POROUS MEDIUM MAGNETOHYDRODYNAMIC FLOW AND HEAT TRANSFER OF TWO IMMISCIBLE FLUIDS

by

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The magnetohydordynamic flow and heat transfer of two viscous incompressible fluids through porous medium has been investigated in the paper. Fluids flow through porous medium between two parallel fixed isothermal plates in the presence of an inclined magnetic and perpendicular electric field. Fluids are electrically conducting, while the channel plates are insulated. The general equations that describe the discussed problem under the adopted assumptions are reduced to ordinary differential equations and closed-form solutions are obtained. Solutions with appropriate boundary conditions for velocity and temperature fields have been obtained. The analytical results for various values of the Hartmann number, load factor, viscosity and porosity parameter have been presented graphically to show their effect on the flow and heat transfer characteristics.

Key words: *MHD flow, heat transfer, porous media, immiscible fluids*

Introduction

The flow and heat transfer of electrically conducting fluids in channels and circular pipes under the effect of a transverse magnetic field occurs in MHD generators, pumps, accelerators, and flow-meters and have applications in nuclear reactors, filtration, geothermal systems and others.

The interest in the outer magnetic field effect on heat-physical processes appeared seventy years ago. Blum [1] carried out one of the first works in the field of mass and heat transfer in the presence of a magnetic field. The requirements of modern technology have stimulated the interest in fluid flow studies, which involve the interaction of several phenomena. One of these phenomena is certainly viscous flow of electrically conducting fluid through porous medium in the present of magnetic field. The mathematical theory of the flow of fluid through a porous medium was initiated by Darcy [2]. For the steady flow, he assumed that viscous forces were in equilibrium with external forces due to pressure difference and body forces.

The flow and temperature distribution through porous channels is of great importance in range of scientific and engineering domains, including earth science, nuclear engineering, and metallurgy. Cunningham and Williams [3] reported several geophysical applications of flow in porous medium. McWhirter *et al.* [4] reported the experimental results of

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the MHD flow in a porous medium required for the design of a blanket of liquid metal around a thermonuclear fusion-fission hybrid reactor. Research results presented by Prescott and Incropera [5] and Lehmann *et al.* [6] show that applied permanent magnetic field during the solidification process modify the intensity of the inter-dendritic flow of the metallic liquid in the mushy zone, *i. e.* a porous medium. This technique allows the reduction of micro-macro segregation occurring during casting processes.

The flow and heat transfer of a viscous incompressible electrically conducting fluid between two infinite parallel insulating plates has been studied by many researchers [7-10]. Several analytical and numerical works in the literature are also devoted to the study of the MHD flow of a conducting fluid through a porous medium between two parallel fixed plates. Alpher [11] examined an incompressible laminar flow and convection heat transfer between parallel plates through a transverse magnetic field. Cox [12] discussed a 2-D incompressible viscous fluid between two parallel porous walls with symmetric and asymmetric suction. Tawil and Sundarammal [13] presented results of MHD flow under stochastic porous media, while Yih [14] examined the radiation effects on natural convection over a cylinder embedded in porous media. Vidhya and Sundarammal [15] considered an incompressible viscous fluid flow and temperature distribution in a porous medium between two vertical parallel plates and the problem is analyzed analytically. Geindreau and Auriault [16] studied tensorial filtration law in rigid porous media for steady-state slow flow of an electrically conducting, incompressible and viscous Newtonian fluid in the presence of a magnetic field. Recently, Singh and Rakesh [17] have investigated the heat and mass transfer MHD flow through porous medium.

All the mentioned studies pertain to a single-fluid model. Most of the problems relating to the petroleum industry, geophysics, plasma physics, magneto-fluid dynamics, *etc.*, involve multi-fluid flow situations. The problem concerning the flow of immiscible fluids has a definite role in chemical engineering and in medicine [18]. There have been some experimental and analytical studies on hydrodynamic aspects of the two-fluid flow reported in the literature. Bird *et al.* [19] obtained an exact solution for the laminar flow of two immiscible fluids between parallel plates. Bhattacharya [20] investigated the flow of two immiscible fluids between two rigid parallel plates with a time-dependent pressure gradient. Later, Mitra [21] analyzed the unsteady flow of two electrically conducting fluids between two rigid parallel plates. The physical situation discussed in [21] is one possible case. Another physical phenomenon is the case in which the two immiscible conducting fluids flow past permeable beds. Chamkha [22] reported analytical solutions for flow of two-immiscible fluids in porous and non-porous parallel-plate channels. The findings of a study of this physical phenomenon have a definite bearing on petroleum and chemical technologies and on biomechanics.

These examples show the importance of knowledge of the laws governing immiscible multi-phase flows for proper understanding of the processes involved. In modeling such problems, the presence of a second immiscible fluid phase adds a number of complexities as to the nature of interacting transport phenomena and interface conditions between the phases. Lohrasbi and Sahai [23] studied two-phase MHD flow and heat transfer in a parallel plate channel with one of the fluids being electrically conducting. Following the ideas of Alireza and Sahai [24], Malashetty *et al.* [25, 26] have studied the two-fluid MHD flow and heat transfer in an inclined channel, and flow in an inclined channel containing porous and fluid layer.

