KINK DEGENERACY AND ROGUE POTENTIAL FLOW FOR THE (3+1)-DIMENSIONAL GENERALIZED KADOMTSEV-PETVIASHVILI EQUATION

by

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The breather-type kink soliton, breather-type periodic soliton solutions and rogue potential flow for the (3+1)-dimensional generalized Kadomtsev-Petviashvili equation are obtained by using the extended homoclinic test technique and homoclinic breather limit method, respectively. Furthermore, some new non-linear phenomena, such as kink and periodic degeneracy, are investigated and the new rational breather solutions are found out. Meanwhile, we also obtained the rational potential solution and it is just a rogue wave. These results enrich the variety of the dynamics of higher-dimensional non-linear wave field.

Key words: *generalized (3+1)-dimensional Kadomtsev-Petviashvili equation, homoclinic breather limit method, rational breather solutions, kink degeneracy, rogue potential flow*

Introduction

In recent years, solitary wave solutions of non-linear evolution equations play an important role in non-linear science fields, especially in non-linear physical science, since they can provide much physical information and more insight into the physical aspects of the problem and thus lead to further applications [1]. It is well known there are many of methods for finding special solutions of non-linear partial differential equations, such as the inverse scattering method [1, 2], the homogeneous balance method [2, 3], the Darboux transformation method [4, 5], Hirotas bi-linear method [6, 7], the variable separation approach [8], the extended tanh-method [9, 10], the Lie group method [11, 12], the extended homoclinic test approach (EHTA) [13-16], and so on.

In this work, the (3+1)-D generalized Kadomtsev-Petviashvili (KP) equation:

$$u_{xxxy} + 3(u_x u_y)_x + u_{tx} + u_{ty} - u_{zz} = 0 \tag{1}$$

will be considered, where $u: R_x \times R_y \times R_z \times R_t \to R.$ When $y = x$, the equation reduces to the KP equation, and so we call it a generalized KP (GKP) equation. Some other similar or variable-coefficient generalizations for the KP equation are studied in the references [17-19]. Exact solutions of the GKP equation have been studied by means of some effective approaches, such as solitary wave solution [20], multiple soliton solutions [21], Wronskian and Grammian

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solutions [22], and so on. But to our best knowledge, rational breather solutions to the (3+1)-D GKP equation (1) have not been reported in previous literatures.

In this manuscript, a novel approach of seeking for the rational breather-wave solution, the homoclinic breather limit method [23, 24], is proposed and applied to solve the (3+1)-D GKP equation.

Homoclinic breather limit method

Consider a high dimensional non-linear evolution equation of the general form:

$$P(u, u_t, u_x, u_y, u_z, u_{xx}, u_{yy}, u_{zz} \cdots) = 0 \tag{2}$$

where $u = u(x, y, z, t)$ and P is a polynomial of u and its derivatives.

The basic idea of the extended homoclinic test method can be expressed in the following four steps:

Step 1. By Painleve analysis, a transformation:

$$u = T(f) \tag{3}$$

is made for the new and unknown function *f*.

Step 2. By using the transformation in step 1, the original equation can be converted into Hirota's bi-linear form:

$$G(D_t, D_x, D_y, D_z; f) = 0 \tag{4}$$

where the D operator [25] is defined by:

$$Q(D_x, D_y, D_z, D_t, \cdots)F(x, y, z, t, \cdots)G(x, y, z, t, \cdots) =$$

$$= Q(\partial_x - \partial_{x'}, \partial_y - \partial_{y'}, \partial_z - \partial_{z'}, \partial_t - \partial_{t'}, \cdots)F(x, y, z, t, \cdots)G(x', y', z', t', \cdots)|_{x'=x, y'=y, z'=z, t'=t, \cdots} \tag{5}$$

where Q is a polynomial of $D_x, D_y, D_z, \cdots$.

Step 3. Solve the previous equation to get homoclinic breather wave solution by using the EHTA [26].

