ON THREE-DIMENSIONAL INCOMPRESSIBLE NAVIER-STOKES FLUID ON CANTOR SETS IN SPHERICAL CANTOR TYPE CO-ORDINATE SYSTEM

by

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This paper addresses the systems of the incompressible Navier-Stokes equations on Cantor sets without the external force involving the fractal heat-conduction problem vial local fractional derivative. The spherical Cantor type co-ordinate method is used to transfer the incompressible Navier-Stokes equation from the Cantorian co-ordinate system into the spherical Cantor type co-ordinate system.

Key words: *incompressible Navier-Stokes equation, heat-conduction problem, spherical Cantor type co-ordinate method, local fractional derivative*

Introduction

Local fractional calculus was used to solve the non-differentiable problems in heat [1-7] and fluid [8-12] flows. The fractal Cauchy stress tensor of the incompressible Navier-Stokes fluid on Cantor sets was written in the form [11]:

$$\mathbf{J}_\omega = -p\vec{\mathrm{I}} + \mu(\nabla^\omega \upsilon + \upsilon\nabla^\omega) \tag{1}$$

and

$$\nabla^\omega \upsilon = 0 \tag{2}$$

where p is the thermodynamic pressure, $\vec{\mathrm{I}}$ – the unit vector in the local fractional field, μ – the shear moduli of viscosity, υ – the fractal fluid velocity, and ∇^ω – the local fractional operator in the 3-D fractal space [1, 5, 7, 8, 11].

The systems of the incompressible Navier-Stokes equations on Cantor sets was expressed as [11, 12]:

$$\nabla^\omega \upsilon = 0 \tag{3a}$$

$$\rho\frac{D^\omega \upsilon}{Dt^\omega} = \rho\left(\frac{\partial^\omega \upsilon}{\partial t^\omega} + \upsilon\nabla^\omega \upsilon\right) = \nabla^\omega \mathbf{J}_\omega + \rho b \tag{3b}$$

$$\rho\frac{D^\omega(\theta+\varphi)}{Dt^\omega} = -\nabla^\omega(p\upsilon) + \upsilon(\nabla^\omega \mathrm{J}) + \rho b\upsilon + \kappa^{2\xi}\nabla^\omega q \tag{3c}$$

which, by using eqs. (1) and (2), could be written [11]:

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$$\nabla^{\omega}\upsilon = 0 \tag{4a}$$

$$\rho\frac{\partial^{\omega}\upsilon}{\partial t^{\omega}} = -\nabla^{\omega}p + \mu\nabla^{2\omega}\upsilon - \rho\upsilon\nabla^{\omega}\upsilon + \rho b \tag{4b}$$

$$\rho\left[\frac{\partial^{\omega}(\theta+\varphi)}{\partial t^{\omega}} + \upsilon\nabla^{\omega}(\theta+\varphi)\right] = -\nabla^{\omega}(p\upsilon) + \mu\nabla^{2\omega}\upsilon + \rho b\upsilon + \kappa^{2\omega}\nabla^{\omega}q \tag{4c}$$

where υ is the fractal fluid velocity, ρ – the fluid density, p – the thermodynamic pressure, φ – the kinetic energy (KE) per unit of mass, θ – the internal energy (IE) per unit of mass, b – the external force (EF) per unit of mass, $\kappa^{2\omega}$ represents the fractal heat-conduction coefficient, q – the fractal temperature field, and $\nabla^{2\omega}$ – the local fractional Laplace operator in the 3-D fractal space [1, 5, 7, 8, 11].

The Cantor type spherical co-ordinate method was reported in [1, 5, 8]. In this paper, the aim is to investigate the systems of the incompressible Navier-Stokes equations on Cantor sets in spherical Cantor type co-ordinate system.