Keeping in view the wide area of practical importance of multi-fluid flows as previously mentioned, the objective of the present study is to investigate the MHD flow and heat transfer of two immiscible fluids through the porous medium in the presence of applied electric and inclined magnetic field.

Mathematical model

As mentioned in the introduction, the problem of the MHD flow and heat transfer of incompressible and electrically conductive fluid through porous medium has been considered in this paper. Fully developed flow takes place between parallel plates that are at a distance $2h$, as shown in fig. 1.

Electrically conductive fluids flow through the porous medium due to the constant pressure gradient, while the fluids are exposed to the externally applied inclined magnetic and perpendicular electric field. The problem is analyzed for electrically insulated channel plates, while their temperature is maintained at constant values T_{w1} and T_{w2}. The fluids in the two regions have been assumed immiscible and incompressible and the flow has been steady, 1-D and fully developed. Both fluids flow through homogeneous and isotropic porous medium of permeability, κ. Furthermore, the two fluids have different kinematic viscosities ν_1 and ν_2 and densities ρ_1 and ρ_2. The physical model, shown in fig. 1, consists of two infinite parallel plates extending in the x- and z-direction.

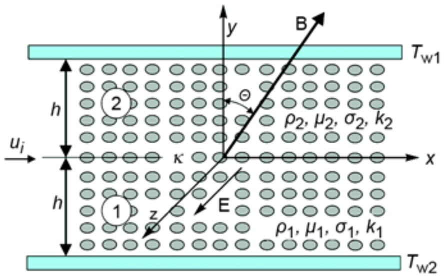

Figure 1. Physical model and co-ordinate system

The region 1 $-h \le y \le 0$ has been occupied by a fluid of viscosity, μ_1, electrical conductivity, σ_1, and thermal conductivity, k_1, and the region 2: $0 \le y \le h$ has been filled by a layer of different fluid of viscosity ,μ_2, thermal conductivity, k_2, and electrical conductivity, σ_2.

A uniform magnetic field of the strength, B_0, has been applied in the direction making an angle, θ, to the y axis, electric field acts in the direction of z-axis while both fluids flow with velocity u_i in x-direction:

$$\vec{\mathrm{v}}_i = [u_i(y), 0, 0] \tag{1}$$

$$\vec{\mathrm{B}} = \left(B_0\sqrt{1-\lambda^2}, B_0\lambda, 0\right) \tag{2}$$

$$\vec{\mathrm{E}} = (0, 0, E) \tag{3}$$

where $\lambda = \cos\theta$.

The described MHD fluid flow and heat transfer problem is mathematically presented with a continuity equation:

$$\nabla\vec{\mathrm{v}}_i = 0 \tag{4}$$

momentum equation for flow in the porous medium:

$$\rho_i \frac{\partial\vec{\mathrm{v}}_i}{\partial t} + \rho_i(\vec{\mathrm{v}}_i\nabla)\vec{\mathrm{v}}_i = -\nabla p_i + \mu_i\nabla^2\vec{\mathrm{v}}_i - \varepsilon_i \frac{\mu_i}{\kappa}\vec{\mathrm{v}}_i + \vec{\mathrm{j}}_i \times \vec{\mathrm{B}} \tag{5}$$

and an energy equation:

$$\rho_i c_{pi}\left(\frac{\partial T_i}{\partial t} + \vec{\mathrm{v}}_i\nabla T_i\right) = k_i\nabla^2 T_i + \mu_i\Phi_i + \varepsilon_i \frac{\mu_i}{\kappa}\vec{\mathrm{v}}_i\vec{\mathrm{v}}_i + \frac{\vec{\mathrm{j}}_i^{\,2}}{\sigma_i} \tag{6}$$

In previous equations and in following boundary conditions used symbols are: i denotes fluid 1 or 2, ε_i is equal to 1 for porous medium or 0 for *clean* medium, κ is the porous medium permeability, c_{pi} – the specific heat capacity, T_i – the thermodynamic temperature, Φ_i – the dissipation function. The fourth term on the right hand side of eq. (5) is the magnetic body force, and $\vec{\mathrm{j}}$ is the current density vector defined by:

$$\vec{\mathrm{j}}_i = \sigma_i (\vec{\mathrm{E}} + \vec{\mathrm{v}}_i \times \vec{\mathrm{B}}) \tag{7}$$

Finally, the momentum and energy equation for described flow and heat transfer problem takes the following form:

$$-\frac{\partial p_i}{\partial x} + \mu_i \frac{\mathrm{d}^2 u_i}{\mathrm{d}y^2} - \varepsilon_i \frac{\mu_i}{\kappa} u_i - \lambda \sigma_i B_0 (E + \lambda B_0 u_i) = 0 \tag{8}$$

$$k_i \frac{\mathrm{d}^2 T_i}{\mathrm{d}y^2} + \mu_i \left(\frac{\mathrm{d}u_i}{\mathrm{d}y} \right)^2 + \varepsilon_i \frac{\mu_i}{\kappa} u_i^2 + \sigma_i (E + \lambda B_0 u_i)^2 = 0 \tag{9}$$