Step 4. Let the period of periodic wave go to infinite in homoclinic breather wave solution, we can obtain a rational breather-wave solution.

Step 5. Solving potential of the breather wave solution in the step 3 and let p tends to zero, we can obtain a rational homoclinic (heteroclinic) wave and this wave is just a rogue wave.

Applications

Kink degeneracy and new rational breather solution

By using Painleve test we can assume that the solution of eq. (1):

$$u(x, y, z, t) = 2(\ln f)_x \tag{6}$$

where $f(x, y, z, t)$ is an unknown real function.

Substituting eq. (6) into eq. (1) we obtain the following bi-linear form:

$$(D_x^3 D_y + D_t D_x + D_t D_y - D_z^2) f \cdot f = 0 \tag{7}$$

where $D_x D_t f\, f = 2(f f_{xt} - f_x f_t)$, $\quad D_x^3 D_y f \cdot f = 2(f f_{xxxy} - f_{xxx} f_y + 3 f_{xx} f_{xy} - 3 f_x f_{xxy})$

With regard to eq. (7), using the homoclinic test technique we can seek the solution in the form:

$$f = \mathrm{e}^{-p_1(\xi)} + \delta_1 \cos[p(\eta)] + \delta_2 \mathrm{e}^{p_1(\xi)} \tag{8}$$

where $\xi = x + a_1 y + b_1 z + c_1 t + d, \eta = x + a_2 y + b_2 z + c_2 t, a_1, b_1, c_1, a_2, b_2, c_2, d, p_1, p, \delta_1, \delta_2$ are real constants to be determined.

Substituting eq. (8) into eq. (7), and equating all the coefficients of different powers of $\mathrm{e}^{-\xi}, \mathrm{e}^{\xi}$, $\cos(\eta), \sin(\eta)$ and constant term to zero, we can obtain a set of algebraic equations for $a_i, b_i, c_i, p_i, \delta_j (i = 1, 2; j = 1, 2)$.

Solving the system with the aid of MAPLE, we get the following results:

$$\begin{cases} -4\delta_2 p_1^2 b_1^2 - \delta_1^2 p^2 a_2 c_2 - \delta_1^2 p^2 c_2 + 4\delta_1^2 p^4 a_2 + \delta_1^2 p^2 b_2^2 + 4\delta_2 p_1^2 a_1 c_1 + 16\delta_2 p_1^4 a_1 + 4\delta_2 p_1^2 c_1 = 0 \\ -a_2 p_1^2 - 3a_1 p_1^2 - a_1 c_2 - a_2 c_1 + 2b_1 b_2 + p^2 a_1 - c_1 - c_2 + 3p^2 a_2 = 0 \\ p^4 a_2 + p_1^2 c_1 + p_1^4 a_1 - p^2 c_2 - p_1^2 b_1^2 - 3p^2 p_1^2 a_1 + p_1^2 a_1 c_1 - p^2 a_2 c_2 - 3p_1^2 p^2 a_2 + p^2 b_2^2 = 0 \end{cases} \tag{9}$$

If $a_1 = 0, b_2 = 0, p_1 = p$, solving eq. (9) yields:

$$\begin{cases} c_1 = \dfrac{2p^2 a_2^2 + 4p^2 a_2 + b_1^2}{2a_2 + a_2^2 + 2} \\ c_2 = -\dfrac{2p^2 a_2^2 + b_1^2 + b_1^2 a_2}{2a_2 + a_2^2 + 2} \\ \delta_2 = -\dfrac{1}{4}\dfrac{\delta_1^2 (8p^2 a_2 + 6p^2 a_2^3 + 2a_2 b_1^2 + b_1^2 a_2^2 + 10p^2 a_2^2 + b_1^2)}{2p^2 a_2^2 + 4p^2 a_2 - 2a_2 b_1^2 - b_1^2 a_2^2 - b_1^2} \end{cases} \tag{10}$$

where a_2, b_1, δ_1, d, and p are some free real constants.

