THE INFLUENCE OF SECONDARY FLOW IN A TWO-PHASE GAS-SOLID SYSTEM IN STRAIGHT CHANNELS WITH A NON-CIRCULAR CROSS-SECTION

by

Saša M. MILANOVIĆ^{*}, Miloš M. JOVANOVIĆ, Boban D. NIKOLIĆ, and Vladislav A. BLAGOJEVIĆ

Faculty of Mechanical Engineering, University of Nis, Nis, Serbia

Original scientific paper DOI: 10.2298/TSCI16S5419M

The paper considers two-phase gas-solid turbulent flow of pneumatic transport in straight horizontal channels with a non-circular cross-section. During turbulent flow, a specific flow phenomenon, known as secondary flow, occurs in these channels in the cross-sectional plane. The existence of strong temperature gradients in the cross-sectional plane of the channel or the cases of curved channels result in the appearance of the secondary flow of the first kind. However, in straight channels with a non-circular cross-section, in the developed turbulent flow mode, a secondary flow, known as Prandtl's secondary flow of the second kind, is induced. The paper presents a numerical simulation of a developed two-phase turbulent flow by using the PHOENICS 3.3.1 software package. Reynolds stress model was used to model the turbulence. The paper provides the data on the changes in turbulent stresses in the channel cross-section as well as the velocities of solid particles transported along the channel.

Key words: computer simulation, pneumatic transport, solid particles, two-phase flow, secondary flow

Introduction

Engineering practice provides frequent examples of two-phase gas-solid type flows in channels with a non-circular cross-section. The most common examples of this type of flow occur in the systems of pneumatic transport of granular material, air conditioning, and ventilation systems, process, and energy systems. Generally, two-phase flows are characterized by a special complex of flow phenomena which are the consequences of the interaction between the gas and solid phase, chemical reactions between the phases, elaborate heat flows with the volumetric effects of gas radiation and surface effects of particle radiation. Such flows have an important influence in the complete mechanism of mass, momentum, and heat transfer in the channel and its environment.

The paper considers two-phase gas-solid type flow in straight channels with a noncircular cross-section. The movement of solid particles along the channel is made possible by the action of an air flow. Solid particles, *i. e.* transported material, move due to the fact that air flow act on them by aerodynamic forces, which become strong enough at appropriate air velocities for the material particles to be carried by gas flow. In this case, the flow in the chan-

^{*} Corresponding author: msasa@masfak.ni.ac.rs

nels is highly turbulent, which is the main cause of all obstacles in shedding some light on the process of momentum, heat, and mass transfer in two-phase flows.

Apart from the basic flow along the channel, which is turbulent in itself, in these channels a specific flow phenomenon occurs in the developed turbulent flow, *i. e.* secondary flow in the plane of the channel cross-section. The mechanisms which lead to the occurrence of these secondary flows are different. In curved channels, where the centrifugal force acts perpendicularly on the primary flow direction inducing pressure difference in the channel cross-section, which results in the appearance of secondary flow. This mechanism is characteristic of both the turbulent and laminar flow in channels with a circular cross-section. This type of secondary flow also appears in the case of the existence of strong temperature gradients in the cross-sectional plane of the channel, *i. e.* one of the channel walls is thermally loaded, while the others are not. Such secondary flows are known as Prandtl's secondary flows of the first kind. However, in completely straight channels with a non-circular cross--section, and only in the developed turbulent flow, a secondary flow known as Prandtl's secondary flow of the second kind is induced in the channel cross-section. Even though the level of velocity of the secondary flow of the second kind is only 2-3% of the average velocity of the main flow, it still has an important influence on the complete mechanisms of momentum, heat and mass transfer in the channel and its environment. The large momentum transfer towards the channel vertices causes large gradients of transverse velocities in the cross-sectional plane of the channel. In open channels, secondary flow moves the fluid with a relatively small momentum towards the central part of the channel and causes a certain depression in the velocity maximum below the free surface of the fluid. In turn, secondary flow produces an increased shear stress towards the channel vertices, which is highly significant in the cases of transport of certain sediments, or when solving the problem of channel erosion. Furthermore, secondary flow greatly influences the intensity of heat transfer from the fluid to the channel wall and vice versa. This flow definitely exerts lesser influence than the secondary flow of the first kind, however, it cannot be neglected, particularly in the cases of the two-phase gas-solid type flow with a high Stokes number, *i. e.* the cases of pneumatic transport of solid particles with a small diameter.

To determine the velocities of solid particles of the transported material, the paper employs a full Reynolds stress model of turbulence, where each component of Reynolds stresses is determined from its own transport differential equation. These equations are not exact conservation equations, but modelled ones [1]. The basic principle used to obtain these equations is to retain the correlations in their original form up to the second order, and to model the terms which contain the third- or higher-order correlations, of same or different physical quantities, using the gradient method, *i. e.* express them through the gradients of known physical quantities and model constants.

Physical model of the gas and solid phase

To generate the secondary flow of the second kind, the paper considers a fully developed turbulent flow, which implies that the velocity profiles, *i. e.* velocities in crosssections, are stable or not changing in a straight channel with a square cross-section whose walls are loaded by uniform temperature flux. It is a known fact that turbulence, which is in its nature a highly unsteady, non-linear, irreversible, stochastic and 3-D phenomenon, constitutes the basis of all processes of momentum, heat and mass transfer. The appearance of Prandtl's secondary flow of the second kind in non-circular straight channels is the consequence of the turbulent flow. The generation of the secondary flow of the second kind de-

