

## HEAT TRANSFER IN MICROPOLAR FLUID FLOW UNDER THE INFLUENCE OF MAGNETIC FIELD

by

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*In this paper, the steady flow and heat transfer of an incompressible electrically conducting micropolar fluid through a parallel plate channel is investigated. The upper and lower plates have been kept at the two constant different temperatures and the plates are electrically insulated. Applied magnetic field is perpendicular to the flow, while the Reynolds number is significantly lower than one i. e. considered problem is in induction-less approximation. The general equations that describe the discussed problem under the adopted assumptions are reduced to ordinary differential equations and three closed-form solutions are obtained. The velocity, micro-rotation and temperature fields in function of Hartmann number, the coupling parameter and the spin-gradient viscosity parameter are graphically shown and discussed.*

**Key words:** micropolar fluid, heat transfer, MHD flow

### Introduction

Since the last century, many researchers have been interested in MHD, due to its important applications in different scientific and technology fields. The outer magnetic and electric field have found application in power plants, flow measurement, nuclear fusion reactor, cooling blankets, accelerators, MHD pumps, etc.

The requirements of modern technology have stimulated the interest in fluid flow studies, which involve the interaction of several phenomena. One of these phenomena is certainly viscous flow of electrically conducting micropolar fluid in the presence of a magnetic field. The theory of thermo-micropolar fluids has been developed by Eringen [1], taking into account the effect of micro-elements of fluids on both the kinematics and conduction of heat. The concept of micropolar fluid is introduced in an attempt to explain the behavior of a certain fluid containing polymeric additives and naturally occurring fluids such as the phenomenon of the flow of colloidal fluids, real fluid with suspensions, liquid crystals and animal blood, etc.

The flow and heat transfer of a viscous incompressible electrically conducting fluid between two infinite parallel insulating plates has been studied by many researchers [2-4] due to its important applications in the further development of MHD technology. The MHD de-

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vices for liquid metals attracted the attention of metallurgist [5]. It was shown that the effect of magnetic field could be very helpful in modernization of technological processes. The increasing interest in the study of MHD phenomena is also related to the development of fusion reactors where plasma is confined by a strong magnetic field [6]. Many exciting innovations were put forth in the areas of MHD propulsion [7], remote energy deposition for drag reduction [8], MHD control of flow and heat transfer in the boundary layer [9, 10].

All the cited studies are limited to classical Newtonian fluids. There are many fluids which are important from the industrial point of view, and display non-Newtonian behavior. Due to complexity of such fluids, several models have been proposed but the micropolar model is the most prominent one.

Eringen [1] initiated the concept of micropolar fluids to characterize the suspensions of neutrally buoyant rigid particles in a viscous fluid. The micropolar fluids exhibit micro-rotational and microinertial effects and support body couple and couple stresses. It may be noted that micropolar fluids take care of the micro-rotation of fluid particles by means of an independent kinematic vector called micro-rotation vector.

According to the theory of micropolar fluids proposed by Eringen [1] it is possible to recover the inadequacy of Navier-Stokes theory to describe the correct behavior of some types of fluids with microstructure such as animal blood, muddy water, colloidal fluids, lubricants and chemical suspensions. In the mathematical theory of micropolar fluids there is, in general, six degrees of freedom, three for translation and three for micro-rotation of micro-elements. Extensive reviews of the theory and applications can be found in the review articles [11, 12] and in the recent books [13, 14].

The research interest in the MHD flows of micropolar fluids has increased substantially over the past decades due to the occurrence of these fluids in industrial and magneto-biological processes. These flows take into account the effect arising from the local structure and micro-motions of the fluid elements. A comprehensive review of the subject and applications of micropolar fluid mechanics was given by Chamkha *et al.* [15] and Bachok *et al.* [16].

The MHD heat transfer of micropolar fluid can be divided in two parts. One contains problems in which the heating is an incidental byproduct of electromagnetic fields as in MHD generators *etc.*, and the second consists of problems in which the primary use of electromagnetic fields is to control the heat transfer, Toshivo *et al.* [17]. Heat transfer in micropolar fluid flow in the presence of magnetic field has gained considerable attention in recent years because of its various applications in contemporary technology. These applications include liquid crystals [18], blood flow in lungs or in arteries [19], flow and thermal control of polymeric processing [20].

Basic ideas and techniques for both steady and unsteady flow problems of Newtonian and non-Newtonian fluids are given by Ashrat *et al.* [21]. The basic equations governing the flow of couple stress fluids are non-linear in nature and even of higher order than the Navier-Stokes equations. Thus an exact solution of these equations is not easy to find. Different numerical, perturbation techniques and a reasonable simplification are commonly used for obtaining solutions of these equations [22, 23].

Keeping in view the wide area of practical importance of micropolar fluid flow and heat transfer as previously mentioned, the objective of the present study is to investigate the MHD flow and heat transfer characteristics of a viscous electrically conducting incompressible micropolar fluid in a parallel plate channel. Viscous dissipation and Joule heating effects have also been taken into account. The effects of the governing parameters on the flow and heat transfer aspects of the problem are discussed.

## Mathematical model

The problem of laminar MHD flow and heat transfer of an incompressible electrical-conducting micropolar fluid between parallel plates is considered. The MHD channel flow analysis is usually performed assuming the fluid constant electrical conductivity and treating the problem as an 1-D one: with these two main assumptions, the governing equations are considerably simplified and they can be solved analytically.

The physical model shown in fig. 1, consists of two infinite parallel plates extending in the  $x$ - and  $z$ -direction. Fully developed flow takes place between parallel plates that are at a distance  $h$ , as shown in fig. 1. Electrically conductive fluid flows through the channel due to the constant pressure gradient and magnetic field is applied perpendicular to the flow direction. A uniform magnetic field of the strength  $B$  is applied in the  $y$ -direction. The upper and lower plates have been kept at the two constant temperatures  $T_1$  and  $T_2$ , respectively. The fluid velocity vector  $\vec{v}$  and magnetic field induction vector  $\vec{B}$  are:

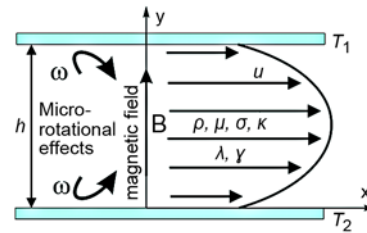


Figure 1. Physical model and co-ordinate system

$$\vec{v} = u\vec{i} \quad (1)$$

$$\vec{B} = B\vec{j} \quad (2)$$

Described laminar MHD flow and heat transfer is mathematically presented with following equations:

$$(\mu + \lambda) \frac{d^2 u^*}{dy^{*2}} + \lambda \frac{d\omega^*}{dy^*} - \sigma B^2 u^* - \frac{dp}{dx} = 0 \quad (3)$$

$$\gamma \frac{d^2 \omega^*}{dy^{*2}} - \lambda \frac{du^*}{dy^*} - 2\lambda \omega^* = 0, \quad (4)$$

$$k \frac{d^2 T^*}{dy^{*2}} + (\mu + \lambda) \left( \frac{du^*}{dy^*} \right)^2 + \sigma B^2 u^{*2} = 0 \quad (5)$$