Substituting eq. (10) into eq. (8) and take $\delta_1 > 0, M > 0,$ we have:

$$f(x,y,z,t) = |\delta_1| \sqrt{M} \cosh\left[p(x + b_1 z + H_1 t + d) + \ln\left(\frac{1}{2} \left|\delta_1 \sqrt{M}\right| \right) \right] + \delta_1 \cos[p(x + a_2 y - L_1 t)] \tag{11}$$

where

$$H_1 = \frac{2p^2 a_2^2 + 4p^2 a_2 + b_1^2}{2a_2 + a_2^2 + 2}, \qquad L_1 = \frac{2p^2 a_2^2 + b_1^2 a_2 + b_1^2}{2a_2 + a_2^2 + 2}$$

and

$$M = \frac{8p^2 a_2 + 6p^2 a_2^3 + 2a_2 b_1^2 + b_1^2 a_2^2 + 10p^2 a_2^2 + b_1^2}{-2p^2 a_2^2 - 4p^2 a_2 + 2a_2 b_1^2 + b_1^2 a_2^2 + b_1^2}$$

Substituting eq. (11) into eq. (6) yields the exact breather-type kink soliton solutions of the (3+1)-D GKP equation:

$$u(x,y,z,t)=$$
$$=\frac{2\left\{p|\delta_1|\sqrt{M}\sinh\left[p(x+b_1z+H_1t+d)+\ln\left(\frac{1}{2}\left|\delta_1\sqrt{M}\right|\right)\right]-\delta_1\sin[p(x+a_2y-L_1t)]\right\}}{|\delta_1|\sqrt{M}\cosh\left[p(x+b_1z+H_1t+d)+\ln\left(\frac{1}{2}\left|\delta_1\sqrt{M}\right|\right)\right]+\delta_1\cos[p(x+a_2y-L_1t)]} \tag{12}$$

The solution $u(x,y,z,t)$ represented by eq. (12) is breather-type kink-wave. It is generated by the interaction between the soliton of variable $X = p(x + b_1z + H_1t + d) + \ln(1/2|\delta_1\sqrt{M}|)$ and the periodic wave of variable $Y = p(x+a_2y-L_1t)$, fig 1(a).

Especially, if we choose $\delta_1 = -2$ and let $p \to 0$ in eq. (12), we can get the rational breather solutions:

$$u(x,y,z,t)=\frac{4b_1^2(a_2+1)^2(A+B)}{b_1^2(a_2+1)^2(A^2+B^2)+6a_2(a_2^2+2a_2+1)} \tag{13}$$

where $A = x + b_1z + \frac{b_1^2 t}{2a_2+a_2^2+2} + d$, $B = x + a_2y - \frac{b_1^2(a_2+1)t}{2a_2+a_2^2+2}, a_2 > 0.$

The solution $u(x,y,z,t)$ represented by eq. (13) is a new rational breather solution. Notice u tends to zero in eq. (13) when the $t \to \pm\infty$, so it is no longer the kinky. Such a surprising feature of weakly dispersive long-wave is firstly obtained. Meanwhile, this shows that kink is degenerated when the period of breather wave tends to infinite in the breather kink-wave, fig. 1(b). This is a new non-linear phenomenon up to now.