The flow and thermal boundary conditions have been unchanged by the addition of electromagnetic fields. The no slip conditions require that the fluid velocities are equal to the velocities of walls and boundary conditions on temperature are isothermal conditions. In addition, the fluid velocity, sheer stress and heat flux must be continuous across the interface $y = 0$. Equations, which represent these conditions for fluids in regions 1 and 2, are:

$$u_2(h) = 0, \quad u_1(-h) = 0, \quad u_1(0) = u_2(0) \tag{10}$$

$$\mu_1 \frac{\mathrm{d}u_1}{\mathrm{d}y} = \mu_2 \frac{\mathrm{d}u_2}{\mathrm{d}y}, \quad y = 0 \tag{11}$$

$$T_1(-h) = T_{\mathrm{w}2}, \quad T_2(h) = T_{\mathrm{w}1}, \quad T_1(0) = T_2(0) \tag{12}$$

$$k_1 \frac{\mathrm{d}T_1}{\mathrm{d}y} = k_2 \frac{\mathrm{d}T_2}{\mathrm{d}y}, \quad y = 0 \tag{13}$$

It is convenient to transform eqs. (8) and (9) and boundary conditions (10) to (13) to a non-dimensional form. The following transformations have been used:

$$y^* = \frac{y}{h}, \quad u_i^* = \frac{u_i}{U}, \quad P = -\frac{\partial p_i}{\partial x}, \quad \mathrm{Ha}_i = B_0 h \sqrt{\frac{\sigma_i}{\mu_i}}, \quad \gamma_i = \frac{\mu_1}{\mu_i},$$
$$K = \frac{E}{B_0 U}, \quad \Lambda = \frac{h^2}{\kappa}, \quad U = \frac{h^2 P}{\mu_1}, \quad G_i = \frac{P}{\dfrac{\mu_i U}{h^2}}, \tag{14}$$
$$\delta = \frac{k_1}{k_2}, \quad \Theta_i = \frac{T_i - T_{\mathrm{w}2}}{T_{\mathrm{w}1} - T_{\mathrm{w}2}}, \quad \mathrm{Pr}_i = \frac{\mu_i c_{pi}}{k_i}, \quad \mathrm{Ec}_i = \frac{U^2}{c_{pi}(T_{\mathrm{w}1} - T_{\mathrm{w}2})}$$

where U is the referent velocity, Ha_i – the Hartmann number, K – the loading factor, Λ – the porosity parameter, Pr_i – the Prandtl number, and Ec_i – the Eckert number.

The dimensionless governing equations, boundary and interface conditions now take the following form:

$$\frac{d^2u_i^*}{dy^{*2}} - \varepsilon_i \Lambda u_i^* - \lambda Ha_i^2 (K + \lambda u_i^*) + G_i = 0 \tag{15}$$

$$\frac{d^2\Theta_i}{dy^{*2}} + \Pr_i \text{ Ec}_i \left[\left(\frac{du_i^*}{dy^*} \right)^2 + \varepsilon_i \Lambda u_i^{*2} + \text{Ha}_i^2 (K + u_i^* \lambda)^2 \right] = 0 \tag{16}$$

$$u_1^*(-1) = 0, \quad u_2^*(1) = 0, \quad u_1^*(0) = u_2^*(0), \quad \gamma_2 \frac{du_1^*}{dy^*} = \frac{du_2^*}{dy^*} \quad \text{for } y^* = 0 \tag{17}$$

$$\Theta_1(-1) = 0, \quad \Theta_2(1) = 1, \quad \Theta_1(0) = \Theta_2(0), \quad \delta \frac{d\Theta_1}{dy^*} = \frac{d\Theta_2}{dy^*} \quad \text{for } y^* = 0 \tag{18}$$

Velocity and temperature distribution

The solutions of eqs. (15) and (16) with boundary and interface conditions (17) and (18) have the following forms:

$$u_i^*(y^*) = A_i \exp(\omega_i y^*) + B_i \exp(-\omega_i y^*) + \Omega_i \tag{19}$$

$$\Theta_i(y^*) = -\Pr_i \text{ Ec}_i \left[\frac{1}{2} A_i^2 \exp(2\omega_i y^*) + \frac{1}{2} B_i^2 \exp(-2\omega_i y^*) \right] -$$
$$- \Pr_i \text{ Ec}_i [L_i \exp(\omega_i y^*) + M_i \exp(-\omega_i y^*) + N_i y^{*2} + C_i y^* + D_i] \tag{20}$$

where:

$$\omega_i^2 = \varepsilon_i \Lambda + \lambda^2 \text{ Ha}_i^2, \quad \Omega_i = \frac{\gamma_i - \lambda K \text{ Ha}_i^2}{\omega_i^2} \tag{21}$$