S1420

pends, above all, on the turbulent fluctuations of the velocity field, while the existence of Reynolds stress gradients promotes the secondary flow of fluids. In the case of a stationary and incompressible flow, the transport equation for the vorticity component perpendicular to the plane of the cross-section Ω_1 has the following form:

$$\underbrace{\overbrace{U_{1} \frac{\partial \Omega_{1}}{\partial x_{1}} + U_{2} \frac{\partial \Omega_{1}}{\partial x_{2}} + U_{3} \frac{\partial \Omega_{1}}{\partial x_{3}} - \overbrace{\Omega_{1} \frac{\partial U_{1}}{\partial x_{1}} - \Omega_{2} \frac{\partial U_{1}}{\partial x_{2}} - \Omega_{3} \frac{\partial U_{1}}{\partial x_{3}}}_{A_{3}} = \\
= \underbrace{\frac{\partial}{\partial x_{1}} \left(\frac{\partial \overline{u_{1} u_{2}}}{\partial x_{3}} - \frac{\partial \overline{u_{1} u_{3}}}{\partial x_{2}} \right)}_{A_{3}} + \underbrace{\frac{\partial^{2}}{\partial x_{2} \partial x_{3}} \left(\overline{u_{3} u_{3}} - \overline{u_{2} u_{2}} \right)}_{A_{4}} - \underbrace{\left(\frac{\partial^{2}}{\partial x_{3}^{2}} - \frac{\partial^{2}}{\partial x_{2}^{2}} \right)}_{A_{5}} + \underbrace{v \left(\frac{\partial^{2} \Omega_{1}}{\partial x_{1}^{2}} + \frac{\partial^{2} \Omega_{2}}{\partial x_{2}^{2}} + \frac{\partial^{2} \Omega_{1}}{\partial x_{3}^{2}} \right)}_{A_{6}} \qquad (1)$$

where the vorticity vector components are defined:

$$\Omega_1 = \frac{\partial U_2}{\partial x_3} - \frac{\partial U_3}{\partial x_2}, \quad \Omega_2 = \frac{\partial U_1}{\partial x_3} - \frac{\partial U_3}{\partial x_1} \quad \text{and} \quad \Omega_3 = \frac{\partial U_2}{\partial x_1} - \frac{\partial U_1}{\partial x_2}$$

The physical sense of the terms of transport eq. (1), for the vorticity component perpendicular to the plane of the cross-section Ω_1 is: A_1 – the convective transport of the vorticity component Ω_1 in the main fluid flow, A_2 – the effects of vortex elongation or compression caused by the gradients of the averaged main flow velocity, which is in principle the main promoter of the secondary flow of the first kind, A_3 , A_4 , and A_5 – the influence of turbulent stresses on the production or destruction of the vorticity component Ω_1 , and A_6 – the process of viscous dissipation of the vorticity component Ω_1 .

For a fully developed turbulent flow, transport eq. (1) for Ω_1 the vorticity component perpendicular to the plane of the cross-section, when all gradients along the axis of the main flow are equal to zero, has the following form:

$$\underbrace{U_2 \frac{\partial \Omega_1}{\partial x_2} + U_3 \frac{\partial \Omega_1}{\partial x_3}}_{A_4} = \underbrace{\frac{\partial^2}{\partial x_2 \partial x_3} \left(\overline{u_3 u_3} - \overline{u_2 u_2}\right)}_{A_4} - \underbrace{\left(\frac{\partial^2}{\partial x_3^2} - \frac{\partial^2}{\partial x_2^2}\right)}_{A_5} \overline{u_2 u_3}}_{A_5} + \underbrace{\nu \left(\frac{\partial^2 \Omega_2}{\partial x_2^2} + \frac{\partial^2 \Omega_1}{\partial x_3^2}\right)}_{A_6} \quad (2)$$

By analyzing the terms of the eq. (2) for turbulent vorticity Ω_1 , one can reach a conclusion that the turbulent terms A_4 and A_5 are of the same order, have a dominant role, opposite signs and are separately much larger than the convective term A_1 . The viscous term A_6 is negligibly small, except in the wall zone of channel vertices. This leads to the conclusion that the difference lies between the turbulent terms A_4 and A_5 which are of the same order as the convective term A_1 , which finally implies that that difference between the relatively large turbulent terms is the mechanism which generates secondary flow, [1]. In other words, secondary flow is the consequence of transverse gradients of primary shear stresses in the area of channel vertices. This is why it is crucial to model all Reynolds stresses as accurately as possible for the purpose of realistic simulation of the secondary flows of the second kind in a straight channel with a square cross-section during a developed turbulent flow.

Two-phase flows are characterized by a complex of a large number of mutually connected, in themselves elaborate phenomena, which are the consequences of the influence between the phases. In the consideration of such flows with the interaction between the phases, a combined approach to the solution of the flow field is adopted in flow modelling. The gas phase is solved by applying the Euler approach – the concept of continuum, while the solid phase is solved by applying the Lagrange approach – the concept of monitoring particles trajectories. The interphase interaction between the gas and solid phase is obtained through the iterative procedure of problem solution:

First step: At the beginning of the integration of conservation equations, the gas phase is first solved without the presence of interphase terms.

Second step: After a certain number of iterations, the obtained gas current field is *frozen* and particles are *run* through it. On the basis of the obtained particles trajectories, the interphase terms of the interaction between the solid and gas phase are determined.

Third step: Particles trajectories are *frozen* and the gas phase flow field is solved again, but now with the interphase terms obtained in the previous step.

Fourth step: If the solution convergence is not achieved, steps two and three are repeated successively until the pre-set criterion of the solution convergence is reached.

To define a mathematical model of the gas phase, the following assumptions are adopted: the flow is steady, 3-D, incompressible, isothermal and chemically inert.

To define a mathematical model of the solid phase, the following assumptions are adopted: particles are of varying dimensions, particles do not change their mass while travelling through the channel, particles have a constant temperature in the channel, the influence of particle collisions is neglected, particles lose a certain degree of momentum upon hitting the channel walls and internal obstacles, particles move stochastically, *i. e.* the turbulent flow field of the gas flow modulates deterministic particles trajectories which are obtained from the averaged values of the gas flow velocities.