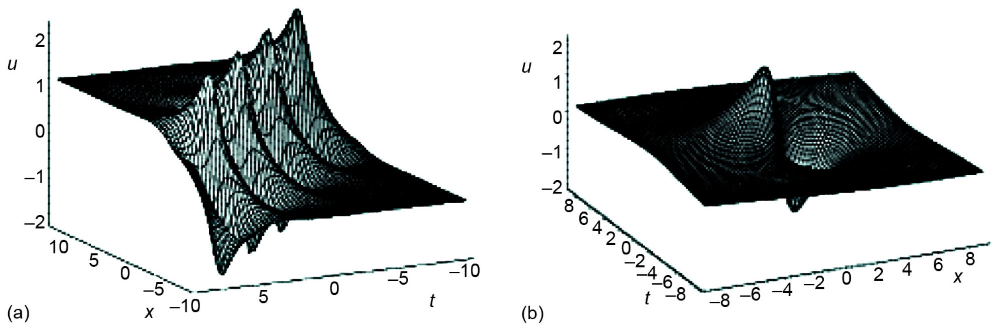

Figure 1. (a) The breather-type kink soliton solutions as $b_1 = 2, p = 1/2, p_1 = 1/2, \delta_1 = 1, d = y = z = 0$; (b) the rational breather solution as $a_2 = 1, b_1 = 2, d = y = z = 0$

Periodic degeneracy and new rational breather solution

By choosing the special test function [27, 28] in homoclinic breather limit method to the (3+1)-D GKP equation, we obtain the breather-type periodic soliton solutions and rational breather solutions.

We suppose that the solution of eq. (7) is:

$$f(x,y,z,t)=\mathrm{e}^{-p_1(y+b_1z+ct+d)}+\delta_1\cos[p(x+b_2z)]+\delta_2\mathrm{e}^{p_1(y+b_1z+ct+d)} \tag{14}$$

where $b_1, b_2, c, d, \delta_1, \delta_2, p,$ and p_1 are some free real constants.

Substituting eq. (14) into eq. (7), and equating all the coefficients of different powers of $e^{-p_1(y+b_1z+ct+d)}, e^{p_1(y+b_1z+ct+d)}$, $\sin[p(x+b_2z)]$, $\cos[p(x+b_2z)]$, and constant term to zero, we can obtain a set of algebraic equations for $c, b_i, \delta_i, p_i (i=1,2)$, namely:

$$\begin{cases} p_1^2c + p^2b_2^2 - p_1^2b_1^2 = 0, \\ -p^2 + c - 2b_1b_2 = 0, \\ \delta_1^2p^2b_2^2 + 4\delta_2p_1^2c - 4\delta_2p_1^2b_1^2 = 0 \end{cases} \tag{15}$$

Solving the system with the aid of MAPLE, we get the following results:

$$c = \frac{b_1(2b_2^3 + b_1p_1^2)}{p_1^2 + b_2^2}, \quad p = \sqrt{-\frac{2b_2b_1 - b_1^2}{p_1^2 + b_2^2}}p_1, \quad \delta_2 = \frac{1}{4}\delta_1^2 \tag{16}$$

Substituting eq. (16) into eq. (7) and take $b_1^2 > 2b_2b_1$, we have:

$$f(x,y,z,t) = |\delta_1|\cosh\left[p_1(y + b_1z + H_2t + d) + \ln\left(\frac{1}{2}|\delta_1|\right)\right] + \delta_1\cos\left[\sqrt{L_2}p_1(x + b_2z)\right] \tag{17}$$

where $H_2 = \dfrac{b_1(2b_2^3 + b_1p_1^2)}{p_1^2 + b_2^2}$ and $L_2 = -\dfrac{2b_2b_1 - b_1^2}{p_1^2 + b_2^2}$.

Substituting eq. (17) into eq. (6) yields breather-type periodic soliton solutions of the (3+1)-D GKP equation:

$$u(x,y,z,t) = -\frac{2\delta_1\sqrt{L_2}p_1\sin[\sqrt{L_2}p_1(x + b_2z)]}{|\delta_1|\cosh\left[p_1(y + b_1z + H_2t + d) + \ln\left(\frac{1}{2}|\delta_1|\right)\right] + \delta_1\cos\left[\sqrt{L_2}p_1(x + b_2z)\right]} \tag{18}$$

The solution $u(x,y,z,t)$ represented by eq. (18) can be considered as a soliton of variable $X = p_1(x + b_1z + H_2t + d) + \ln(1/2|\delta_1|)$ spread along the direction of variable $Y = (L_2)^{1/2}p_1(x + a_2z)$, fig. 2(a).