$$A_1 = -B_1 \exp(2\omega_1) - \Omega_1 \exp(\omega_1), \quad A_2 = -\frac{R_1 R_6 + R_3 R_4}{R_1 R_5 + R_2 R_4} \tag{22}$$

$$B_1 = \frac{R_2}{R_1} A_2 + \frac{R_3}{R_1}, \quad B_2 = -A_2 \exp(2\omega_2) - \Omega_2 \exp(\omega_2) \tag{23}$$

$$R_1 = 1 - \exp(2\omega_1), \quad R_2 = 1 - \exp(2\omega_2) \tag{24}$$

$$R_3 = \Omega_2[1 - \exp(\omega_2)] - \Omega_1[1 - \exp(\omega_1)] \tag{25}$$

$$R_4 = \gamma_2 \omega_1 [1 + \exp(2\omega_1)], \quad R_5 = \omega_2 [1 + \exp(2\omega_2)] \tag{26}$$

$$R_6 = \gamma_2 \omega_1 \Omega_1 \exp(\omega_1) + \omega_2 \Omega_2 \exp(\omega_2) \tag{27}$$

$$L_i = \frac{2}{\omega_i^2}(\omega_i^2 \Omega_i + \lambda K \,\mathrm{Ha}_i^2) A_i, \qquad M_i = \frac{2}{\omega_i^2}(\omega_i^2 \Omega_i + \lambda K \,\mathrm{Ha}_i^2) B_i \tag{28}$$

$$N_i = \frac{1}{2}[\Omega_i(\omega_i^2 \Omega_i + 2\lambda K \,\mathrm{Ha}_i^2) + K^2 \,\mathrm{Ha}_i^2] \tag{29}$$

$$R_7 = \frac{1}{2} A_1^2 \exp(-2\omega_1) + \frac{1}{2} B_1^2 \exp(2\omega_1) + L_1 \exp(-\omega_1) + M_1 \exp(\omega_1) + N_1 \tag{30}$$

$$R_8 = \frac{\mathrm{Pr}_2 \,\mathrm{Ec}_2}{\mathrm{Pr}_1 \,\mathrm{Ec}_1} \tag{31}$$

$$R_9 = \frac{\mathrm{Pr}_2 \,\mathrm{Ec}_2}{\mathrm{Pr}_1 \,\mathrm{Ec}_1}\left(\frac{1}{2} A_2^2 + \frac{1}{2} B_2^2 + L_2 + M_2\right) - \frac{1}{2} A_1^2 - \frac{1}{2} B_1^2 - L_1 - M_1 \tag{32}$$

$$R_{10} = \frac{1}{2} A_2^2 \exp(2\omega_2) + \frac{1}{2} B_2^2 \exp(-2\omega_2) + L_2 \exp(\omega_2) + M_2 \exp(-\omega_2) + N_2 \tag{33}$$

$$R_{11} = \frac{1}{\mathrm{Pr}_2 \,\mathrm{Ec}_2} + R_{10}, \qquad R_{12} = \frac{1}{\delta} \frac{\mathrm{Pr}_2 \,\mathrm{Ec}_2}{\mathrm{Pr}_1 \,\mathrm{Ec}_1} \tag{34}$$

$$R_{13} = \frac{\omega_2}{\delta} \frac{\mathrm{Pr}_2 \,\mathrm{Ec}_2}{\mathrm{Pr}_1 \,\mathrm{Ec}_1} (A_2^2 - B_2^2 + L_2 - M_2) - \omega_1 (A_1^2 - B_1^2 + L_1 - M_1) \tag{35}$$

$$C_1 = D_1 + R_7, \qquad C_2 = -D_2 - R_{11} \tag{36}$$

$$D_1 = R_8 D_2 + R_9, \qquad D_2 = \frac{R_{13} - R_7 - R_9 - R_{11} R_{12}}{R_8 + R_{12}} \tag{37}$$

With the aid of the expressions for velocity and temperature distribution, following important characteristics of the flow and heat transfer are derived:

– the flow rate:

$$q = \int_{-1}^{1} u_i^*(y^*) \,\mathrm{d}y^* = \int_{-1}^{1} [A_i \exp(\omega_i y^*) + B_i \exp(-\omega_i y^*) + \Omega_i] \,\mathrm{d}y^* \tag{38}$$

$$q = \frac{A_1}{\omega_1}[1 - \exp(-\omega_1)] + \frac{B_1}{\omega_1}[\exp(\omega_1) - 1] + \frac{A_2}{\omega_2}[\exp(\omega_2) - 1] + \frac{B_2}{\omega_2}[1 - \exp(-\omega_2)] + \Omega_1 + \Omega_2 \tag{39}$$