Mathematical model of the gas phase

The mathematical model of the gas phase is formed for a 3-D fully developed turbulent flow in a straight channel with a square cross-section, and this fully developed turbulent flow implies that the velocity profiles in cross-sections are stable, *i. e.* not changing. Steady, incompressible, turbulent flow is assumed where the channel walls have a constant temperature, different from the environmental temperature. Gravity volume force and temperature buoyancy effects are neglected. In such a case, the general equation of momentum, heat and mass transfer for the gas phase is identical to the generally known field conservation equation (Reynolds) for a single-phase fluid with the addition of the interphase term, [2]:

$$\frac{\partial}{\partial t}(\rho \,\Phi) + U_j \,\frac{\partial}{\partial x_j}(\rho \,\Phi) - \frac{\partial}{\partial x_j} \left(\Gamma_{\phi} \,\frac{\partial \Phi}{\partial x_j}\right) = S_{\phi} + S_{\phi}^{IF} \tag{3}$$

Under the conditions of isothermal flow, the effects of buoyant flow, caused by the temperature gradients, are negligible, and the density of the gas phase can be considered constant. Thus, according to the adopted assumptions of the physical model, the averaged equations of momentum, heat and mass conservation have the following forms:

continuity equation

$$\frac{\partial U_j}{\partial x_j} = 0 \tag{4}$$

– momentum equation

$$U_{j}\frac{\partial U_{i}}{\partial x_{j}} - \frac{\partial}{\partial x_{j}}\left(\nu\frac{\partial U_{i}}{\partial x_{j}}\right) = -\frac{1}{\rho}\frac{\partial P}{\partial x_{j}} - \frac{\partial \overline{u_{i}u_{j}}}{\partial x_{j}} + S_{Ui}^{IF}$$
(5)

energy equation

$$U_{j} \frac{\partial T}{\partial x_{j}} - \frac{\partial}{\partial x_{j}} \left(a \frac{\partial T}{\partial x_{j}} \right) = -\frac{\partial \overline{\partial u_{j}}}{\partial x_{j}}$$
(6)

Turbulent models

The starting basis for the formation of a stress model of turbulence is the transport equation which defines the dynamics of Reynolds stresses [1, 3], and which can be presented in the following form for an incompressible fluid (Rotta):

$$\frac{\frac{a}{\partial \overline{u_{i}u_{j}}}}{\partial t} + \overline{U_{k}} \frac{\frac{\partial \overline{u_{i}u_{j}}}{\partial x_{k}}}{\partial x_{k}} = \left(\overline{f_{i}u_{j}} + \overline{f_{j}u_{i}}\right) - \left(\overline{u_{i}u_{k}} \frac{\partial U_{j}}{\partial x_{k}} + \overline{u_{j}u_{k}} \frac{\partial U_{i}}{\partial x_{k}}\right) - \frac{2v \overline{\frac{\partial u_{i}}{\partial x_{k}}} \frac{\partial u_{j}}{\partial x_{k}}}{e} + \frac{\overline{p}\left(\frac{\partial u_{i}}{\partial x_{j}} + \frac{\partial u_{j}}{\partial x_{i}}\right)}{f} - \frac{\partial}{\partial x_{k}}\left[\overline{\frac{u_{i}u_{j}u_{k}}{g^{1}}} + \frac{\overline{p}\left(u_{i}\delta_{jk} + u_{j}\delta_{ik}\right)}{g^{2}} - v \frac{\partial}{\partial x_{k}} \frac{\overline{u_{i}u_{j}u_{k}}}{g^{3}}\right] \qquad (7)$$

The terms in the eq. (7) have the following physical interpretation: a is the local change in turbulent stresses, b – the convective change in turbulent stresses, c – the generation of turbulent stresses due to the action of the fluctuating component of volume forces, d – the generation of turbulent stresses due to the deformation of the main flow, e – the viscous dissipation of turbulent stresses, f – the redistribution between certain components of turbulent stresses due to the fluctuating pressure, and g – the diffusion transport of turbulent stresses due to the fluctuation of velocity (term g^1), pressure (term g^2) and molecular transport (term g^3).

The stress turbulence model implies a simultaneous solution of transport eq. (7), with the momentum eq. (5) in the Reynolds averaged form. However, in its exact form only these terms can be treated: a, b, d, and g^3 , with the addition of term c in specific situations, while the other terms: e, f, g^1 , and g^2 , represent the correlations which have to be modelled in the function of the available dependent variables.

Dependent variables that are available for the modelling of unknown terms, for which the transport equations are solved, in the stress turbulence model are: the averaged velocity U_i , the turbulent stresses $\rho u_i u_j$ and the velocity of the dissipation of the turbulent kinetic energy: $\varepsilon = v(\partial u_i/\partial x_k)^2$. The velocity of the dissipation of the turbulent kinetic energy is a phenomenon linked to the smallest vortex structure, but which, above all, depends on the energy dominant vortices, thus it can also be used as a variable which defines the size of these vortices. However, for the purpose of characterization of the time scope of turbulence, an additional variable is introduced, this being the kinetic turbulence energy, k, itself.