The no-slip conditions require that the fluid velocities are equal to the plate's velocities, boundary conditions on temperature are isothermal conditions and there is no micro-rotation at the plates. The fluid and thermal boundary conditions for this problem are represented by equations:

$$\begin{aligned} u^* = 0, \quad \omega^* = 0, \quad T^* = T_2 \quad \text{for } y^* = 0, \\ u^* = 0, \quad \omega^* = 0, \quad T^* = T_1 \quad \text{for } y^* = h \end{aligned} \quad (6)$$

In previous general equations and boundary conditions, used symbols are common for the theory of MHD flows.

Now the following transformations have been used to transform previous equations to non-dimensional form:

$$y = \frac{y^*}{h}, \quad U = \frac{h^2 P}{\mu}, \quad P = -\frac{\partial p}{\partial x} = \text{const.}, \quad \theta = \frac{T - T_2}{T_1 - T_2}, \quad K = \frac{\lambda}{\mu},$$

$$\Gamma = \frac{\gamma}{\mu h^2}, \quad \text{Ha} = Bh \sqrt{\frac{\sigma}{\mu}}, \quad \text{Pr} = \frac{\mu c_p}{k}, \quad \text{Ec} = \frac{U^2}{c_p (T_1 - T_2)} \quad (7)$$

Equations (3)-(5) get the following form:

$$(1 + K) \frac{d^2 u}{dy^2} + K \frac{d\omega}{dy} - \text{Ha}^2 u + 1 = 0 \quad (8)$$

$$\Gamma \frac{d^2 \omega}{dy^2} - K \frac{du}{dy} - 2K\omega = 0 \quad (9)$$

$$\frac{d^2 \theta}{dy^2} + (1 + K) \text{Pr Ec} \left( \frac{du}{dy} \right)^2 + \text{Ec Pr Ha}^2 u^2 = 0 \quad (10)$$

The boundary dimensionless conditions for previous equations are:

$$u = 0, \quad \omega = 0, \quad T = T_2 \quad \text{for } y = 0,$$

$$u = 0, \quad \omega = 0, \quad T = T_1 \quad \text{for } y = 1 \quad (11)$$

After basic mathematical transformations from eqs. (8) and (9), the equation for velocity is:

$$u^{iv} - au''' + bu - d = 0 \quad (12)$$

where

$$A = \frac{K}{1 + K}, \quad B^* = \frac{\text{Ha}^2}{1 + K}, \quad C = \frac{1}{1 + K}, \quad D^* = \frac{K}{\Gamma}, \quad E = \frac{2K}{\Gamma} \quad (13)$$

The solution of eq. (12) has three possible cases, and there are three corresponding solutions for temperature and micro rotation, which are given with the following equations.

For the first case solutions are:

$$u = C_1 \exp(\delta_1 y) + C_2 \exp(\delta_2 y) + C_3 \exp(\delta_3 y) + C_4 \exp(\delta_4 y) + \frac{d}{b} \quad (14)$$

$$\omega = C_1 \mathcal{D}_1 \exp(\delta_1 y) + C_2 \mathcal{D}_2 \exp(\delta_2 y) + C_3 \mathcal{D}_3 \exp(\delta_3 y) + C_4 \mathcal{D}_4 \exp(\delta_4 y) \quad (15)$$

$$\theta = -\text{Pr Ec} \left[ \frac{1}{4\delta_1^2} {}^1C \exp(2\delta_1 y) + \frac{1}{4\delta_2^2} {}^2C \exp(2\delta_2 y) + \frac{1}{4\delta_3^2} {}^3C \exp(2\delta_3 y) + \right.$$

$$\left. + \frac{1}{4\delta_4^2} {}^4C \exp(2\delta_4 y) + \frac{{}^2D}{(\delta_1 + \delta_3)^2} \exp[(\delta_1 + \delta_3)y] + \frac{{}^3D}{(\delta_1 + \delta_4)^2} \exp[(\delta_1 + \delta_4)y] + \right.$$

$$\left. + \frac{{}^1E}{(\delta_2 + \delta_3)^2} \exp[(\delta_2 + \delta_3)y] + \frac{{}^2E}{(\delta_2 + \delta_4)^2} \exp[(\delta_2 + \delta_4)y] + \right.$$

$$+ \frac{1}{\delta_1^2} {}^1F \exp(\delta_1 y) + \frac{1}{\delta_2^2} {}^2F \exp(\delta_2 y) + \frac{1}{\delta_3^2} {}^3F \exp(\delta_3 y) + \frac{1}{\delta_4^2} {}^4F \exp(\delta_4 y) + \left[ \frac{1}{2} {}^1Dy^2 + \frac{1}{2} {}^3Ey^2 + \frac{1}{2} {}^5Fy^2 + {}^1H_1 y + {}^1H_2 \right] \quad (16)$$

For the second case solutions are:

$$u = (C_5 + C_6 y) \exp(\xi_1 y) + (C_7 + C_8 y) \exp(\xi_2 y) + \frac{d}{b} \quad (17)$$

$$\omega = (E_{13} + E_{14} y) \exp(\xi_1 y) + (E_{15} + E_{16} y) \exp(\xi_2 y) \quad (18)$$

$$\theta = -\text{Pr Ec} \left[ (\Omega_{28} + \Omega_{29} y + \Omega_{30} y^2) \exp(2\xi_1 y) + (\Omega_{31} + \Omega_{32} y + \Omega_{33} y^2) \exp(2\xi_2 y) + (\Omega_{34}^* + \Omega_{35}^* y) \exp(\xi_1 y) + (\Omega_{36}^* + \Omega_{37}^* y) \exp(\xi_2 y) + \Omega_{38}^* y^2 + \Omega_{39}^* y^3 + \Omega_{40}^* y^4 + {}^2H_1 y + {}^2H_2 \right] \quad (19)$$

