THE LOCAL FRACTIONAL SERIES EXPANSION SOLUTION FOR LOCAL FRACTIONAL KORTEWEG-DE VRIES EQUATION

by

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In this paper, the local fractional series expansion method is used to find the series solution for the local fractional Korteweg-de Vries equation.

Key words: *local fractional series expansion method local fractional derivative, Korteweg-de Vries equation, series solution*

Introduction

The local fractional Korteweg-de Vries equation in the special parameters were written in the form [1, 2]:

$$\frac{\partial^{\beta}\Pi(\eta,\tau)}{\partial\tau^{\beta}} - \Pi(\eta,\tau)\frac{\partial^{\beta}\Pi(\eta,\tau)}{\partial\eta^{\beta}} + \frac{\partial^{3\beta}\Pi(\eta,\tau)}{\partial\eta^{3\beta}} = 0 \tag{1}$$

where $\partial^{\beta}/\partial\tau^{\beta}$, $\partial^{\beta}/\partial\eta^{\beta}$, and $\partial^{3\beta}/\partial\eta^{3\beta}$ are the local fractional partial derivatives [1] and $\Pi(\eta,\tau)$ is the non-differentiable wave function. In this paper, we consider the linear local fractional Korteweg-de Vries equation [1, 2]:

$$\frac{\partial^{\beta}\Pi(\eta,\tau)}{\partial\tau^{\beta}} + \frac{\partial^{\beta}\Pi(\eta,\tau)}{\partial\eta^{\beta}} + \frac{\partial^{3\beta}\Pi(\eta,\tau)}{\partial\eta^{3\beta}} = 0 \tag{2a}$$

with the initial value condition:

$$\Pi(\eta,0) = E_{\beta}(\eta^{\beta}) \tag{2b}$$

where $\Pi(\eta,\tau)$ is the wave speed of the fractal wave, and $E_{\beta}(\eta^{\beta})$ is the Mittag-Leffler function defined on Cantor sets [1].

The local fractional derivative was used to model the electric circuit [3], damped vibration [4], population [5], heat flow [6], and others [7-15]. The local fractional series expansion method (LFSEM) was proposed in [13], and the extensions of the technology were developed in [14, 15]. The main aim of the article is to consider a new application of the LFSEM to solve the linear local fractional Korteweg-de Vries equation.

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The non-differentiable solution for linear local fractional Korteweg-de Vries equation

Following the idea [13], we consider the multi-term separated functions of independent variables η and τ, namely:

$$\Pi(\eta,\tau)=\sum_{k=0}^{\infty}\Phi_k(\tau)\Theta_k(\eta) \tag{3}$$

where $\Phi_k(\tau)$ and $\Theta_k(\eta)$ are two non-differentiable functions.

Let

$$\Pi(\eta,\tau)=\sum_{k=0}^{\infty}\frac{\tau^{k\beta}}{\Gamma(1+k\beta)}\Theta_k(\eta) \tag{4}$$

then we have:

$$\frac{\partial^{\beta}\Pi(\eta,\tau)}{\partial\tau^{\beta}}=\sum_{k=0}^{\infty}\frac{\tau^{k\beta}}{\Gamma(1+k\beta)}\Theta_{k+1}(\eta) \tag{5}$$

$$\frac{\partial^{\beta}\Pi(\eta,\tau)}{\partial\eta^{\beta}}=\sum_{k=0}^{\infty}\frac{\tau^{k\beta}}{\Gamma(1+k\beta)}\frac{\partial^{\beta}\Theta_k(\eta)}{\partial\eta^{\beta}} \tag{6}$$

and

$$\frac{\partial^{3\beta}\Pi(\eta,\tau)}{\partial\eta^{3\beta}}=\sum_{k=0}^{\infty}\frac{\tau^{k\beta}}{\Gamma(1+k\beta)}\frac{\partial^{3\beta}\Theta_k(\eta)}{\partial\eta^{3\beta}} \tag{7}$$