The transport differential equation for turbulent temperature fluxes θu_i has the following form, [1]:

$$\frac{a}{D\overline{\partial u_{i}}} = \overline{-u_{i}u_{k}} \frac{\partial T}{\partial x_{k}} = \overline{-\overline{u_{k}u_{k}}} \frac{\partial T}{\partial x_{k}} = \overline{-\overline{\partial u_{k}}} \frac{\partial U_{i}}{\partial x_{k}} = \overline{-g_{i}\beta\overline{\theta}^{2}} + \overline{\frac{\partial}{\partial \theta}} \frac{\partial \overline{\theta}_{i}}{\partial x_{i}\rho} = \overline{-(a+v)} \frac{\overline{\partial \theta}}{\partial x_{k}} \frac{\partial \overline{u_{i}}}{\partial x_{k}} + \frac{\partial}{\partial x_{k}} \left(\underbrace{a\frac{\overline{\partial \theta}}{\partial x_{k}}u_{i}}_{D_{\theta i}} + v\frac{\overline{\partial u_{i}}}{\partial x_{k}} \theta}_{D_{\theta i}} - \overline{\frac{\partial p}{\partial \theta_{i}}} - \underbrace{\frac{\partial p}{\partial x_{k}}}_{D_{\theta i}} \theta}_{D_{\theta i}} \right) = D_{\theta i}$$
(8)

The terms in the above equation have the following physical interpretation: *a* is the local change in turbulent temperature fluxes, $P_{\partial i}^{T}$ – the generation of turbulent temperature fluxes using the gradient of averaged temperature, $P_{\partial i}^{U}$ – the generation of turbulent temperature fluxes using the gradient of averaged velocity, $B_{\partial i}$ – the generation of turbulent temperature fluxes using the buoyancy forces, $D_{\partial i}^{v}$ – the viscous dissipation of temperature fluxes, $D_{\partial i}^{\theta}$ – the turbulent diffusion due to velocity fluctuations, $D_{\partial i}^{p}$ – the turbulent diffusion due to pressure fluctuations, $\mathcal{P}_{\partial i}$ – the redistributive pressure-temperature correlation, and $\varepsilon_{\partial i}$ – the velocity of the dissipation of turbulent temperature fluxes.

velocity of the dissipation of turbulent temperature fluxes. The production terms $P_{\theta i}^T$, $P_{\theta i}^U$, and $B_{\theta i}$, generated by the gradient of averaged temperature, the gradient of averaged velocity and the buoyancy forces, respectively, can be retained in the original form, while the terms $D_{\theta i} = D_{\theta i}^{\nu} + D_{\theta i}^{\theta} + D_{\theta i}^{p}$, $\Phi_{\theta i}$, and $\varepsilon_{\theta i}$, have to be modelled.

The modelling of the terms in transport eq. (7) leads to the closed form of the transport equation for Reynolds stresses, which reads:

$$U_{k} \frac{\partial \overline{u_{i}u_{j}}}{\partial x_{k}} = \frac{\partial}{\partial x_{k}} \left(C_{g}^{'} \frac{k}{\varepsilon} \overline{u_{k}u_{n}} \frac{\partial \overline{u_{i}u_{j}}}{\partial x_{n}} \right) + P_{ij} + \Phi_{ij,1} + \Phi_{ij,2} + \Phi_{ij,2} - \frac{2}{3} \delta_{ij} \varepsilon$$
(9)

the modelled terms have the following forms in the previous transport equation:

$$\begin{split} P_{ij} &= -\overline{u_i u_k} \frac{\partial U_j}{\partial x_k} - \overline{u_j u_k} \frac{\partial U_i}{\partial x_k}, \quad \varPhi_{ij,1} = -C_1 \frac{\varepsilon}{k} \left(\overline{u_i u_j} - \frac{2}{3} \delta_{ij} k \right), \quad D_{ij} = -\overline{u_i u_k} \frac{\partial U_k}{\partial x_j} - \overline{u_j u_k} \frac{\partial U_k}{\partial x_j}, \\ \varPhi_{ij,2} &= -\alpha \left(P_{ij} - \frac{2}{3} \delta_{ij} P \right) - \beta k \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \gamma \left(D_{ij} - \frac{2}{3} \delta_{ij} P \right), \\ P &= -\overline{u_i u_j} \frac{\partial U_i}{\partial x_j}, \quad f_z = \frac{C_{\mu}^{0.75} k^{1.5}}{0.417 \cdot \varepsilon} \frac{1}{x_{nz}}, \quad \varPhi_{ij,z} = \varPhi_{ij,z1} + \varPhi_{ij,z2}, \end{split}$$

$$\begin{split} \Phi_{ij,z1} &= C_{z1} \left(\overline{u_k u_m} \delta_{ij} n_k n_m - \frac{3}{2} \overline{u_k u_i} n_k n_j - \frac{3}{2} \overline{u_k u_j} n_k n_i \right) f_z \frac{k}{\varepsilon}, \\ \Phi_{ij,z2} &= C_{z2} \left(\Phi_{km,2} \delta_{ij} n_k n_m - \frac{3}{2} \Phi_{ki,2} n_k n_j - \frac{3}{2} \Phi_{kj,2} n_k n_i \right) f_z \end{split}$$

The closing of the stress model for Reynolds stresses, eq. (9) is performed by an additional transport differential equation for the dissipation of the kinetic energy of turbulence, so that ε is the dissipation of the kinetic energy of turbulence appears as a new additional variable which is determined from its own transport equation:

$$U_{k} \frac{\partial \varepsilon}{\partial x_{k}} = \frac{\partial}{\partial x_{k}} \left(C_{\varepsilon} \frac{k}{\varepsilon} \overline{u_{k} u_{j}} \frac{\partial \varepsilon}{\partial x_{j}} \right) + \frac{\varepsilon}{k} (C_{\varepsilon 1} P - C_{\varepsilon 2} \varepsilon)$$
(10)

The empirical coefficients of the turbulence model for Reynolds stresses are shown in tab 1.

Table 1.