For the third case solutions are:

$$u = [C_9 \cos(\beta_1 y) + C_{10} \sin(\beta_1 y)] \exp(\alpha_1 y) + [C_{11} \cos(\beta_1 y) + C_{12} \sin(\beta_1 y)] \exp(-\alpha_1 y) + \frac{d}{b} \quad (20)$$

$$\omega = [P_3^* \sin(\beta_1 y) + P_4^* \cos(\beta_1 y)] \exp(\alpha_1 y) + [P_5^* \sin(\beta_1 y) + P_6^* \cos(\beta_1 y)] \exp(-\alpha_1 y) \quad (21)$$

$$\theta = -\text{Pr Ec} \left\{ \left[ \frac{1}{2\alpha_1} \Omega_{45} + \frac{1}{2} (\chi_1 \Omega_{47} - \chi_2 \Omega_{49}) \cos(2\beta_1 y) + \frac{1}{2} (\chi_2 \Omega_{47} + \chi_1 \Omega_{49}) \sin(2\beta_1 y) \right] \exp(2\alpha_1 y) + \left[ \frac{1}{2\alpha_1} \Omega_{46} - \frac{1}{2} (\chi_1 \Omega_{48} + \chi_2 \Omega_{50}) \cos(2\beta_1 y) + \frac{1}{2} (\chi_2 \Omega_{48} - \chi_1 \Omega_{50}) \sin(2\beta_1 y) \right] \exp(-2\alpha_1 y) - \frac{1}{2\beta_1} \Omega_{51} \sin(2\beta_1 y) - \frac{1}{2\beta_1} \Omega_{52} \cos(2\beta_1 y) + [(\Omega_{53} \chi_1 - \Omega_{55} \chi_2) \cos(\beta_1 y) + (\Omega_{53} \chi_2 + \Omega_{55} \chi_1) \sin(\beta_1 y)] \exp(\alpha_1 y) + [(\Omega_{54} \chi_1 - \Omega_{56} \chi_2) \cos(\beta_1 y) - (\Omega_{54} \chi_2 + \Omega_{56} \chi_1) \sin(\beta_1 y)] \exp(-\alpha_1 y) + \frac{1}{2} \Omega_{57} y^2 + {}^3H_1 y + {}^3H_2 \right\} \quad (22)$$

where the constants from eqs. (14)-(22) are given in the Appendix.

## Results and discussion

The previous section defined the mathematical model for the steady flow and heat transfer of an incompressible electrically conducting micropolar fluid between two infinite horizontal parallel plates under a constant pressure gradient and applied magnetic field. Obtained solutions for the velocity, micro-rotation, temperature fields and shear stress in func-

tion of Hartmann number, the coupling parameter and the spin-gradient viscosity parameter are presented graphically.

Even though all the effects are present simultaneously, the motion may be assumed to be affected by viscous action, which is measured by  $\mu$ , the effect of couple stresses, measured by  $\gamma$ , and the direct coupling of the microstructure to the velocity field, measured by  $\lambda$ .

Figures 2 to 4 show the effect of the spin-gradient viscosity parameter on the distribution of velocity, micro-rotation and temperature fields.

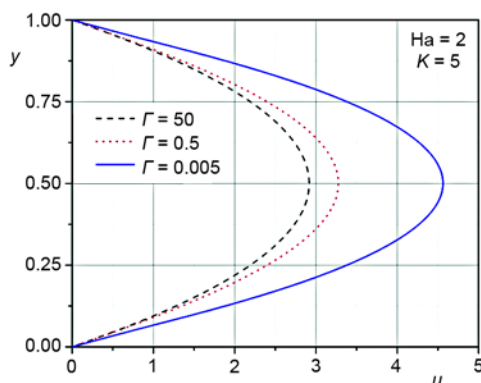


Figure 2. Velocity profiles for different values of the spin-gradient viscosity parameter

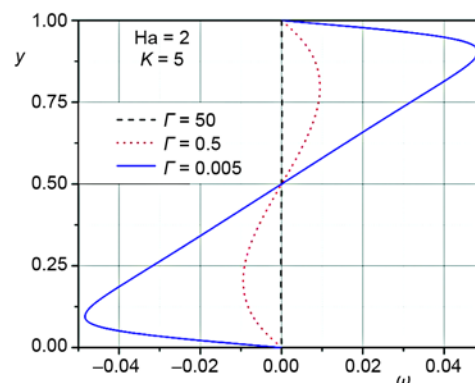


Figure 3. Micro-rotation in function of the spin-gradient viscosity parameter

Figure 2 shows the effect of the spin-gradient viscosity parameter on velocity, which predicts that the velocity increases as the spin-gradient viscosity parameter decrease. When the viscous effects are much larger than the couple stress effects, ratio  $\gamma/\mu$  is small, becoming zero when  $\gamma = 0$ , which is equivalent to the case of MHD flow in the channel.

This fact leads to the conclusion that the increase of gyro-viscosity  $\gamma$  reduces the flow compared to the viscous fluid case.

Micro-rotation in function of the spin-gradient viscosity parameter is shown in fig. 3. Increasing of the spin-gradient viscosity parameter causes a decrease in absolute values of micro-rotation.

Figure 4 show the effect of the spin-gradient viscosity parameter on dimensionless temperature field.

Increasing of the spin-gradient viscosity parameter causes a decrease of dimensionless temperature over the entire width of the channel. Increase of the spin gradient viscosity reduces the amount of energy transformed in the fluid. As gyro-viscosity increases, the dominant heat transfer mechanism is conduction.

The effect of Hartmann number on the velocity and micro rotation is shown in figs. 5 and 6. It can be seen from those figures that the velocity, as it is expected, decreases for large values of Hartmann number. This happens because of the imposing of a magnetic field normal to the flow direction, which creates a Lorentz force opposite to the flow direction. Similarly, the micro-rotation decreases with the increase of Hartman number, *i. e.* the magnetic field reduces the expected behavior of micropolar fluids.

For lower values of Hartmann number, viscous dissipation increases the temperature while for increase of magnetic field intensity only Joule heating effect increases the temperature, as shown in fig. 7.