Especially, if we choose $\delta_1 = -2$ in eq. (18), while let $p_1 \to 0$, we can get the rational breather solution:

$$u(x,y,z,t) = \frac{4b_1(b_1 - 2b_2)(x + b_2z)}{(b_1^2 - 2b_1b_2)(x + b_2z)^2 + b_2^2(y + b_1z + 2b_1b_2t + d)^2} \tag{19}$$

where the solution $u(x,y,z,t)$ represented by eq. (19) is breather wave and no longer has periodic feature. Here periodic degeneracy occurs when the period of periodic wave tends infinity. This is a strange and interesting physical phenomenon to the evolution of 2-D flow of shallow-water waves having small amplitudes. It is observed that the periodic feature of the solutions disappeared when the p_1 tends to zero. More importantly, we obtained a new rational breather wave solution, fig. 2(b).

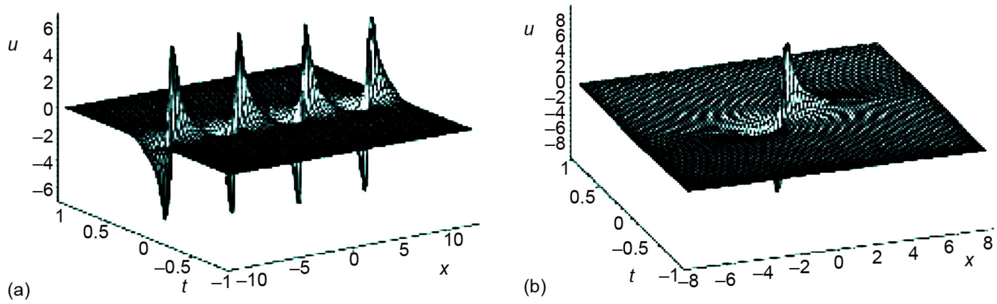

Figure 2. (a) The kinky periodic-soliton solution as $b_1 = 2, b_2 = -5, \delta_1 = 2, p_1 = 1, d = y = z = 0$; (b) the rational breather solution as $b_1 = 1, b_2 = -5, p_1 = 1, d = y = z = 0$

Kinky periodic degradation and new rational breather solution

By choosing special test function in homoclinic breather limit method to the (3+1)-D GKP equation, we obtain a kinky periodic-wave solution and a new rational breather solution.

We suppose that the solution of eq. (7) is:

$$f(x,y,z,t) = e^{-p_1(x+b_1z+d)} + \delta_1 \cos[p(y+b_2z+ct)] + \delta_2 e^{p_1(x+b_1z+d)} \tag{20}$$

where $b_1, b_2, c, d, \delta_1, \delta_2, p,$ and p_1 are some free real constants.

Substituting eq. (20) into eq. (7), and equating all the coefficients of different powers of $e^{p_1(x+b_1z+d)}$, $e^{-p_1(x+b_1z+d)}$, $\sin[p(y+b_2z+ct)]$, $\cos[p(y+b_2z+ct)]$ and constant term to zero, we can obtain a set of algebraic equations for $c, b_i, \delta_i (i = 1, 2)$.

Solving the system with the aid of MAPLE, we get the results:

$$c = -p_1^2 + 2b_1b_2, \quad p = b_1\sqrt{-\frac{1}{-p_1^2 + 2b_1b_2 - b_2^2}}\,p_1, \quad \delta_2 = \frac{1}{4}\delta_1^2 \tag{21}$$

Substituting eq. (21) into eq. (20) and taking $p_1^2 + b_2^2 > 2b_1b_2$, we have:

$$f(x,y,z,t) = |\delta_1|\cosh\left[p_1(x+b_1z) + \ln\left(\frac{1}{2}|\delta_1|\right)\right] + \delta_1 \cos\left[\sqrt{L_3}\,p_1b_1(y+b_2z+H_3t)\right] \tag{22}$$

where $L_3 = \dfrac{1}{P_1^2 - 2b_1b_2 + b_2^2}$ and $H_3 = -p_1^2 + 2b_1b_2$.