C'_{g}	C_1	C_2	C_{z1}	C_{z2}	C_{μ}	$C_{arepsilon}$	$C_{arepsilon 1}$	$C_{arepsilon 2}$
0.21	1.50	0.40	0.50	0.06	0.09	0.15	1.44	1.90

The modelled forms of the terms in eq. (8) lead to the closed form of the transport equation for turbulent temperature fluxes:

$$U_{k} \frac{\partial \overline{\partial u_{i}}}{\partial x_{k}} = \frac{\partial}{\partial x_{k}} \left(C_{\theta} \frac{k}{\varepsilon} \overline{u_{k} u_{j}} \frac{\partial \overline{\partial u_{i}}}{\partial x_{j}} \right) + P_{\theta i} + \Phi_{\theta i} + \Phi_{\theta i, z} + \varepsilon_{\theta i}$$
(11)

the modelled terms have the following forms in the previous transport equation:

$$P_{\theta i} = -\overline{u_k u_i} \frac{\partial T}{\partial x_k} - \overline{\theta u_k} \frac{\partial U_i}{\partial x_k}, \quad \Phi_{\theta i} = -C_{\theta 1} \frac{\varepsilon}{k} \overline{\theta u_i} + C_{\theta 2} \overline{\theta u_k} \frac{\partial U_i}{\partial x_k},$$
$$\Phi_{\theta i,z} = -C_{\theta 1z} \frac{\varepsilon}{k} \overline{\theta u_k} n_i n_k f_z$$
Table 2

The empirical coefficients of the model of turbulent temperature fluxes are shown in tab. 2.

ľ	$C_{ heta}$	$C_{ heta 1}$	$C_{ heta 2}$	$C_{\theta z1}$	
	0.11	2.45	0.66	0.80	

Mathematical model of the solid phase

The presence of solid particles in gas flows which are encountered in the vast majority of engineering processes complicate the problem to a large extent, both because of the need to model the flow of the discrete phase and due to the interaction between the phases. The presence of particles creates aerodynamic resistances which causes the change in the momentum of both phases. The mathematical model of the solid phase is based on the Lagrange concept of problem solution, which is closer to reality and enables a more realistic picture and more reliable prediction of the movement of solid particles in fluid turbulence thanks to a larger amount of information. The Lagrange concept implies the monitoring of trajectories of the transported material solid particles. On the basis of this concept, one can determine the positions – trajectories of solid particles, their momentum, velocity, temperature as well as mass change along those trajectories, and finally determine the interphase terms in motion eq. (5).

Solid particle positions are determined by solving the following motion equation for each group of particles:

$$\frac{\mathrm{d}x_{\mathrm{p}}}{\mathrm{d}t} = \tilde{U}_{\mathrm{p}} \tag{12}$$

The current velocity of solid particles \tilde{U}_{p} is determined from the solid phase momentum equation:

$$m_{\rm p} \frac{\mathrm{d}U_{\rm p}}{\mathrm{d}t} = \underbrace{\mathfrak{R}_{\rm p}(\tilde{U} - \tilde{U}_{\rm p})}_{A} + \underbrace{m_{\rm p}bg}_{B} - \underbrace{V_{\rm p}\nabla P}_{C}$$
(13)

In eq. (13), the first term on the right side of the equals sign, A, represents the force of resistance to the movement of particles in relation to the gas phase and is the dominant force which acts on the solid particles in the direction of the flow, causing their motion. The second term, B, represents the gravitational force, while the third term, C, represents the buoyancy force. The gravitational and buoyancy forces are perpendicular to the direction of the particle movement, *i. e.* to the direction of the force of resistance to the relative movement, but are of opposite senses, thus it can be assumed that their actions on a solid particle of the transported material are in equilibrium when solving this problem. Since the vertical forces are in balance, *i. e.* they do not affect the movement of solid particles, only the force of resistance reaction acts on the particles of the transported material, which causes the solid particles to move along the channel. The other forces that act on the solid particles are neglected: the force due to the increase in the pressure gradient, Basset, Saffman and Magnus force.

In the previous equation, in the expression for the force of resistance reaction, \Re_p – represents the function of the resistance of a solid particle and it is determined from the following expression:

$$\Re_{\rm p} = 0.5\rho A_{\rm p} C_D \left| \tilde{U} - \tilde{U}_{\rm p} \right| \tag{14}$$

where $C_{\rm D}$ is the coefficient of the resistance of a solid particle, determined from the expression:

$$C_D = \frac{24}{\text{Re}} (1+0.15 \,\text{Re}^{0.687}) + \frac{0.42}{1+4.25 \cdot 10^4 \,\text{Re}^{-1.16}}$$
(15)

and applied for spherical particles of the transported material as well as for the Reynolds number $\text{Re} < 10^5$.

Integration of the particle momentum equation

The integration of the momentum equation of the transported material particles is conducted in the following sequences in the second iterative step of problem solution:

- the Lagrange integration time step is determined,
- a particle is launched on the basis of its initial and boundary conditions,

- particle characteristics are determined for each new position of the particle, and

- interphase terms are determined.

Determination of the Lagrange integration time step, Δt_L

The Lagrange integration time step is determined from the expression:

$$\Delta t_L = \max[t_0, \min(t_1, t_2, t_3)]$$
(16)

where t_0 is the minimal time step (usually this time step is not shorter than 10^{-7} seconds), t_1 – the time step obtained when the minimal time of a particle passing through a numerical cell is divided by the minimal number of Lagrange steps (usually this number is not greater than 10), t_2 – the impulse relaxation time, and t_3 – the maximal time step.

Particles launching

After determining the Lagrange time step, Δt_L , the particles are *launched* by integrating the particle trajectories eq. (12). The integral of this equation is:

$$x_{\rm p}^n = x_{\rm p}^0 + \tilde{U}_{\rm p}^0 \Delta t_L \tag{17}$$

where *n* is the value of the positional particle vector at the end of the time step, and 0 – the value of the positional particle vector at the beginning of the time step Δt_L . The same symbols relate to the particle velocity. At the integration start, the values marked with 0 represent the initial and boundary conditions.

Determination of particle characteristics

The characteristics of solid particles are determined by solving the particle momentum eq. (13), which can be written in the general form as:

$$\frac{\mathrm{d}X}{\mathrm{d}t} = A - B\tilde{X} \tag{18}$$

where \tilde{X} is the respective current particle variable, while A and B are the constants.

