{t\,max}$ – maximum theoretical flow rate, [m^3s^{-1}]
T_A – absolute temperature, [K]
v – velocity of fluid in the section *vena contracta*, [ms^{-1}]

Greek symbols

α – pressure-viscosity coefficient, [$Pa\cdot s^{-1}$]
μ – dynamic viscosity of fluid, [Pa·s]
μ_0 – dynamic viscosity at the atmospheric pressure, [Pa·s]
μ_0^* – dynamic viscosity of the hydraulic fluid at a working temperature (at the entrance of the clearance), [Pas]
ν_0 – kinematic viscosity at the atmospheric pressure, [m^2s^{-1}]
ρ – fluid density, [kgm^{-3}]

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Paper submitted: May 17, 2016
Paper revised: July 20, 2016
Paper accepted: July 28, 2016