– the shear stress at the plates:

$$\tau_i = \mu_i \left(\frac{\mathrm{d}u_i}{\mathrm{d}y}\right)_{y^* = -1;1} = \mu_i \frac{U}{h} \left(\frac{\mathrm{d}u_i^*}{\mathrm{d}y^*}\right)_{y^* = -1;1} \tag{40}$$

$$\tau_1(-1) = \frac{\mu_1 U \omega_1}{h} [A_1 \exp(-\omega_1) - B_1 \exp(\omega_1)],$$

$$\tau_2(1) = \frac{\mu_2 U \omega_2}{h} [A_2 \exp(\omega_2) - B_2 \exp(-\omega_2)] \tag{41}$$

– the mean temperature:

$$\Theta_{sr} = \int_{-1}^{1} \Theta_i(y^*)\,\mathrm{d}y^* \tag{42}$$

$$\Theta_{1sr} = \mathrm{Ec}_1\,\mathrm{Pr}_1 \left[\frac{B_1^2 - A_1^2}{4\omega_1} + \frac{M_1 - L_1}{\omega_1} \right] \tag{43}$$

$$\Theta_{2sr} = \mathrm{Ec}_1\mathrm{Pr}_1 \left[\frac{B_1^2\exp(2\omega_1) - A_1^2\exp(-2\omega_1)}{4\omega_1} + \right.$$

$$\left. \frac{M_1\exp(\omega_1) - L_1\exp(-\omega_1)}{\omega_1} + D_1 - \frac{C_1}{2} + \frac{N_1}{3} \right] \tag{44}$$

$$\Theta_{3sr} = \mathrm{Ec}_2\,\mathrm{Pr}_2 \left[\frac{B_2^2\exp(-2\omega_2) - A_2^2\exp(2\omega_2)}{4\omega_2} + \right.$$

$$\left. + \frac{M_2\exp(-\omega_2) - L_2\exp(\omega_2)}{\omega_2} - D_2 - \frac{C_2}{2} - \frac{N_2}{3} \right] \tag{45}$$

$$\Theta_{4sr} = \mathrm{Ec}_2\,\mathrm{Pr}_2 \left(\frac{B_2^2 - A_2^2}{4\omega_2} + \frac{M_2 - L_2}{\omega_2} \right) \tag{46}$$

$$\Theta_{sr} = (\Theta_{1sr} - \Theta_{2sr}) + (\Theta_{3sr} - \Theta_{4sr}) \tag{47}$$

– dimensionless heat transfer coefficient – Nusselt number on the plates:

$$\mathrm{Nu}_{\mathrm{lp}} = \left.\frac{\mathrm{d}\Theta_1}{\mathrm{d}y^*}\right|_{y^*=-1} = -\,\mathrm{Ec}_1\,\mathrm{Pr}_1[B_1^2\omega_1\exp(2\omega_1) + A_1^2\omega_1\exp(-2\omega_1) +$$

$$+L_1\omega_1\exp(-\omega_1) - M_1\omega_1\exp(\omega_1) - 2N_1 + C_1] \tag{48}$$

$$\mathrm{Nu}_{\mathrm{up}} = \left.\frac{\mathrm{d}\Theta_2}{\mathrm{d}y^*}\right|_{y^*=1} = -\,\mathrm{Ec}_2\,\mathrm{Pr}_2[B_2^2\omega_2\exp(-2\omega_1) + A_2^2\omega_2\exp(2\omega_1) +$$

$$+L_2\omega_2\exp(\omega_2) - M_2\omega_2\exp(-\omega_2) - 2N_2 + C_2] \tag{49}$$

Results and discussion

In this paper, results for steady MHD flow and heat transfer of two viscous incompressible fluids through porous medium are presented and discussed for various parametric conditions. The part of obtained results has been presented graphically in figs. 2 to 13. Figures 2 and 3 present the effect of the Hartmann number on velocity and temperature field. In fig. 2 the velocity profiles over the channel height for several values of the Hartmann number are shown. It can clearly be seen that as the Hartmann number is increased, the velocity profiles become flatter. In the center of the duct, often called the core, the Lorentz force acts in the direction opposite to the flow direction and tends to retard the flow. The main balance of forces is established between the Lorentz force and the driving pressure gradient, while the load factor K = 0 (applied external electric field is zero). The influence of the Hartmann number on the velocity profiles is more pronounced in the channel region 2 containing the fluid with higher electrically conductivity compared to that in region 1. For large values of Hartmann number flow can be almost completely stopped in the region 2.