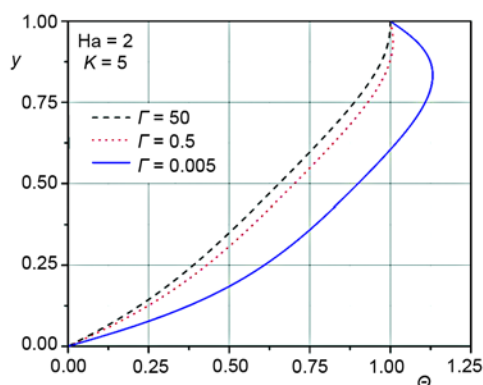


Figure 4. Dimensionless temperature in function of the spin-gradient viscosity parameter

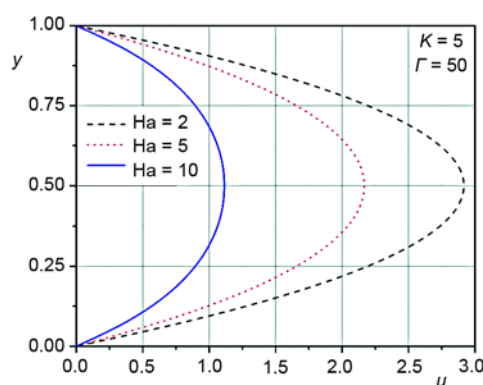


Figure 5. Velocity profiles for different values of Hartmann number

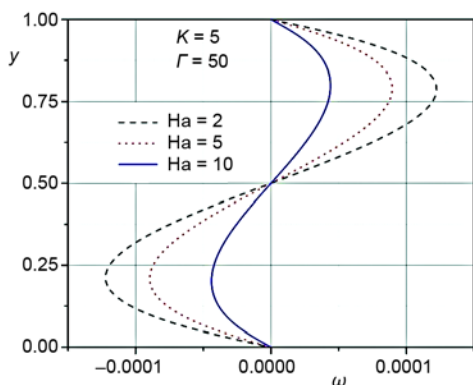


Figure 6. Micro-rotation for different values of Hartmann number

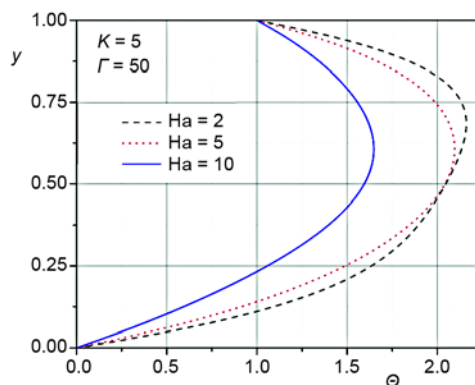


Figure 7. Dimensionless temperature as a function of Hartmann number

Figure 8 shows the effect of the coupling parameter on velocity. From figs. 8 and 9 it can be observed that the increase in coupling parameter  $K$  decreases the velocity but increases the micro-rotation, which means, as expected, that the resistance of fluid increases with the increase of  $K$ . In the limit  $K \rightarrow 0$  results correspond to the case viscous fluid.

The micro-rotation component,  $\omega$ , increases near the plates with increasing  $K$ , showing a reverse rotation near boundaries. An effect of coupling of the microstructure to the velocity field is more pronounced near the domain boundaries.

Figure 10 shows the influence of the coupling parameter on the dimensionless temperature. Increasing of the coupling parameter causes a decrease of dimensionless temperature over the entire height of the channel.

Figures 11 to 13 presents the influence of Hartmann number, coupling parameter and spin gradient viscosity parameter on shear stress which is in case of micropolar fluid defined in following way:  $\tau = (\mu + \lambda)(du/dy) + \lambda\omega$ .

Increase of Hartmann number decreases the shear stress while coupling parameter increases it. In the case of spin gradient viscosity there is no significant change of shear stress, the increase of shear stress with increase of spin viscosity of micropolar fluid is nearly

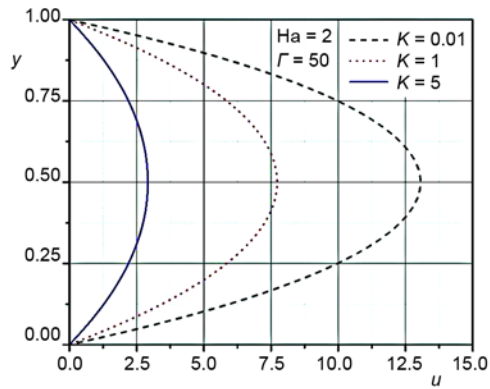


Figure 8. Velocity profiles for different values of the coupling parameter

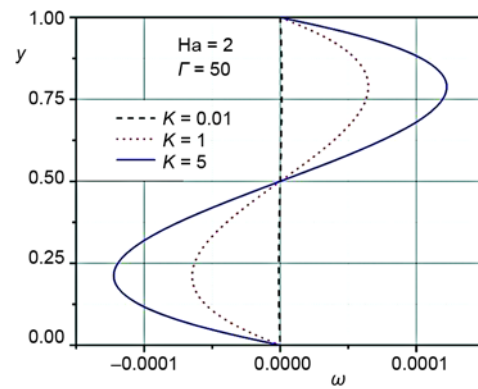


Figure 9. Micro-rotation for different values of the coupling parameter

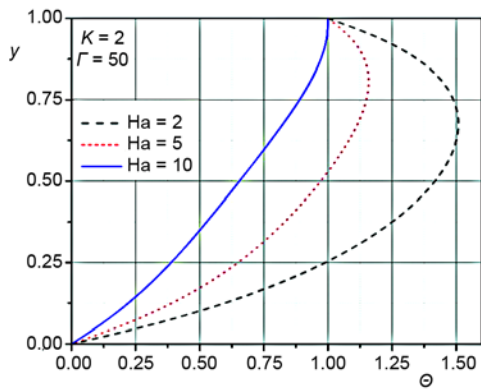


Figure 10. Dimensionless temperature as a function of the coupling parameter

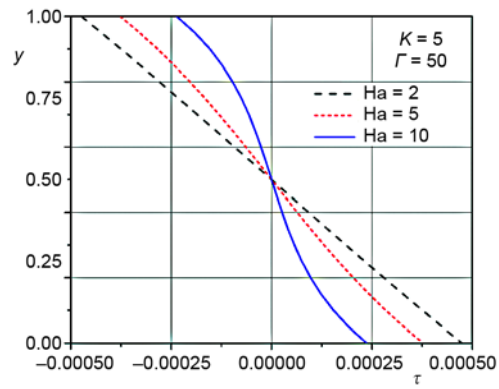


Figure 11. Shear stress as a function of Hartmann number

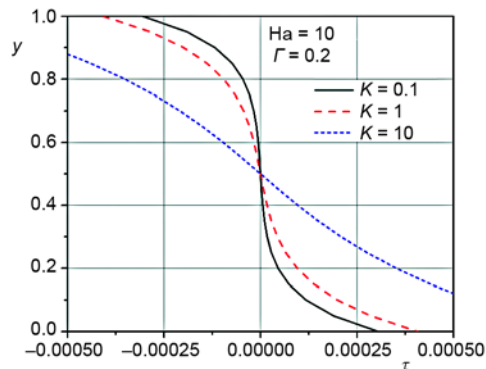


Figure 12. Shear stress as a function of the coupling parameter

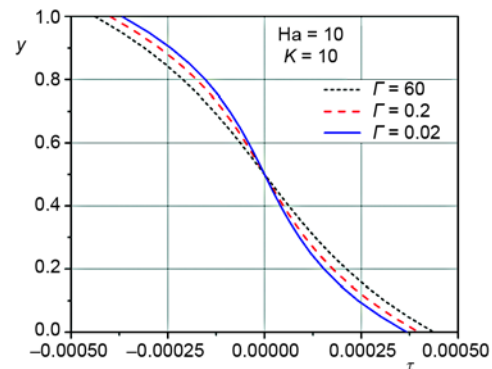


Figure 13. Shear stress as a function of the spin-gradient viscosity parameter

negligible. The influence of the vortex viscosity is significantly higher than the spin gradient viscosity, and this effect is particularly pronounced near the walls.