Therefore, from eqs. (5)-(7) we can structure the formula:

$$\sum_{k=0}^{\infty}\frac{\tau^{k\beta}}{\Gamma(1+k\beta)}\Theta_{k+1}(\eta)+\sum_{k=0}^{\infty}\frac{\tau^{k\beta}}{\Gamma(1+k\beta)}\frac{\partial^{\beta}\Theta_k(\eta)}{\partial\eta^{\beta}}+\sum_{k=0}^{\infty}\frac{\tau^{k\beta}}{\Gamma(1+k\beta)}\frac{\partial^{3\beta}\Theta_k(\eta)}{\partial\eta^{3\beta}}=0 \tag{8}$$

which leads to the iterative relationship:

$$\Theta_{k+1}(\eta)+\frac{\partial^{\beta}\Theta_k(\eta)}{\partial\eta^{\beta}}+\frac{\partial^{3\beta}\Theta_k(\eta)}{\partial\eta^{3\beta}}=0 \tag{9}$$

with the initial value condition:

$$\Theta_0(\eta)=E_{\beta}(\eta^{\beta}) \tag{10}$$

We rewrite eq. (9):

$$\begin{cases}\Theta_{k+1}(\eta)=-\left[\dfrac{\partial^{\beta}\Theta_k(\eta)}{\partial\eta^{\beta}}+\dfrac{\partial^{3\beta}\Theta_k(\eta)}{\partial\eta^{3\beta}}\right],\\ \Theta_0(\eta)=E_{\beta}(\eta^{\beta})\end{cases} \tag{11}$$

Thus, from eq. (11) we obtain the series solution of non-differentiable type:

$$\Theta_1(\eta) = -\left[\frac{\partial^\beta \Theta_0(\eta)}{\partial \eta^\beta} + \frac{\partial^{3\beta} \Theta_0(\eta)}{\partial \eta^{3\beta}}\right] = -2E_\beta(\eta^\beta)\,, \tag{12}$$

$$\Theta_2(\eta) = -\left[\frac{\partial^\beta \Theta_1(\eta)}{\partial \eta^\beta} + \frac{\partial^{3\beta} \Theta_1(\eta)}{\partial \eta^{3\beta}}\right] = 2^2 E_\beta(\eta^\beta) \tag{13}$$

$$\Theta_3(\eta) = -\left[\frac{\partial^\beta \Theta_2(\eta)}{\partial \eta^\beta} + \frac{\partial^{3\beta} \Theta_2(\eta)}{\partial \eta^{3\beta}}\right] = -2^3 E_\beta(\eta^\beta) \tag{14}$$

$$\Theta_4(\eta) = -\left[\frac{\partial^\beta \Theta_3(\eta)}{\partial \eta^\beta} + \frac{\partial^{3\beta} \Theta_3(\eta)}{\partial \eta^{3\beta}}\right] = 2^4 E_\beta(\eta^\beta) \tag{15}$$

and so on.

Thus, we find the non-differentiable solution of eq. (2a) in the series form:

$$\Pi(\eta,\tau) = \sum_{k=0}^{\infty} (-2)^k \frac{\tau^{k\beta}}{\Gamma(1+k\beta)} E_\beta(\eta^\beta) \tag{16}$$

In view of eq. (16), we have the closed solution of the non-differentiable type:

$$\Pi(\eta,\tau) = E_\beta(-2\tau^\beta)E_\beta(\eta^\beta) \tag{17}$$

and the corresponding graph of eq. (17) is illustrated in fig. 1.

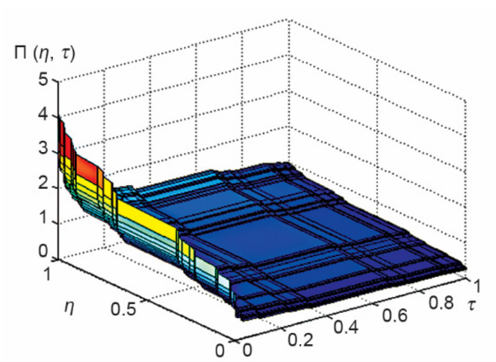

Figure 1. The non-differentiable solution of the linear local fractional Korteweg-de Vries equation *(for color image see journal web-site)*

Conclusion

In this work, we employed the LFSEM to solve the linear local fractional Korteweg-de Vries equation. The non-differentiable solution of the linear local fractional Korteweg-de Vries equation in the closed and series forms was obtained. The result is given to show the efficiency of the technology to solve the local fractional partial differential equations in fluid flows.

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Nomenclature

η – space co-ordinate, [m]
β – fractal order, [–]
$\Pi(\eta,\ \tau)$ – wave speed, [ms^{-1}]
τ – time, [s]

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