The spherical Cantor type co-ordinate method

For $R \in (0,+\infty)$, $\eta \in (0,\pi)$, $\theta \in (0,2\pi)$, and $\mu^{2\omega}+\eta^{2\omega}+\sigma^{2\omega}=R^{2\omega}$ the Cantor type spherical co-ordinate system is written in the form [1, 8]:

$$\begin{cases} \mu^{\omega} = R^{\omega}\cos_{\varepsilon}(\vartheta^{\omega})\cos_{\varepsilon}(\theta^{\omega}), \\ \eta^{\omega} = R^{\omega}\cos_{\varepsilon}(\vartheta^{\omega})\sin_{\varepsilon}(\theta^{\omega}), \\ \sigma^{\omega} = R^{\omega}\sin^{\omega}(\vartheta^{\omega}) \end{cases} \tag{5}$$

A local fractional vector (LFV) takes in the form:

$$\begin{aligned} \vec{\mathrm{r}} &= \mathrm{R}^{\omega}\cos_{\omega}(\vartheta^{\omega})\cos_{\omega}(\theta^{\omega})\vec{\mathrm{e}}_1^{\omega} + \mathrm{R}^{\omega}\cos_{\omega}(\vartheta^{\omega})\sin_{\omega}(\theta^{\omega})\vec{\mathrm{e}}_2^{\omega} + \mathrm{R}^{\omega}\sin_{\omega}(\vartheta^{\omega})\vec{\mathrm{e}}_3^{\omega} \\ &= \mathrm{r}_R\vec{\mathrm{e}}_R^{\omega} + \mathrm{r}_{\vartheta}\vec{\mathrm{e}}_{\vartheta}^{\omega} + \mathrm{r}_{\theta}\vec{\mathrm{e}}_{\theta}^{\omega} \end{aligned} \tag{6}$$

where $(\vec{\mathrm{e}}_1^{\omega},\vec{\mathrm{e}}_2^{\omega},\vec{\mathrm{e}}_3^{\omega})$ and $(\vec{\mathrm{e}}_R^{\omega},\vec{\mathrm{e}}_{\vartheta}^{\omega},\vec{\mathrm{e}}_{\theta}^{\omega})$ are the LFV in two fractal vector spaces.

Therefore, the local fractional gradient and Laplace operators in the Cantor type spherical co-ordinates system were written in the forms:

$$\nabla^{\omega}\phi(R,\vartheta,\theta) = \vec{\mathrm{e}}_R^{\omega}\frac{\partial^{\omega}\phi}{\partial R^{\omega}} + \vec{\mathrm{e}}_{\vartheta}^{\omega}\frac{1}{R^{\omega}}\frac{\partial^{\omega}\phi}{\partial\vartheta^{\omega}} + \vec{\mathrm{e}}_{\theta}^{\omega}\frac{1}{R^{\omega}}\frac{1}{\sin_{\omega}(\vartheta^{\omega})}\frac{\partial^{\omega}\phi}{\partial\theta^{\omega}} \tag{7}$$

$$\begin{aligned} \nabla^{2\omega}\phi(R,\vartheta,\theta) = {} & \frac{\partial^{2\omega}\phi}{\partial R^{2\omega}} + \frac{1}{R^{2\omega}}\frac{1}{\sin_{\omega}(\vartheta^{\omega})}\frac{\partial^{\omega}}{\partial\vartheta^{\omega}}\left[\sin_{\omega}(\vartheta^{\omega})\frac{\partial^{\omega}\phi}{\partial\vartheta^{\omega}}\right] + \\ & + \frac{2}{R^{\omega}}\frac{\partial^{\omega}\phi}{\partial R^{\omega}} + \frac{1}{R^{2\omega}}\frac{1}{\sin_{\omega}^{2}(\vartheta^{\omega})}\frac{\partial^{2\omega}\phi}{\partial\theta^{2\omega}} \end{aligned} \tag{8}$$

where the LFV are given by the expressions:

$$\vec{\mathrm{e}}_R^{\omega} = \sin_{\omega}(\vartheta^{\omega})\cos_{\omega}(\theta^{\omega})\vec{\mathrm{e}}_1^{\omega} + \sin_{\omega}(\vartheta^{\omega})\sin_{\omega}(\theta^{\omega})\vec{\mathrm{e}}_2^{\omega} + \cos_{\omega}(\vartheta^{\omega})\vec{\mathrm{e}}_3^{\omega} \tag{9a}$$

$$\vec{e}_{\vartheta}^{\omega} = \cos_{\omega}(\vartheta^{\omega})\cos_{\omega}(\theta^{\omega})\vec{e}_{1}^{\omega} + \cos_{\omega}(\vartheta^{\omega})\sin_{\omega}(\theta^{\omega})\vec{e}_{2}^{\omega} - \sin_{\omega}(\vartheta^{\omega})\vec{e}_{3}^{\omega} \tag{9b}$$

$$\vec{e}_{\theta}^{\omega} = -\sin_{\omega}(\theta^{\omega})\vec{e}_{1}^{\omega} + \cos_{\omega}(\theta^{\omega})\vec{e}_{2}^{\omega} \tag{9c}$$