## Conclusion

In this paper, the steady flow and heat transfer of an incompressible electrically conducting micropolar fluid through a parallel plate channel is investigated. The upper and lower plates have been kept at the two constant different temperatures and the plates are electrically insulated. Applied magnetic field is perpendicular to the flow, while the Reynolds number is significantly lower than one, *i. e.* considered problem is in induction-less approximation. The general equations that describe the discussed problem under the adopted assumptions are reduced to ordinary differential equations and closed-form solutions are obtained. Effects of Hartmann number, the coupling parameter and the spin-gradient viscosity parameter on the heat and mass transfer have been analyzed. The influences of each of the governing parameters on dimensionless velocity, dimensionless temperature and micro-rotation are discussed by means of graphs.

## Acknowledgment

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## Appendix

For the first case, eqs. (14)-(16), constants are:

$$C_1 = -\frac{1}{\mathcal{D}_1}(\mathcal{D}_2 C_2 + \mathcal{D}_3 C_3 + \mathcal{D}_4 C_4); \quad C_2 = -\frac{1}{\mathfrak{R}_1}(\mathfrak{R}_2 C_3 + \mathfrak{R}_3 C_4);$$

$$C_3 = -\frac{1}{S_1}\left(S_2 C_4 + \frac{d}{b}\right); \quad C_4 = \frac{\mathcal{T}_1 - S_1}{\mathcal{T}_2 S_1 - \mathcal{T}_1 S_2} \frac{d}{b};$$

$$\mathfrak{T}_1 = \frac{1}{E}\left[\frac{B^*}{A} - D^*\right]; \quad \mathfrak{T}_2 = \frac{1}{EA}; \quad \mathcal{D}_i = \delta_i(\mathfrak{T}_1 - \delta_i^2 \mathfrak{T}_2) \quad i = 1, 2, 3, 4$$

$$\mathcal{M}_i = 1 - \mathcal{D}_{i+1}/\mathcal{D}_i; \quad \mathcal{N}_i = \exp \delta_{i+1} - \frac{\mathcal{D}_{i+1}}{\mathcal{D}_1} \exp \delta_1; \quad \mathfrak{R}_i = \mathcal{D}_{i+1}(\exp \delta_{i+1} - \exp \delta_1); \quad i = 1, 2, 3$$

$$\mathcal{T}_i = \mathcal{N}_{i+1} - \frac{\mathcal{N}_1}{\mathfrak{R}_1} \mathfrak{R}_{i+1}; \quad S_i = \mathcal{M}_{i+1} - \frac{\mathcal{M}_1}{\mathfrak{R}_1} \mathfrak{R}_{i+1}; \quad i = 1, 2$$

$$\begin{aligned} {}^1\mathcal{D}_1 &= \frac{{}^1C}{4\delta_1^2} + \frac{{}^2C}{4\delta_2^2} + \frac{{}^3C}{4\delta_3^2} + \frac{{}^4C}{4\delta_4^2} + \frac{{}^2D}{(\delta_1 + \delta_3)^2} + \frac{{}^3D}{(\delta_1 + \delta_4)^2} + \\ &+ \frac{{}^1E}{(\delta_2 + \delta_3)^2} + \frac{{}^2E}{(\delta_2 + \delta_4)^2} + \frac{{}^1F}{\delta_1^2} + \frac{{}^2F}{\delta_2^2} + \frac{{}^3F}{\delta_3^2} + \frac{{}^4F}{\delta_4^2}; \end{aligned}$$

$$\begin{aligned} {}^1\mathcal{D}_2 &= \frac{1}{4\delta_1^2} {}^1C \exp(2\delta_1) + \frac{1}{4\delta_2^2} {}^2C \exp(2\delta_2) + \frac{1}{4\delta_3^2} {}^3C \exp(2\delta_3) + \frac{1}{4\delta_4^2} {}^4C \exp(2\delta_4) + \\ &+ \frac{1}{2} {}^1D + \frac{{}^2D}{(\delta_1 + \delta_3)^2} \exp(\delta_1 + \delta_3) + \frac{{}^3D}{(\delta_1 + \delta_4)^2} \exp(\delta_1 + \delta_4) + \frac{{}^1E}{(\delta_2 + \delta_3)^2} \exp(\delta_2 + \delta_3) + \end{aligned}$$

$$+ \frac{{}^2E}{(\delta_2 + \delta_4)^2} \exp(\delta_2 + \delta_4) + \frac{1}{2} {}^3E + \frac{1}{\delta_1^2} {}^1F \exp(\delta_1) + \frac{1}{\delta_2^2} {}^2F \exp(\delta_2) +$$

$$+ \frac{1}{\delta_3^2} {}^3F \exp(\delta_3) + \frac{1}{\delta_4^2} {}^4F \exp(\delta_4) + \frac{1}{2} {}^5F;$$

$${}^1H_2 = -{}^1\mathcal{H}_1, \quad {}^1H_1 = {}^1\mathcal{H}_1 - {}^1\mathcal{H}_2 - \frac{1}{\text{Pr Ec}}; \quad {}^iC = C_i^2[(1+K)\delta_i^2 + \text{Ha}^2]; \quad i = 1, 2, 3, 4$$

$${}^iD = 2C_1C_{i+1}[(1+K)\delta_1\delta_{i+1} + \text{Ha}^2]; \quad i = 1, 2, 3$$

$${}^iE = 2C_2C_{i+2}[(1+K)\delta_2\delta_{i+2} + \text{Ha}^2]; \quad i = 1, 2$$

$${}^3E = 2C_3C_4[(1+K)\delta_3\delta_4 + \text{Ha}^2], \quad {}^iF = 2\frac{d}{b} \text{Ha}^2 C_i; \quad i = 1, 2, 3, 4 \quad {}^5F = \frac{d^2}{b^2} \text{Ha}^2$$