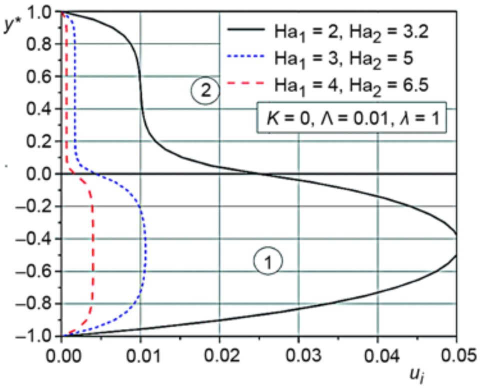

Figure 2. Effect of Hartmann number on velocity

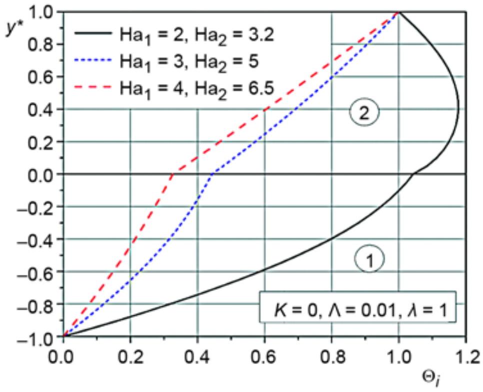

Figure 3. Effect of Hartmann number on non-dimensional temperature

Figure 3 shows the influence of the Hartmann number on the dimensionless temperature. Several interesting observations can readily be made. First, it should be recalled that, in the solution, both viscous heating and Joule heating were included in the analysis. As expected, the stronger the magnetic field, the more the flow is retarded in the both fluid regions of the channel. With the increase of the Hartmann number temperature in the middle of the channel significantly decreases, while near the plate's increases mainly due to viscous heating resulted from large shear stresses. A MHD effect on the thermal characteristics of the flow is manifested in redistribution of the internal heat sources (viscous dissipation and Joule heating) under the influence of the magnetic field. The effect of increasing the Hartmann number on temperature profiles (fig. 3) in both of the parallel-plate channel regions was in equalizing the fluid temperatures.

Figures 4 and 5 show the effect of the magnetic field inclination angle on the distribution of velocity, and temperature. Figure 4 shows the effect of the angle of inclination on velocity which predicts that the velocity increases as the inclination angle increases. These results are expected because the application of a transverse magnetic field normal to the flow direction has a tendency to create a drag-like Lorentz force which has a decreasing effect on the flow velocity. In fig. 5, the dimensionless temperature distribution as a function of y^*, for various values of applied magnetic field inclination angle, is shown. It can be seen from figs. 4 and 5 that the magnetic field flattens out the velocity and temperature profiles and reduces the flow energy transformation as the inclination angle decreases, for the $K = 0$ (short-circuit condition).

In fig. 6 velocity profiles are displayed with the variations in porosity parameter Λ. From this figure, it is noticed that the velocity of the fluid increases from with the increase in

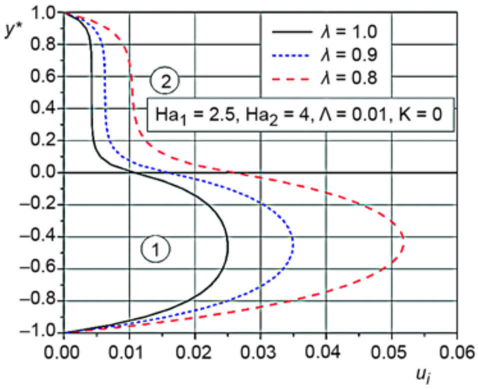

Figure 4. Effect of magnetic field inclination angle on velocity

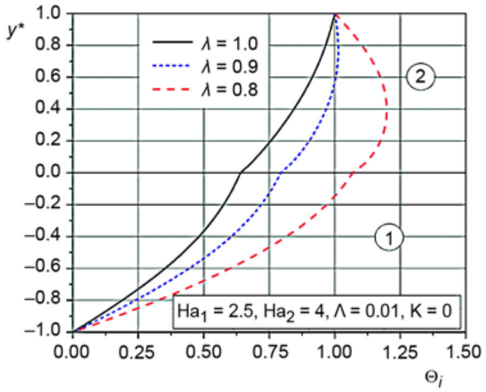

Figure 5. Effect of magnetic field inclination angle on non-dimensional temperature

the values of the porosity parameter. Physically, an increase in the permeability of porous medium leads the rise in the flow of fluid through it. When the holes of the porous medium become large, the resistance of the medium may be neglected. An effect of porosity parameter, Λ, on temperature is presented in fig. 7. From this figure, it is noticed that the temperature of the fluid slightly increases with the increase in the values of the porosity parameter, Λ. This is due to the balance of Joule heating and viscous heating. For small values of porosity parameter, only Joule heating is pronounced and for higher values of porosity parameter viscous heating increase temperature significantly in both fluids region.