As far as the particle momentum is concerned, which is the only variable in this case, momentum eq. (13) can be written in the following form:

$$\frac{\mathrm{d}U_{\mathrm{p}}}{\mathrm{d}t} = \frac{\Re_{\mathrm{p}}U}{m_{\mathrm{p}}} + bg - \frac{1}{\rho_{\mathrm{p}}}\nabla P - \frac{\Re_{\mathrm{p}}}{m_{\mathrm{p}}}\tilde{U}_{\mathrm{p}}$$
(19)

from which the constants A and B can be defined $A = (\Re_p \tilde{U})m_p + bg - 1/\rho_p \nabla P$, $B = \Re_p/m_p$.

Determination of interphase terms

The fundamental problem in the analysis of the two-phase gas-solid turbulent flow is the treatment of the mutual exchange of momentum, energy and mass between the phases due to the chaotic movement of the particles shifting from one vortex to another. When modelling a two-phase turbulent flow, the presence of the solid phase, *i. e.* the transported material particles, causes the appearance of additional sources in momentum, energy and mass conservation in the gas phase equations. The basic principle of determination of interphase interaction terms is grounded in the division of the flow field into numerical cells and the consideration of each cell as a control volume [4]. Thus the change in momentum during the passing through the observed numerical cell is taken as the source or sink of the momentum in the gas phase, *i. e.* the transporting air.

The mathematical model of the gas phase is established on the basis of the models developed for monophasic flow with a correction due to the presence of solid particles. Namely, when a particle passes through the entire numerical cell, it experiences an interphase interaction of the change in momentum, energy or mass. The interphase interaction terms describe a change in the momentum of the transported material particles, *i. e.* a change in momentum of the gas phase due to the presence of solid particles in it. These interphase interaction terms have to be added to the momentum equation of the gas phase, thus they need to be determined in an appropriate manner. For example, if a particle moves at a velocity higher than that of the environment, *i. e.* the gas phase, the particle will slow down due to the interaction of the phases, *i. e.* the velocity of the particle will drop while the velocity of the gas phase in the immediate surroundings will increase. The interphase interaction term in eq. (5) represents the force of resistance to the movement of a solid particle in the air flow, *i. e.* the force of resistance to the flow of the fluid around the solid particle. This force is of the same magnitude and direction as the force of resistance to relative movement of solid particles, *i. e.* the force of resistance reaction that causes the movement of particles, but of opposite sign. Here, as previously mentioned above, it is assumed that the forces which are perpendicular to the direction of the movement of particles are in equilibrium, and that their influence on particles is neglected. This means that the mutual influence between the phases is, in fact, given by the magnitude of axial forces, so that the interphase interaction term is determined by solving the particle movement equation:

$$m_{\rm p} \frac{\mathrm{d}U_{\rm p}}{\mathrm{d}t} = S_{Ui}^{IF} \tag{20}$$

The integration of partial differential eq. (20) is performed for each numerical cell, *i. e.* control volume, bearing in mind the Lagrange time step so as not to skip any cells, that is, to carry out the integration for every numerical cell and the entire flow space along the channel. Interphase interaction terms appear in the momentum eq. (5) of the gas phase, they are equal to the change in the particle momentum for each numerical cell, and they describe the change in momentum of the gas phase due to the presence of solid particles. They are determined from the expression:

$$S_{U_i}^{IF} = \frac{\pi}{6} \sum \eta \left[\rho_p^0 \tilde{U}_{p,i}^0 (D_p^0)^3 - \rho_p^n \tilde{U}_{p,i}^n (D_p^n)^3 \right]$$
(21)

Numerical model

A numerical model was formed for a developed two-phase gas-solid turbulent flow in a horizontal straight channel with a square cross-section, and with the side dimension of 200 mm. In order to form fully developed turbulent flow with stable velocities, and to induce and emphasize the secondary flow, known as Prandtl's secondary flow of the second kind, an $80 D_h$, *i. e.* 18 m long channel was taken.

The selected numerical grid along the cross-section of the channel is non-uniform, which accounts for different sizes of numerical cells. There are fewer numerical cells in the central part of the channel and they have greater surfaces, while there are more of them closer to the walls and the vertices of the channel and their surface is smaller. The total mass flow of

the transported material (particles) set through the cross-section of the channel at the entrance, is established as partial flow in the numerical cells proportional to the cell surface.

Since the numerical cells differ in their surfaces, *i. e.* the cells close to the channel

walls and vertices are smaller, the mass flows of the transported material (particles) are also smaller in them, contrary to the numerical cells situated in the central part of the channel that are larger in themselves and that have larger mass flows. Thus, if one observes the entire cross-section of the channel, one gets a steady distribution of transported solid particles along the entire cross-section. During the simulations, the fineness of the numerical grid was examined, and the paper presents the results from the highest resolution numerical grid above which the fineness of the grid did not affect the obtained results: NX = 40, NY = 40, and NZ = 180, fig. 1.

As already mentioned, the mechanisms which lead to the occurrence of secondary flows in turbulent flow are different. The influence of Prandtl's secondary flow of the second kind, which is induced in



Figure 1. Numerical grid for the channel cross-section

straight channels with a non-circular cross-section during a developed turbulent flow, can not be neglected, regardless of its size, particularly in the cases of the two-phase gas-solid flow, where the transported material solid particles are of a relatively small diameter, *i. e.* there is a two-phase flow with a high Stokes number. To notice the effects of the secondary flow of the second kind in the cross-section of the observed channel, the study includes three simulation cases of the two-phase gas-solid turbulent flow, where the transporting fluid was always air while different particles were used as the transported material, namely, quartz, ash, and flour.

Firstly, solid quartz particles of 0.5 mm in diameter and 2500 kg/m³ in density were considered as the transported material. Secondly, ash, with the particle diameter of 0.14 mm and density of 1800 kg/m³, and flour, with the particle diameter of 0.20 mm and density of 1410 kg/m³, were used as the other transported materials.