For the second case, eqs. (17)-(19), constants are:

$$C_5 = -\left(C_7 + \frac{d}{b}\right); \quad C_7 = -\frac{1}{E_7}(E_8C_8 + E_9), \quad C_8 = \frac{E_9E_{10} - E_7E_{12}}{E_7E_{11} - E_8E_{10}},$$

$$\mathfrak{T}_1 = \frac{1}{E} \left[ \frac{B^*}{A} - D^* \right]; \quad \mathfrak{T}_2 = \frac{1}{EA}; \quad F_i = \mathfrak{T}_1 - 3\mathfrak{T}_2\xi_i^2; \quad i = 1, 2 \quad E_i = \xi_i(\mathfrak{T}_1 - \mathfrak{T}_2\xi_i^2); \quad i = 1, 2$$

$$G_i = \xi_i(\mathfrak{T}_1 - \xi_i^2\mathfrak{T}_2) \exp \xi_i; \quad i = 1, 2, \quad H_i = [\mathfrak{T}_1(1 + \xi_i) - \mathfrak{T}_2\xi_i^2(3 + \xi_i)] \exp \xi_i; \quad i = 1, 2,$$

$$E_4 = E_1 \frac{d}{b}; \quad E_5 = \exp \xi_2 - \exp \xi_1; \quad E_6 = (1 - \exp \xi_1) \frac{d}{b}; \quad G_3 = G_2 - G_1, \quad G_4 = G_1 \frac{d}{b};$$

$$E_7 = E_5 - \frac{E_3}{F_1} \exp \xi_1; \quad E_8 = \exp \xi_2 - \frac{F_2}{F_1} \exp \xi_1; \quad E_9 = E_6 + \frac{E_4}{F_1} \exp \xi_1; \quad E_{10} = G_3 - \frac{H_1}{F_1} E_3;$$

$$E_{11} = H_2 - \frac{H_1}{F_1} F_2; \quad E_{12} = \frac{H_1}{F_1} E_4 - G_4; \quad E_{13} = E_1C_5 + F_1C_6; \quad E_{14} = E_1C_6; \quad E_{16} = E_2C_8;$$

$${}^2\mathcal{H}_1 = \Omega_{28} + \Omega_{31} + \Omega_{34}^* + \Omega_{36}^*;$$

$${}^2\mathcal{H}_2 = (\Omega_{28} + \Omega_{29} + \Omega_{30}) \exp(2\xi_1) + (\Omega_{31} + \Omega_{32} + \Omega_{33}) \exp(2\xi_2) +$$

$$+ (\Omega_{34}^* + \Omega_{35}^*) \exp(\xi_1) + (\Omega_{36}^* + \Omega_{37}^*) \exp(\xi_2) + \Omega_{38}^* + \Omega_{39}^* + \Omega_{40}^*;$$

$${}^2H_1 = {}^2\mathcal{H}_1 - {}^2\mathcal{H}_2 - \frac{1}{\text{Pr Ec}}; \quad {}^2H_2 = -{}^2\mathcal{H}_1;$$

$$\Omega_1 = (1+K)(C_6^2 + \xi_1^2 C_5^2 + 2C_5C_6\xi_1) + \text{Ha}^2 C_5^2;$$

$$\Omega_2 = (1+K)(2C_5C_6\xi_1^2 + 2\xi_1C_6^2) + 2\text{Ha}^2 C_5C_6;$$

$$\begin{aligned}
 \Omega_3 &= (1+K)(\xi_1^2 C_6^2 + \text{Ha}^2 C_6^2); \quad \Omega_4 = (1+K)(C_8^2 + \xi_2^2 C_7^2 + 2\xi_2 C_7 C_8) + C_7^2 \text{Ha}^2; \\
 \Omega_5 &= (1+K)(2C_7 C_8 \xi_2^2 + 2\xi_2 C_8^2) + 2C_7 C_8 \text{Ha}^2; \quad \Omega_6 = (1+K)\xi_2^2 C_8^2 + C_8^2 \text{Ha}^2; \\
 \Omega_7 &= (1+K)(2C_6 C_8 + 2C_6 C_7 \xi_2 + 2C_5 C_8 \xi_1 + 2\xi_1 \xi_2 C_5 C_7) + 2C_5 C_7 \text{Ha}^2; \\
 \Omega_8 &= (1+K)(2C_6 C_8 \xi_2 + 2C_6 C_8 \xi_1 + 2\xi_1 \xi_2 C_6 C_7 + 2\xi_1 \xi_2 C_5 C_8) + 2\text{Ha}^2 (C_6 C_7 + C_5 C_8); \\
 \Omega_{10} &= 2 \frac{d}{b} \text{Ha}^2 C_5; \quad \Omega_{12} = 2 \frac{d}{b} \text{Ha}^2 C_7; \quad \Omega_{13} = 2 \frac{d}{b} \text{Ha}^2 C_8; \quad \Omega_{14} = \frac{d^2}{b^2} \text{Ha}^2; \\
 \Omega_{15} &= \frac{\Omega_1}{2\xi_1} - \frac{\Omega_2}{4\xi_1^2} + \frac{\Omega_3}{4\xi_1^3}; \quad \Omega_{16} = \frac{\Omega_2}{2\xi_1} - \frac{\Omega_3}{2\xi_1^2}; \quad \Omega_{17} = \frac{\Omega_3}{2\xi_1}; \quad \Omega_{18} = \frac{\Omega_4}{2\xi_2} - \frac{\Omega_5}{4\xi_2^2} + \frac{\Omega_6}{4\xi_2^3}; \\
 \Omega_{19} &= \frac{\Omega_5}{2\xi_2} - \frac{\Omega_6}{2\xi_2^2}; \quad \Omega_{20} = \frac{\Omega_6}{2\xi_2}; \quad \Omega_{21}^* = \frac{\Omega_{10}}{\xi_1} - \frac{\Omega_{11}}{\xi_1^2}; \quad \Omega_{22}^* = \frac{\Omega_{11}}{\xi_1}; \quad \Omega_{23}^* = \frac{\Omega_{12}}{\xi_2} - \frac{\Omega_{13}}{\xi_2^2}; \\
 \Omega_{24}^* &= \frac{\Omega_{13}}{\xi_2}; \quad \Omega_{28} = \frac{\Omega_{15}}{2\xi_1} - \frac{\Omega_{16}}{4\xi_1^2} + \frac{\Omega_{17}}{4\xi_1^3}; \quad \Omega_{29} = \frac{\Omega_{16}}{2\xi_1} - \frac{\Omega_{17}}{2\xi_1^2}; \quad \Omega_{30} = \frac{\Omega_{17}}{2\xi_1}; \\
 \Omega_{31} &= \frac{\Omega_{18}}{2\xi_2} - \frac{\Omega_{19}}{4\xi_2^2} + \frac{\Omega_{20}}{4\xi_2^3}; \quad \Omega_{32} = \frac{\Omega_{19}}{2\xi_2} - \frac{\Omega_{20}}{2\xi_2^2}; \quad \Omega_{33} = \frac{\Omega_{20}}{2\xi_2}; \quad \Omega_{34}^* = \frac{\Omega_{21}^*}{\xi_1} - \frac{\Omega_{22}^*}{\xi_1^2}; \\
 \Omega_{35}^* &= \frac{\Omega_{22}^*}{\xi_1}; \quad \Omega_{36}^* = \frac{\Omega_{23}^*}{\xi_2} - \frac{\Omega_{24}^*}{\xi_2^2}; \quad \Omega_{37}^* = \frac{\Omega_{24}^*}{\xi_2}; \\
 \Omega_{38}^* &= \frac{\Omega_7 + \Omega_{14}}{2}; \quad \Omega_{39}^* = \frac{\Omega_8}{6}; \quad \Omega_{40}^* = \frac{\Omega_9}{12}.
 \end{aligned}$$