Transferring the 3-D incompressible Navier-Stokes fluid on Cantor sets

With the help of eqs. (4a)-(4c), we present the systems of the incompressible Navier-Stokes equations on Cantor sets without the external force in the Cantorian co-ordinate system:

$$\nabla^{\omega}\upsilon = 0 \tag{10a}$$

$$\rho\frac{\partial^{\omega}\upsilon}{\partial t^{\omega}} = -\nabla^{\omega}p + \mu\nabla^{2\omega}\upsilon - \rho\upsilon\nabla^{\omega}\upsilon \tag{10b}$$

$$\rho\left[\frac{\partial^{\omega}(\theta+\varphi)}{\partial t^{\omega}} + \upsilon\nabla^{\omega}(\theta+\varphi)\right] = -\nabla^{\omega}(p\upsilon) + \mu\nabla^{2\omega}\upsilon + \kappa^{2\omega}\nabla^{\omega}q \tag{10c}$$

In view of eqs. (7) and (8), eq. (10a) in the spherical Cantor type co-ordinate system is written in the form:

$$e_{R}^{\omega}\frac{\partial^{\omega}\upsilon_{R}(R,\vartheta,\theta)}{\partial R^{\omega}} + e_{\vartheta}^{\omega}\frac{1}{R^{\omega}}\frac{\partial^{\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial\vartheta^{\omega}} + e_{\theta}^{\omega}\frac{1}{R^{\omega}}\frac{1}{\sin_{\omega}(\vartheta^{\omega})}\frac{\partial^{\omega}\upsilon_{\theta}(R,\vartheta,\theta)}{\partial\theta^{\omega}} = 0 \tag{11a}$$

Similarly, eq. (10b) in the spherical Cantor type co-ordinate system is written as:

$$\begin{aligned}\rho\frac{\partial^{\omega}\upsilon_{R}(R,\vartheta,\theta)}{\partial t^{\omega}} = &-\frac{\partial^{\omega}p_{R}(R,\vartheta,\theta)}{\partial R^{\omega}} + \mu\frac{\partial^{2\omega}\upsilon_{R}(R,\vartheta,\theta)}{\partial R^{2\omega}} + \frac{2\mu}{R^{\omega}}\frac{\partial^{\omega}\upsilon_{R}(R,\vartheta,\theta)}{\partial R^{\omega}} + \\ &+\frac{\mu}{R^{2\omega}}\frac{1}{\sin_{\omega}(\vartheta^{\omega})}\frac{\partial^{\omega}}{\partial\vartheta^{\omega}}\left[\sin_{\omega}(\vartheta^{\omega})\frac{\partial^{\omega}\upsilon_{R}(R,\vartheta,\theta)}{\partial\vartheta^{\omega}}\right] + \frac{\mu}{R^{2\omega}}\frac{1}{\sin_{\omega}^{2}(\vartheta^{\omega})}\frac{\partial^{2\omega}\upsilon_{R}(R,\vartheta,\theta)}{\partial\theta^{2\omega}} - \\ &-\upsilon_{R}(R,\vartheta,\theta)\frac{\partial^{\omega}\upsilon_{R}(R,\vartheta,\theta)}{\partial R^{\omega}} + \upsilon_{\vartheta}(R,\vartheta,\theta)\frac{1}{R^{\omega}}\frac{\partial^{\omega}\upsilon_{R}(R,\vartheta,\theta)}{\partial\vartheta^{\omega}} + \\ &+\upsilon_{\theta}(R,\vartheta,\theta)\frac{1}{R^{\omega}}\frac{1}{\sin_{\omega}(\vartheta^{\omega})}\frac{\partial^{\omega}\upsilon_{R}(R,\vartheta,\theta)}{\partial\theta^{\omega}}\end{aligned} \tag{12a}$$