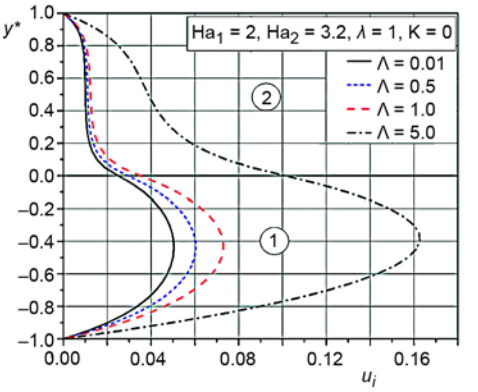

Figure 6. Effect of porosity parameter on velocity

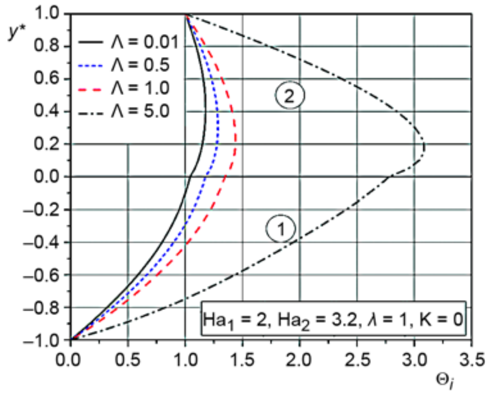

Figure 7. Effect of porosity parameter on non-dimensional temperature

Of particular significance is the analysis when the loading factor, K, is different from zero (value of K define the system as generator, flow-meter or pump), while the Hartmann number is constant. The introduction of parameter K modifies the usual Hartmann flow. In addition, for a given Hartmann number, the relationship between pressure gradient and mean flow or flow rate is altered by K. In the case when $K \neq 0$ the external electric field plays the role of a supplementary pressure gradient. Figure 8 shows the effect of the K on velocity, which predicts the possibility to change the flow direction. Unlike the short circuit case, an interaction between fluids at the interface is significantly expressed.

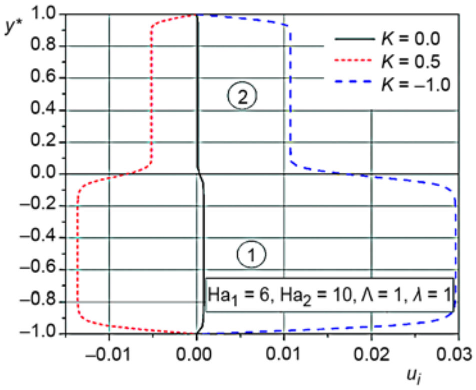

Figure 8. Effect of loading factor on velocity

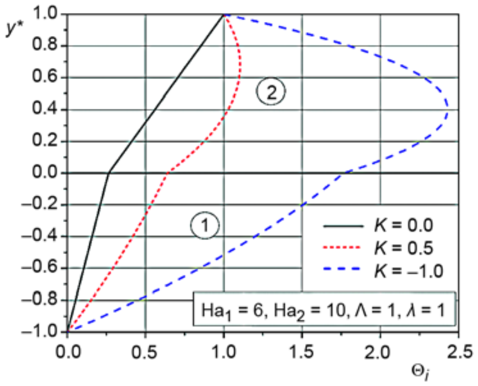

Figure 9. Effect of loading factor on non-dimensional temperature

In the figure 9 the temperature distribution as a function of y^*, for various values of K, is shown. For K = 0, and high intensity of magnetic field the temperature distribution in both fluid regions is almost linear *i. e.* temperature is affected only by conduction. In the case when $K \neq 0$ heat transfer is affected by the viscous dissipation and Joule heating. Viscous dissipation dominates in the regions near the plates and at the interface of fluids. Towards the middle of the each fluid region, temperature rises as a result of Joule heating. Figure 9 illustrates that with the increase in the K the heat transfer between the fluids increases.

The numerical values of shear stress at the lower and upper plate are presented in figs. 10 and 11 for different values of Hartmann and porosity parameter, while the loading factor take the positive, zero or negative values. Figure 10 presents the effect of the Hartmann number. Increasing of Hartmann number cause a decrease of shear stress for the short circuit case (K = 0). In the case when external electric field play the role of additional pressure gradient shear stress at both plates are increased, and for high magnetic field intensity stresses at both plates reach some constant value.

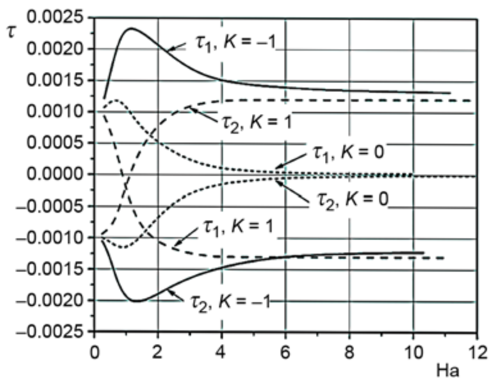

Figure 10. Effect of Hartmann number on shear stress

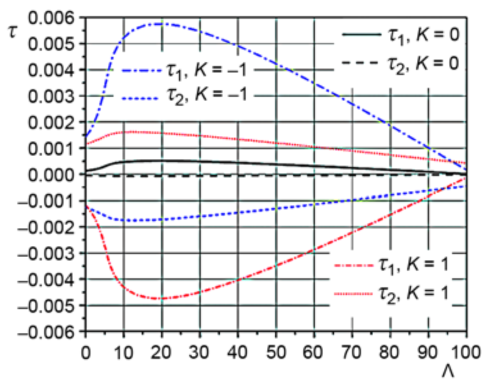

Figure 11. Effect of porosity parameter on shear stress

Figure 11 presents the effect of the porosity parameter on shear stress, while the Hartmann numbers take constant values. Increasing of porosity parameter primarily cause a increase of shear stress at both plates because of velocity increase. For high values of porosity parameter resistance of porous medium can be neglected and shear stress at the plates decreases for all values of loading factor, K. It is interesting to note increased shear stress in fluid region 1 with higher viscosity and also increased values for loading factor different from zero.