Substituting eq. (22) into eq. (6), we obtain the kink periodic-soliton solutions of the (3+1)-D GKP equation:

$$u(x,y,z,t) = \frac{2|\delta_1|p_1 \sinh\left[p_1(x+b_1z+d) + \ln\left(\frac{1}{2}|\delta_1|\right)\right]}{|\delta_1|\cosh\left[p_1(x+b_1z+d) + \ln\left(\frac{1}{2}|\delta_1|\right)\right] + \delta_1\cos\left[\sqrt{L_3}\,b_1p_1(y+b_2z+H_3t)\right]} \tag{23}$$

The solution $u(x,y,z,t)$ represented by eq. (23) can be considered as a kink soliton of variable $X = p_1(x + b_1 z + d) + \ln(1/2|\delta_1|)$ spread along the direction of variable $Y = p_1(y + b_2 z + H_3 t)$, fig. 3(a).

If we choose $\delta_1 = -2$ in eq. (23), while let $p_1 \to 0$ we can get the rational breather solution:

$$u(x,y,z,t) = \frac{4b_2(b_2 - 2b_1)(x + b_1 z + d)}{b_1^2(y + b_2 z + 2b_1 b_2 t)^2 + (b_2^2 - 2b_1 b_2)(x + b_1 z + d)^2} \tag{24}$$

where the solution $u(x,y,z,t)$ represented by eq. (24) is breather wave and no longer has periodic kink feature. Here periodic kink degeneracy occurs when the period of periodic wave tends infinity. It is observed that the periodic kink feature of the solution disappeared when the p_1 tends to zero. More importantly, we obtained a new rational breather wave solution, fig. 3(b).

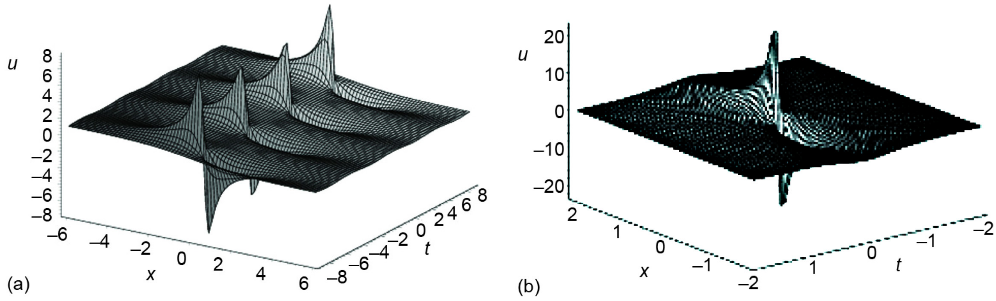

Figure 3. (a) The breather-type periodic soliton solutions as $b_1 = 1$, $b_2 = 5$, $p_1 = 1/2$, $\delta_1 = 2$, $d = y = z = 0$; (b) the rational breather solution as $b_1 = 1$, $b_2 = 5$, $d = y = z = 0$

Potential solutions and rogue wave

We solve the potential of solution by the eq. (12) and let p tends to zero, we can obtain a rational homoclinic (heteroclinic) wave and this wave is just a rogue wave.

Solving the potential of solution (12) and taking $\delta_1 = -2$, we have:

$$\phi = -[u(x,y,z,t)]_x =$$

$$= \frac{2p^2\left\{1 - M + 2\sqrt{M}\sinh\left[p(x + b_1 z + H_1 t + d) + \frac{1}{2}\ln|M|\right]\sin[p(x + a_2 y - L_1 t)]\right\}}{\left\{\sqrt{M}\cosh\left[p(x + b_1 z + H_1 t + d) + \frac{1}{2}\ln|M|\right] - \cos[p(x + a_2 y - L_1 t)]\right\}^2} \tag{25}$$