For the third case, eqs. (20)-(22), constants are:

$$\begin{aligned}
 C_9 &= -\left(C_{11} + \frac{d}{b}\right); \quad C_{10} = Q_9^* C_{11} + Q_9 - C_{12}; \quad C_{12} = \frac{Q_{16}^* Q_{15} - Q_{17} Q_{14}^*}{Q_{16} Q_{14}^* - Q_{16}^* Q_{14}}; \\
 \mathfrak{I}_1 &= \frac{1}{E} \left( \frac{B^*}{A} - D^* \right); \quad \mathfrak{I}_2 = \frac{1}{EA}; \quad \alpha_1 = \sqrt{R} \cos \frac{\theta_1}{2}; \quad \beta_1 = \sqrt{R} \sin \frac{\theta_1}{2}; \\
 P &= (\mathfrak{I}_1 - \mathfrak{I}_2 \alpha_1^2 + 3\mathfrak{I}_2 \beta_1^2) \alpha_1; \\
 Q &= (\mathfrak{I}_1 - 3\mathfrak{I}_2 \alpha_1^2 + \mathfrak{I}_2 \beta_1^2) \beta_1; \quad Q_1 = \cos \beta_1 \exp \alpha_1; \quad Q_2 = \sin \beta_1 \exp \alpha_1; \\
 Q_3 &= \cos \beta_1 \exp(-\alpha_1); \quad Q_4 = \sin \beta_1 \exp(-\alpha_1); \\
 P_1 &= \mathfrak{I}_1 - \mathfrak{I}_2 \alpha_1^2 + \mathfrak{I}_2 \beta_1^2; \quad P_2 = 2\mathfrak{I}_2 \alpha_1 \beta_1; \quad P_4 = (P_2 \sin \beta_1 + P_1 \cos \beta_1) \exp(\alpha_1); \\
 P_6 &= (P_1 \sin \beta_1 + P_2 \cos \beta_1) \exp(-\alpha_1); \quad Q_5 = P_3 \alpha_1 + P_4 \beta_1; \\
 Q_6 &= P_4 \alpha_1 - P_3 \beta_1; \quad Q_8 = P_5 \beta_1 - P_6 \alpha_1;
 \end{aligned}$$

$$Q_9 = \frac{P}{Q} \frac{d}{b}; \quad Q_9^* = \frac{2P}{Q}; \quad Q_{10} = Q_3 - Q_1; \quad Q_{11} = (1 - Q_1) \frac{d}{b};$$

$$Q_{12} = Q_6 + Q_7; \quad Q_{13} = Q_6 \frac{d}{b}; \quad Q_{14} = Q_4 - Q_2; \quad Q_{14}^* = Q_2 Q_9^* + Q_{10}; \quad Q_{15} = Q_{11} + Q_2 Q_9;$$

$$Q_{16} = Q_8 - Q_5; \quad Q_{16}^* = Q_5 Q_9^* - Q_{12}; \quad Q_{17} = Q_5 Q_9 - Q_{13};$$

$$P_3^* = P_1(\alpha_1 C_{10} - \beta_1 C_9) + P_2(\beta_1 C_{10} + \alpha_1 C_9); \quad P_4^* = P_1(\beta_1 C_{10} + \alpha_1 C_9) - P_2(\alpha_1 C_{10} - \beta_1 C_9);$$

$$P_5^* = -[P_1(\alpha_1 C_{12} + \beta_1 C_{11}) + P_2(\beta_1 C_{12} - \alpha_1 C_{11})]; \quad P_6^* = P_1(\beta_1 C_{12} - \alpha_1 C_{11}) - P_2(\alpha_1 C_{12} + \beta_1 C_{11});$$

$${}^3\mathcal{B}_1 = \left[ \frac{1}{2\alpha_1} \Omega_{45} + \frac{1}{2} (\Omega_{47} \chi_1 - \Omega_{49} \chi_2) \right] + \left[ \frac{1}{2\alpha_1} \Omega_{46} - \frac{1}{2} (\Omega_{48} \chi_1 + \Omega_{50} \chi_2) \right] - \\ - \frac{1}{2\beta_1} \Omega_{52} + (\Omega_{53} \chi_1 - \Omega_{55} \chi_2) + (\Omega_{54} \chi_1 - \Omega_{56} \chi_2);$$

$$\chi_1 = \frac{\alpha_1}{\alpha_1^2 + \beta_1^2}; \quad \chi_2 = \frac{\beta_1}{\alpha_1^2 + \beta_1^2};$$

$${}^3\mathcal{B}_2 = \left[ \frac{1}{2} (\chi_1 \Omega_{47} - \chi_2 \Omega_{49}) \cos(2\beta_1) + \frac{1}{2} (\chi_2 \Omega_{47} + \chi_1 \Omega_{49}) \sin(2\beta_1) \right] \exp(2\alpha_1) + \\ + \left[ -\frac{1}{2} (\chi_1 \Omega_{48} + \chi_2 \Omega_{50}) \cos(2\beta_1) + \frac{1}{2} (\chi_2 \Omega_{48} - \chi_1 \Omega_{50}) \sin(2\beta_1) \right] \exp(-2\alpha_1) + \\ + \frac{\Omega_{45}}{2\alpha_1} \exp(2\alpha_1) + \frac{\Omega_{46}}{2\alpha_1} \exp(-2\alpha_1) - \frac{\Omega_{51}}{2\beta_1} \sin(2\beta_1) - \frac{\Omega_{52}}{2\beta_1} \cos(2\beta_1) + \\ + [(\Omega_{53} \chi_1 - \Omega_{55} \chi_2) \cos(\beta_1) + (\Omega_{53} \chi_2 + \Omega_{55} \chi_1) \sin(\beta_1)] \exp(\alpha_1) + \\ + [(\Omega_{54} \chi_1 - \Omega_{56} \chi_2) \cos(\beta_1) - (\Omega_{54} \chi_2 + \Omega_{56} \chi_1) \sin(\beta_1)] \exp(-\alpha_1) + \frac{\Omega_{57}}{2};$$