The transported particles were assigned an initial velocity at the entrance to the channel, so as to initiate their movement, and this velocity was chosen as equal to the suspension velocity of the particles which amounted to 2.8 m/s [5]. To form the mathematical model of the ash and flour particle transport, the same conditions were adopted as in the case of quartz particles: the particle entrance velocity of 2.8 m/s (the recommended value of the suspension velocity of ash particles was 0.36 m/s and for flour particles 1.2-1.5 m/s).

Upon the formation of the mathematical model, the transported particles were equally distributed at the channel entrance along the entire cross-section of the channel. To record the behaviour of the transported material, six groups of particles were selected and distributed along the x- and y-axis. Apart from defining the initial velocity of the transported particles of quartz, it was necessary to define the flow velocity of the transporting air at the entrance, which ranged from 12-22 m/s [6], thus the paper adopted the value of 22 m/s, while air pressure was taken as 1 bar with the density of 1.2 kg/m³. The same values of pressure, density, and flow velocity of the transporting air at the entrance were adopted for the transport of ash and flour particles as well (the recommended value of the transporting air velocity for the transport of ash particles was 12-20 m/s and for the transport of flour particles approx. 20 m/s).

To perform a comparative analysis of monophasic and two-phase flows in the defined real problem, the simulation of monophasic flow was conducted under the same boundary conditions for the gas phase as in the simulations of two-phase flow for various types of transport of solid particles. Furthermore, the same fineness of the numerical grid was retained. The dominant parameter which influences the secondary flow of the second kind is the turbulent tangential stress in the cross-sectional plane of the channel, thus it was chosen as the parameter for comparing the effects of two-phase flow on the observed phenomenon.

Results and discussion

The fundamental problem in the consideration of the two-phase gas-solid turbulent flow is based on the change in momentum, energy and mass of the transported material while passing through a certain segment of the flow field. The change in momentum during the passing through an observed numerical control volume is taken as the source or sink of momentum of the continuous - gas phase. In such a case, the mathematical model of the gas phase is established for monophasic flow with the correction due to the presence of solid particles. In the first iterative step only the air current is observed, and conservation equations are solved for it as if there were no dispersed phase. The presence of the dispersed phase causes the appearance of additional sources of the momentum in the gas phase equations through additional terms which describe a change in momentum of solid particles, *i. e.* a change in momentum of the gas phase due to the presence of particles. This is why in the second iterative step the obtained flow field of the gas phase is stopped in time, *i. e. frozen*, and the movement of solid particles is observed within it. This flow field of the gas phase is then used to determine the trajectories of solid particles. Based on thus obtained trajectories, the interphase terms are determined for the interaction between the transporting air and the transported material - particles. So as to include the interphase interaction terms in the analysis, i. e. the influence of one phase on the other, the third iterative step *freezes* the obtained trajectories of transported particles from the previous step and solves the flow field of the gas phase again. The solution of the flow field of the gas phase is now performed by taking into consideration the influence of transported particles through the previously obtained interphase interaction terms. If the solution convergence of 0.1% is achieved, the problem is solved, otherwise the iteration procedure continues simultaneously.

During the simulation of the two-phase turbulent flow, it is assumed that the flow of the transporting gas – air is isothermal, steady and 3-D, and the mass and temperature of solid particles does not change during the transport process, *i. e.* neglecting the influence of collisions between the particles as well as their colliding with the channel walls. The solution of the mathematical model assumes that the temperature of the channel walls does not change along the channel and that it differs from the temperature of the environment.

The formation of the secondary flow of the second kind in a straight channel with a non-circular cross-section during a developed turbulent flow occurs due to the existence of the gradients of Reynolds stresses. The turbulent terms A_4 and A_5 , which contain Reynolds stresses: $\overline{u_2u_2}, \overline{u_3u_3}$, and $\overline{u_2u_3}$, have a dominant role and are of opposite sign in the vorticity eq. (2), whose direction coincides with the basic flow direction. These terms express the influence of turbulent terms can be much larger than the convective term A_1 , in the equation for turbulent vorticity whose direction is perpendicular to the cross-section of the channel. The generation of turbulent normal and shear stresses depends on the size of the velocity gradients of secondary and primary flow. The velocity gradients of secondary flow have a greater influence on the generation of

S1430

turbulent shear stresses $\overline{u_2 u_3}$ than the primary velocity gradients in most of the cross-section during a developed turbulent flow. The difference between the turbulent terms is almost of the same order as the convective term itself, and it is precisely this difference between the relatively large turbulent terms that is the main cause secondary flow formation of the second kind in the cross-section of the channel. Secondary flow launches the small momentum fluid towards the centre of the channel and produces increased shear stresses towards the channel vertices.

It can be said that the vorticity is higher if the production of turbulent stresses is greater, thus it can be concluded that it is exactly that difference between the relatively large turbulent terms A_4 and A_5 that is the mechanism which generates secondary flow. The larger the difference, the more intensive and pronounced the secondary flow. The presence of particles in the gas phase leads to the increase in the difference between the turbulent terms which contain turbulent stresses, thus to a more intensive secondary flow. By observing figs. 2-4 of turbulent stresses, it can be seen that a greater difference between the normal turbulent stresses $\overline{u_3u_3}$ and $\overline{u_2u_2}$ and the shear turbulent stress $\overline{u_2u_3}$ leads to a more pronounced secondary flow. By comparing monophasic and two-phase flow, figs. 2-4, it can be concluded that secondary flow is more intensive and pronounced in the case of two-phase flow, which means that the presence of solid particles in the transporting air current promotes the generation of secondary flow.