$$\begin{aligned}\rho\frac{\partial^{\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial t^{\omega}} = &-\frac{\partial^{\omega}p_{\vartheta}(R,\vartheta,\theta)}{\partial R^{\omega}} + \mu\frac{\partial^{2\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial R^{2\omega}} + \frac{2\mu}{R^{\omega}}\frac{\partial^{\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial R^{\omega}} + \\ &+\frac{\mu}{R^{2\omega}}\frac{1}{\sin_{\omega}(\vartheta^{\omega})}\frac{\partial^{\omega}}{\partial\vartheta^{\omega}}\left[\sin_{\omega}(\vartheta^{\omega})\frac{\partial^{\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial\vartheta^{\omega}}\right] + \frac{\mu}{R^{2\omega}}\frac{1}{\sin_{\omega}^{2}(\vartheta^{\omega})}\frac{\partial^{2\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial\theta^{2\omega}} - \\ &-\upsilon_{R}(R,\vartheta,\theta)\frac{\partial^{\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial R^{\omega}} + \upsilon_{\vartheta}(R,\vartheta,\theta)\frac{1}{R^{\omega}}\frac{\partial^{\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial\vartheta^{\omega}} + \\ &+\upsilon_{\theta}(R,\vartheta,\theta)\frac{1}{R^{\omega}}\frac{1}{\sin_{\omega}(\vartheta^{\omega})}\frac{\partial^{\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial\theta^{\omega}}\end{aligned} \tag{12b}$$

$$\rho\frac{\partial^{\omega}\upsilon_{\theta}(R,\vartheta,\theta)}{\partial t^{\omega}}=-\frac{\partial^{\omega}p_{\theta}(R,\vartheta,\theta)}{\partial R^{\omega}}+\mu\frac{\partial^{2\omega}\upsilon_{\theta}(R,\vartheta,\theta)}{\partial R^{2\omega}}+\frac{2\mu}{R^{\omega}}\frac{\partial^{\omega}\upsilon_{\theta}(R,\vartheta,\theta)}{\partial R^{\omega}}+$$
$$+\frac{\mu}{R^{2\omega}}\frac{1}{\sin_{\omega}(\vartheta^{\omega})}\frac{\partial^{\omega}}{\partial\vartheta^{\omega}}\left[\sin_{\omega}(\vartheta^{\omega})\frac{\partial^{\omega}\upsilon_{\theta}(R,\vartheta,\theta)}{\partial\vartheta^{\omega}}\right]+\frac{\mu}{R^{2\omega}}\frac{1}{\sin_{\omega}^{2}(\vartheta^{\omega})}\frac{\partial^{2\omega}\upsilon_{\theta}(R,\vartheta,\theta)}{\partial\theta^{2\omega}}-$$
$$-\upsilon_{R}(R,\vartheta,\theta)\frac{\partial^{\omega}\upsilon_{\theta}(R,\vartheta,\theta)}{\partial R^{\omega}}+\upsilon_{\vartheta}(R,\vartheta,\theta)\frac{1}{R^{\omega}}\frac{\partial^{\omega}\upsilon_{\theta}(R,\vartheta,\theta)}{\partial\vartheta^{\omega}}+$$
$$+\upsilon_{\theta}(R,\vartheta,\theta)\frac{1}{R^{\omega}}\frac{1}{\sin_{\omega}(\vartheta^{\omega})}\frac{\partial^{\omega}\upsilon_{\theta}(R,\vartheta,\theta)}{\partial\theta^{\omega}}\tag{12c}$$

Equation (10c) in the spherical Cantor type co-ordinate system is written in the forms:

$$\rho\,\frac{\partial^{\omega}(\theta+\varphi)(R,\vartheta,\theta)}{\partial t^{\omega}}+\rho\upsilon_{R}(R,\vartheta,\theta)\,\frac{\partial^{\omega}(\theta+\varphi)(R,\vartheta,\theta)}{\partial R^{\omega}}+$$
$$+\frac{\rho\upsilon_{\vartheta}(R,\vartheta,\theta)}{R^{\omega}}\,\frac{\partial^{\omega}(\theta+\varphi)(R,\vartheta,\theta)}{\partial\vartheta^{\omega}}+\frac{\rho\upsilon_{\theta}(R,\vartheta,\theta)}{R^{\omega}\sin_{\omega}(\vartheta^{\omega})}\,\frac{\partial^{\omega}(\theta+\varphi)(R,\vartheta,\theta)}{\partial\theta^{\omega}}=$$
$$=-\frac{\partial^{\omega}[p\upsilon_{R}(R,\vartheta,\theta)]}{\partial R^{\omega}}-\frac{1}{R^{\omega}}\,\frac{\partial^{\omega}[p\upsilon_{\vartheta}(R,\vartheta,\theta)]}{\partial\vartheta^{\omega}}-\frac{1}{R^{\omega}\sin_{\omega}(\vartheta^{\omega})}\,\frac{\partial^{\omega}[p\upsilon_{\theta}(R,\vartheta,\theta)]}{\partial\theta^{\omega}}+$$
$$+\mu\,\frac{\partial^{2\omega}\upsilon_{R}(R,\vartheta,\theta)}{\partial R^{2\omega}}+\frac{\mu}{R^{2\omega}\sin_{\omega}(\vartheta^{\omega})}\,\frac{\partial^{\omega}}{\partial\vartheta^{\omega}}\left[\sin_{\omega}(\vartheta^{\omega})\,\frac{\partial^{\omega}\upsilon_{R}(R,\vartheta,\theta)}{\partial\vartheta^{\omega}}\right]+$$
$$+\frac{2\mu}{R^{\omega}}\,\frac{\partial^{\omega}\upsilon_{R}(R,\vartheta,\theta)}{\partial R^{\omega}}+\frac{\mu}{R^{2\omega}\sin_{\omega}^{2}(\vartheta^{\omega})}\,\frac{\partial^{2\omega}\upsilon_{R}(R,\vartheta,\theta)}{\partial\theta^{2\omega}}+\kappa^{2\omega}\,\frac{\partial^{\omega}q_{R}(R,\vartheta,\theta)}{\partial R^{\omega}}+$$
$$+\frac{\kappa^{2\omega}}{R^{\omega}}\,\frac{\partial^{\omega}q_{R}(R,\vartheta,\theta)}{\partial\vartheta^{\omega}}+\frac{\kappa^{2\omega}}{R^{\omega}\sin_{\omega}(\vartheta^{\omega})}\,\frac{\partial^{\omega}q_{R}(R,\vartheta,\theta)}{\partial\theta^{\omega}}\tag{13a}$$

$$\rho\,\frac{\partial^{\omega}(\theta+\varphi)(R,\vartheta,\theta)}{\partial t^{\omega}}+\rho\upsilon_{R}(R,\vartheta,\theta)\,\frac{\partial^{\omega}(\theta+\varphi)(R,\vartheta,\theta)}{\partial R^{\omega}}+$$
$$+\frac{\rho\upsilon_{\vartheta}(R,\vartheta,\theta)}{R^{\omega}}\,\frac{\partial^{\omega}(\theta+\varphi)(R,\vartheta,\theta)}{\partial\vartheta^{\omega}}+\frac{\rho\upsilon_{\theta}(R,\vartheta,\theta)}{R^{\omega}\sin_{\omega}(\vartheta^{\omega})}\,\frac{\partial^{\omega}(\theta+\varphi)(R,\vartheta,\theta)}{\partial\theta^{\omega}}=$$
$$=-\frac{\partial^{\omega}[p\upsilon_{R}(R,\vartheta,\theta)]}{\partial R^{\omega}}-\frac{1}{R^{\omega}}\,\frac{\partial^{\omega}[p\upsilon_{\vartheta}(R,\vartheta,\theta)]}{\partial\vartheta^{\omega}}-\frac{1}{R^{\omega}\sin_{\omega}(\vartheta^{\omega})}\,\frac{\partial^{\omega}[p\upsilon_{\theta}(R,\vartheta,\theta)]}{\partial\theta^{\omega}}+$$
$$+\mu\,\frac{\partial^{2\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial R^{2\omega}}+\frac{\mu}{R^{2\omega}\sin_{\omega}(\vartheta^{\omega})}\,\frac{\partial^{\omega}}{\partial\vartheta^{\omega}}\left[\sin_{\omega}(\vartheta^{\omega})\,\frac{\partial^{\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial\vartheta^{\omega}}\right]+$$
$$+\frac{2\mu}{R^{\omega}}\,\frac{\partial^{\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial R^{\omega}}+\frac{\mu}{R^{2\omega}\sin_{\omega}^{2}(\vartheta^{\omega})}\,\frac{\partial^{2\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial\theta^{2\omega}}+\kappa^{2\omega}\,\frac{\partial^{\omega}q_{\vartheta}(R,\vartheta,\theta)}{\partial R^{\omega}}+$$
$$+\frac{\kappa^{2\omega}}{R^{\omega}}\,\frac{\partial^{\omega}q_{\vartheta}(R,\vartheta,\theta)}{\partial\vartheta^{\omega}}+\frac{\kappa^{2\omega}}{R^{\omega}\sin_{\omega}(\vartheta^{\omega})}\,\frac{\partial^{\omega}q_{\vartheta}(R,\vartheta,\theta)}{\partial\theta^{\omega}}\tag{13b}$$