$${}^3H_1 = {}^3\mathcal{B}_1 - {}^3\mathcal{B}_2 - \frac{1}{\text{Pr Ec}}; \quad {}^3H_2 = -{}^3\mathcal{B}_1;$$

$$\Omega_{42} = \beta_1 C_{10} + \alpha_1 C_9; \quad \Omega_{43} = \beta_1 C_{12} - \alpha_1 C_{11}; \quad \Omega_{44} = \alpha_1 C_{12} + \beta_1 C_{11};$$

$$\Omega_{45} = \frac{1}{4\alpha_1} [(1 + K)(\Omega_{41}^2 + \Omega_{42}^2) + \text{Ha}^2 (C_9^2 + C_{10}^2)];$$

$$\Omega_{46} = \frac{1}{4\alpha_1} [(1 + K)(\Omega_{43}^2 + \Omega_{44}^2) + \text{Ha}^2 (C_{11}^2 + C_{12}^2)];$$

$$\begin{aligned}\Omega_{47} &= \frac{1}{4} \frac{\alpha_1}{\alpha_1^2 + \beta_1^2} [(1+K)(\Omega_{42}^2 - \Omega_{41}^2) + \text{Ha}^2 (C_9^2 - C_{10}^2)] - \\ &\quad - \frac{1}{2} \frac{\beta_1}{\alpha_1^2 + \beta_1^2} [(1+K)\Omega_{41}\Omega_{42} + \text{Ha}^2 C_9 C_{10}]; \\ \Omega_{48} &= \frac{1}{4} \frac{\alpha_1}{\alpha_1^2 + \beta_1^2} [(1+K)(\Omega_{44}^2 - \Omega_{43}^2) + \text{Ha}^2 (C_{12}^2 - C_{11}^2)] + \\ &\quad + \frac{1}{2} \frac{\beta_1}{\alpha_1^2 + \beta_1^2} [(1+K)\Omega_{43}\Omega_{44} - \text{Ha}^2 C_{11}C_{12}]; \\ \Omega_{49} &= \frac{1}{2} \frac{\alpha_1}{\alpha_1^2 + \beta_1^2} [(1+K)\Omega_{41}\Omega_{42} + \text{Ha}C_9C_{10}] + \\ &\quad + \frac{1}{4} \frac{\beta_1}{\alpha_1^2 + \beta_1^2} [(1+K)(\Omega_{42}^2 - \Omega_{41}^2) + \text{Ha}^2 (C_9^2 - C_{10}^2)]; \\ \Omega_{50} &= \frac{1}{2} \frac{\alpha_1}{\alpha_1^2 + \beta_1^2} [(1+K)\Omega_{43}\Omega_{44} - \text{Ha}C_{11}C_{12}] + \\ &\quad + \frac{1}{4} \frac{\beta_1}{\alpha_1^2 + \beta_1^2} [(1+K)(\Omega_{43}^2 - \Omega_{44}^2) + \text{Ha}^2 (C_{11}^2 - C_{12}^2)]; \\ \Omega_{51} &= \frac{1}{2\beta_1} [(1+K)(\Omega_{41}\Omega_{43} - \Omega_{42}\Omega_{44}) + \text{Ha}^2 (C_9C_{12} + C_{10}C_{11})]; \\ \Omega_{52} &= \frac{1}{2\beta_1} [(1+K)(\Omega_{42}\Omega_{43} - \Omega_{41}\Omega_{44}) + \text{Ha}^2 (C_9C_{11} - C_{10}C_{12})]; \\ \Omega_{53} &= 2 \frac{d}{b} \text{Ha}^2 (C_9\alpha_1 - C_{10}\beta_1)(\alpha_1^2 + \beta_1^2)^{-1}; \quad \Omega_{54} = 2 \frac{d}{b} \text{Ha}^2 (C_{11}\alpha_1 + C_{12}\beta_1)(\alpha_1^2 + \beta_1^2)^{-1}; \\ \Omega_{55} &= 2 \frac{d}{b} \text{Ha}^2 (C_9\beta_1 + C_{10}\alpha_1)(\alpha_1^2 + \beta_1^2)^{-1}; \quad \Omega_{56} = 2 \frac{d}{b} \text{Ha}^2 (C_{11}\beta_1 - C_{12}\alpha_1)(\alpha_1^2 + \beta_1^2)^{-1}; \\ \Omega_{57} &= (1+K)(\Omega_{42}\Omega_{43} - \Omega_{41}\Omega_{44}) + \text{Ha}^2 \left( C_9C_{11} + C_{10}C_{12} + \frac{d^2}{b^2} \right)\end{aligned}$$

## Nomenclature

$\vec{B}$  – magnetic field vector, [T]  
 $c_p$  – specific heat capacity, [Jkg<sup>-1</sup>K<sup>-1</sup>]  
 $\text{Ha}$  – Hartmann number,  
 $h$  – height of channel, [m]  
 $k$  – thermal conductivity of fluid, [WK<sup>-1</sup>m<sup>-1</sup>]  
 $p$  – pressure, [Pa]  
 $T$  – temperature, [K]  
 $u$  – fluid velocity, [ms<sup>-1</sup>]  
 $x$  – longitudinal co-ordinate, [m]  
 $y$  – transversal co-ordinate, [m]

## Greek symbols

$\gamma$  – spin gradient viscosity, [kgms<sup>-1</sup>]  
 $\theta$  – dimensionless temperature,  
 $\lambda$  – vortex viscosity, [kgm<sup>-1</sup>s<sup>-1</sup>]  
 $\mu$  – dynamic viscosity, [kgm<sup>-1</sup>s<sup>-1</sup>]  
 $\nu$  – kinematic viscosity, [m<sup>2</sup>s<sup>-1</sup>]  
 $\rho$  – density of fluid, [kgm<sup>-3</sup>]  
 $\omega$  – micro-rotation vector, [s<sup>-1</sup>]  
 $\sigma$  – electrical conductivity, [Sm<sup>-1</sup>]

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