Figures 12 and 13 shows the behavior of the Nusselt number on lower and upper plate for different values of magnetic field intensity and porosity parameter. At the both plates and for both fluids Nusselt number decrease for Hartmann number between 1 and 2. This is due to the mutual influence of viscous and Joule heating. Increase of magnetic field intensity in the case when loading factor is different from zero increase the heat transfer. For both plates with increase of magnetic field intensity Nusselt number decrease for the case when loading factor is equal to zero. For the case of Hartmann flow heat transfer is mainly due to the conduction. Increase of porosity parameter decrease the Nusselt number for both fluids and for all values of loading factor. Convective heat transfer is more intense at the upper plate. For short circuit case porosity parameter t have very small influence on heat transfer. External electric field as additional pressure gradient increase significantly the heat transfer. At the upper plate Nusselt number changes rapidly for the case when loading factor is different from zero, and this is also the case for the lower plate, while in the case of short circuit conditions (K = 0) the temperature change near the plates are moderate.

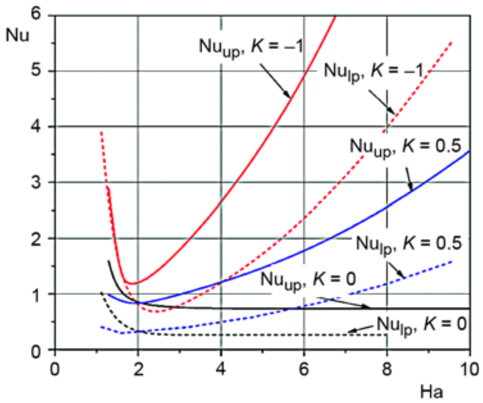

Figure 12. Effect of Hartmann number on heat transfer

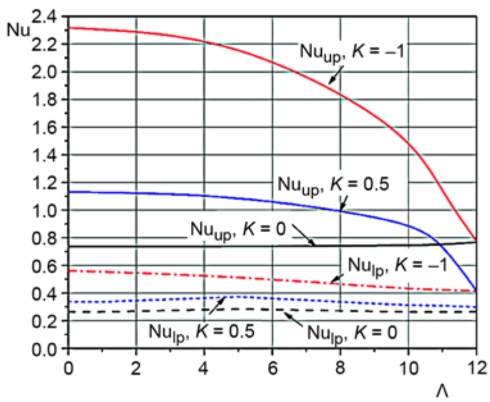

Figure 13. Effect of porosity parameter on heat transfer

Conclusion

In this paper the MHD flow and heat transfer of two viscous incompressible fluids through porous medium has been investigated in the paper. Fluids flow through homogeneous and isotropic porous medium of permeability, κ, between two parallel fixed isothermal plates. A uniform magnetic field has been applied in the direction making an arbitrary angle to the vertical axis, while electric field acts perpendicular to the flow. The general equations that describe the discussed problem under the adopted assumptions are reduced to ordinary differential equations and closed-form solutions are obtained. Effects of Hartmann number and porosity parameter on the heat and mass transfer have been analyzed. The influences of each of the governing parameters on dimensionless velocity, dimensionless temperature, shear stress and Nusselt number are discussed with the aid of graphs.

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Nomenclature

$\vec{B}$ – magnetic field vector, [T]
c_{pi} – specific heat capacity of fluid i [$Jkg^{-1}K^{-1}$]
$\vec{E}$ – electric field vector, [Vm^{-1}]
Ec_i – Eckert number in region i
Ha_i – Hartmann number in region i
h – region height, [m]
$\vec{j}$ – current density vector, [Am^{-2}]
K – load factor
k_i – thermal conductivity of fluid i, [$WK^{-1}m^{-1}$]
Pr_i – Prandtl number of fluid in region i
p – pressure, [Pa]
T – thermodynamic temperature, [K]
u_i – fluid velocity in region i, [ms^{-1}]
$\vec{v}$ – velocity vector [ms^{-1}]
x – longitudinal co-ordinate, [m]
y – transversal co-ordinate, [m]

Greek symbols

γ_i – viscosities ratio of fluids
δ – ratio of thermal conductivities
ε_i – 1 for porous medium, 0 for *clean* medium
Λ – porosity parameter
κ – permeability of porous medium [m^2]
Φ – dissipative function
μ_i – dynamic viscosity in region i, [$kgm^{-1}s^{-1}$]
ν_i – kinematic viscosity in region i, [m^2s^{-1}]
Θ_i – dimensionless temperature in region i
ρ_i – density of fluid in region i, [kgm^{-3}]
σ_i – electrical conductivity region i, [Sm^{-1}]
τ – shear stress, [$kgm^{-1}s^{-2}$]

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