$$\rho \frac{\partial^{\omega}(\theta+\varphi)(R,\vartheta,\theta)}{\partial t^{\omega}}+\rho \upsilon_R(R,\vartheta,\theta)\frac{\partial^{\omega}(\theta+\varphi)(R,\vartheta,\theta)}{\partial R^{\omega}}+$$

$$+\frac{\rho \upsilon_{\vartheta}(R,\vartheta,\theta)}{R^{\omega}}\frac{\partial^{\omega}(\theta+\varphi)(R,\vartheta,\theta)}{\partial \vartheta^{\omega}}+\frac{\rho \upsilon_{\theta}(R,\vartheta,\theta)}{R^{\omega}\sin_{\omega}(\vartheta^{\omega})}\frac{\partial^{\omega}(\theta+\varphi)(R,\vartheta,\theta)}{\partial \theta^{\omega}}=$$

$$=-\frac{\partial^{\omega}[p\upsilon_R(R,\vartheta,\theta)]}{\partial R^{\omega}}-\frac{1}{R^{\omega}}\frac{\partial^{\omega}[p\upsilon_{\vartheta}(R,\vartheta,\theta)]}{\partial \vartheta^{\omega}}-\frac{1}{R^{\omega}\sin_{\omega}(\vartheta^{\omega})}\frac{\partial^{\omega}[p\upsilon_{\theta}(R,\vartheta,\theta)]}{\partial \theta^{\omega}}+$$

$$+\mu\frac{\partial^{2\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial R^{2\omega}}+\frac{\mu}{R^{2\omega}\sin_{\omega}(\vartheta^{\omega})}\frac{\partial^{\omega}}{\partial \vartheta^{\omega}}\left[\sin_{\omega}(\vartheta^{\omega})\frac{\partial^{\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial \vartheta^{\omega}}\right]+$$

$$+\frac{2\mu}{R^{\omega}}\frac{\partial^{\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial R^{\omega}}+\frac{\mu}{R^{2\omega}\sin^2_{\omega}(\vartheta^{\omega})}\frac{\partial^{2\omega}\upsilon_{\vartheta}(R,\vartheta,\theta)}{\partial \theta^{2\omega}}+\kappa^{2\omega}\frac{\partial^{\omega}q_{\vartheta}(R,\vartheta,\theta)}{\partial R^{\omega}}+$$

$$+\frac{\kappa^{2\omega}}{R^{\omega}}\frac{\partial^{\omega}q_{\vartheta}\left(R,\vartheta,\theta\right)}{\partial \vartheta^{\omega}}+\frac{\kappa^{2\omega}}{R^{\omega}\sin_{\omega}(\vartheta^{\omega})}\frac{\partial^{\omega}q_{\vartheta}\left(R,\vartheta,\theta\right)}{\partial \theta^{\omega}} \tag{13c}$$

Conclusion

In this paper, the systems of the incompressible Navier-Stokes equations on Cantor sets without the external force in the spherical Cantor type co-ordinate system were derived from the spherical Cantor type co-ordinate method. The new forms of the systems of the incompressible Navier-Stokes equations on Cantor sets involving the fractal heat-conduction terms were discussed. Observing the equations from the Cantorian co-ordinate system into the spherical Cantor type co-ordinate system has come true.

Nomenclature

b – EF per unit of mass, [Nm^{-3}]
p – thermodynamic pressure, [$Pa\cdot m^{-3}$]
q – fractal temperature field, [Km^{-3}]

Greek symbols

θ – IE per unit of mass, [Jm^{-3}]
$\kappa^{2\omega}$ – heat-conduction coefficient, [$Wm^{-1}k^{-1}$]
ρ – fluid density, [kgm^{-3}]
υ – fractal fluid velocity, [ms^{-1}]
φ – KE per unit of mass, [Jm^{-3}]
ω – fractal dimension, [–]
$\nabla^{2\omega}$ – local fractional Laplace, [–]

Acronyms

EF – external force
IE – internal energy
KE – kinetic energy

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