Figure 2. Distribution of turbulent stresses $\overline{u_2u_3}$ in the cross-section of the channel, in the middle (for color image see journal website)

The movement of solid particles of the transported material in the transporting air flow is such that an increase in velocity leads to a decrease in their acceleration along the channel. The acceleration of particles decreases almost to zero and their velocity becomes uniform. As the turbulent flow along the observed channel develops, the profile of particle velocity gets deformed in such a manner that particle velocity first goes through a linear increase



Figure 3. Distribution of turbulent stresses $\overline{u_2u_2}$ in the cross-section of the channel, in the middle *(for color image see journal website)*



Figure 4. Distribution of turbulent stresses $\overline{u_3u_3}$ in the cross-section of the channel, in the middle *(for color image see journal website)*

in a short part of the path, with that path being longer for the particles which are closer to the channel walls in relation to the particles closer to the centre of the channel, fig. 5. By developing a turbulent flow, velocity profiles of transported particles stabilize, acceleration almost ceases, and solid particles move uniformly, fig. 5, with velocities which are lower than the velocity of the surrounding air. The path along which the velocity of the transported material particles increases is longer for the particles which are closer to the walls, both due to the viscous forces which act on them and the influence of the channel walls themselves, as well as the velocity of the transporting air.

Viscous zones, caused by secondary flow, immediately next to the solid wall, influence the turbulent interactions via their damping action. Here, it is characteristic that the velocity fluctuations perpendicular to the walls are damped, while the velocity fluctuations parallel to the walls are intensified, but also affect the turbulent flow structure itself. With the beginning of the development of turbulent flow, the velocity of the transported material particles increases. By establishing a fully developed turbulent flow, the velocity profile stabilizes, the velocity becomes approximately constant, which means that the acceleration of solid particles ceases and the transported material particles move almost uniformly, fig. 5, tending to the velocity of the transporting air, which they never reach.



Figure 5. Change in the particle velocity along the channel; (a) flour, (b) quartz, (c) ash

Conclusion

The main characteristic of fluid flow, that is encountered in engineering and technical devices as well as natural watercourses and atmospheric flows, is the turbulent momentum, heat and mass transfer. During a developed turbulent flow in straight channels secondary flow is induced in the cross-sectional plane, the so-called Prandtl's secondary flow of the second kind. The paper employed a full Reynolds turbulence stress model to solve a turbulent two-phase flow in a straight horizontal channel. The performed numerical simulations yielded a change in all of the components of turbulent stresses, while the paper only presents the components which affect the formation of the secondary flow as well as the distribution of velocities of the solid particles transported along the channel. The contribution of the paper lies in the acquisition of a reliable and modern engineering tool on the basis of the numerical calculation approach to the complex phenomena of pneumatic transport of granular materials in channels with a non-circular cross-section.

Nomenclature

- cross-section surface, $[m^2]$ A
- heat diffusion coefficient, $(=\lambda/\rho c_p)$ а
- buoyancy force coefficient b
- c_p D- specific heat, $[Jkg^{-1}K^{-1}]$
- solid particles diameter, [m]
- gravity acceleration, [ms⁻²] g
- kinetic turbulence energy, [m²s⁻²] k
- particle mass, [kg] т
- averaged pressure, [Nm⁻²] Ρ
- \tilde{P} – current pressure, [Nm⁻²]
- ∇P continuous phase pressure gradient
- Re Reynolds number
- S - source term
- Т - averaged temperature, [K]
- time step, [s] t
- U– averaged velocity, [ms⁻¹]
- Ũ – current velocity vector, [ms⁻¹]
- $\overline{u_i u_j}$ components of turbulent stresses
- particle volume, [m³]

Greek symbols

- Г - transport, diffusion parameter coefficient
- dissipations of turbulent kinetic energy, [m²s⁻³] ε
- flow of number of particles per one cell, $[s^{-1}]$ n
- $\overline{\theta u_i}$ turbulent temperature flux, [kgKm⁻²s⁻¹]
- λ - heat transfer coefficient, [Wm⁻¹K⁻¹]
- kinematic viscosity, [m²s⁻¹] v
- ρ – density, [kgm⁻³]
- gas phase universal parameter Φ
- vorticity, $[s^{-1}]$ Ω

Subscripts

- i, j, k - vector component
- end of time step п
- particle p
- $\vec{x}, \vec{y}, \vec{z}$ position vector
- 0 - beginning of time step

Superscripts

IF - interphase term of gas and solid phase interaction

References

- [1] Stevanović, Ž., Numerički aspekti turbulentnog prenošenja impulsa i toplote (Numerical Issues in Turbulent Momentum and Heat Transport - in Serbian), Faculty of Mechanical Engineering, University of Nis, Nis, Serbia, 2008
- [2] Milanović, S., Investigation on Pneumatic Transport of Granular Matter of Noncircular Form in Turbulent Two Phase Flow in Straight Channels (in Serbian), Ph. D. thesis, Faculty of Mechanical Engineering, University of Nis, Nis, 2014
- [3] Hanjalić, K., Opšte jednačine transportnih procesa (The General Equations of Transport Processes - in Serbian), Faculty of Mechanical Engineering, University of Sarajevo, Sarajevo, Bosnia and Herzegovina, 1976
- [4] Sijerčić, M., Matematičko modeliranje kompleksnih turbulentnih transportnih procesa (Mathematical Modelling of Complex Turbulent Transport Processes – in Serbian), Yugoslav Society of thermal Engineers - Edition: Scientific Investigation Achievements, Belgrade, 1998
- [5] Bogdanović, B., et al., Leteći pneumatički transport (Flying Pneumatic Transport in Serbian), Faculty of Mechanical Engineering, University of Nis, Nis, Serbia, 2009
- [6] Hanjalić, K., Mathematical Models of Turbulent Transport Processes (in Serbian), Proceedings, Proceedings information, Conference Dedicated to Academician Muhamed Riđanović, Bosnian and Herzegovinian Academy of Science and Art, Sarajevo, Bosnia and Herzegovina, 1984