where $L_1 = \dfrac{2p^2 a_2^2 + b_1^2 a_2 + b_1^2}{2a_2 + a_2^2 + 2}$, $H_1 = \dfrac{2p^2 a_2^2 + 4p^2 a_2 + b_1^2}{2a_2 + a_2^2 + 2}$, and

$$M = \frac{8p^2 a_2 + 6p^2 a_2^3 + 2a_2 b_1^2 + a_2^2 b_1^2 + 10p^2 a_2^2 + b_1^2}{-2p^2 a_2^2 - 4p^2 a_2 + 2a_2 b_1^2 + a_2^2 b_1^2 + b_1^2}$$

The function ϕ is a breather-type periodic soliton, fig. 4(a).

Let $p \to 0$ in eq. (25). We obtain the rational breather wave, and it is just a rogue wave, fig. 4(b), namely:

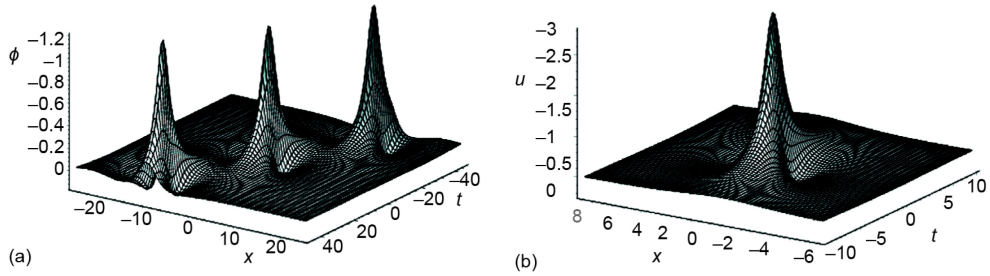

Figure 4. (a) The breather-type periodic soliton ϕ as $a_2 = 1$, $b_2 = 1.7$, $p = 1/4$, $d = y = z = 0$; (b) the $U_{\text{rogue wave}}$ as $a_2 = 1$, $b_2 = 1.7$, $d = y = z = 0$

$$U_{\text{rogue wave}} = -\frac{8b_1^2(a_2+1)^2[6a_2(a_2^2+2a_2+2)-2b_1^2(a_2+1)^2AB]}{[b_1^2(a_2+1)^2(A^2+B^2)+6a_2(a_2^2+2a_2+2)]^2} \tag{26}$$

where

$$A = x + b_1 z + \frac{b_1^2 t}{2a_2 + a_2^2 + 2} + d, \quad B = x + a_2 y - \frac{(1+a_2)b_1^2 t}{2a_2 + a_2^2 + 2}, \quad \text{and } a_2 > 0\,.$$

The solution $U_{\text{rogue wave}}$ contains two waves with different velocities and directions. It is easy to verify that $U_{\text{rogue wave}}$ is a rational breather-type wave. In fact, $U \to 0$ for fixed x as y or $z \to \infty$. So, U is not only a rational breather wave but also a rogue wave solution which has two to three times amplitude higher than its general surrounding waves in a short time.

Remark: Solving the potential of solution eqs. (18) and (23), and let $p_1 \to 0$, we can obtain some analogous results.

Conclusion

In summary, successfully applying the extended homoclinic test method to the (3+1)-D GKP equation, we obtain the exact kink breather, kinky periodic and periodically breather solitary solutions. By using the homoclinic breather limit method proposed in this work, we obtain some new rational breather solutions. Furthermore, we investigate two new physical phenomena, kink and periodic degeneracy. Our results show the variety of the dynamics of high-dimensional systems. Meanwhile, we also obtained the rational potential solution and it is just a rogue wave. This method is simple and straightforward. In the future, we will investigate some other types of non-linear evolution equations and non-integrable systems. What's more, can we obtain similar results to another integrable or non-integrable system with kink breather wave?

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Nomenclature

x, y, z – space co-ordinates, [m]

t – time, [s]

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