

ON LOCAL FRACTIONAL VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS IN FRACTAL STEADY HEAT TRANSFER

by

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In this paper we address the inverse problems for the fractal steady heat transfer described by the local fractional linear and non-linear Volterra integro-differential equations. The Volterra integro-differential equations are presented for investigating the fractal heat-transfer.

Key words: *steady heat transfer, Volterra integro-differential equation, local fractional calculus, fractals*

Introduction

The linear and non-linear differential equations were used to model for the systems in heat-transfer by the conduction, convection, and radiation. The inverse problems for the heat-transfer differential equations transferred into the integro-differential equations in heat-transfer phenomena were considered in [1-7].

Fractal heat transfer via local fractional calculus [8-15] is one of the important topics for the complex phenomena in applied science. The steady linear homogeneous heat-transfer equation in fractal media was considered [14, 15]:

$$\eta \frac{d^{2\alpha} \Lambda(x)}{dx^{2\alpha}} + \mu \Lambda(x) = 0 \quad (1a)$$

where η and μ are two constants. Equation (1a) is called the linear oscillator equation without the driving force.

The steady linear non-homogeneous heat-transfer equation in fractal media was presented [14, 15]:

$$\eta \frac{d^{2\alpha} \Lambda(x)}{dx^{2\alpha}} + \mu \Lambda(x) = \Pi(x) \quad (1b)$$

where η and μ are two constants and $\Pi(x)$ is the driving force. Equation (1b) is called the linear oscillator equation with the driving force.

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The steady homogeneous non-linear heat transfer equation in fractal media was written in the form [15]:

$$\eta \frac{d^{2\alpha} \Lambda(x)}{dx^{2\alpha}} + \mu \Lambda^4(x) = 0 \quad (1c)$$

where η and μ are two constants. Equation (1c) is called the non-linear oscillator equation without the driving force.

The steady homogeneous non-linear heat transfer equation in fractal media is considered [15]:

$$\eta \frac{d^{2\alpha} \Lambda(x)}{dx^{2\alpha}} + \mu \Lambda^4(x) = \Pi(x) \quad (1d)$$

where η and μ are two constants and $\Pi(x)$ is the driving force. Equation (1d) is called the non-linear oscillator equation with the driving force.

The aim of the manuscript is to transfer the steady heat-transfer equations into the local fractional integro-differential equations.

The steady generalized heat-transfer equation in fractal media

In view of eqs. (1a)-(1d), the local fractional ordinary differential equation of $2\alpha^{\text{th}}$ order in heat transfer is written in the form:

$$\frac{d^{2\alpha} \Lambda(x)}{dx^{2\alpha}} = \Theta[x, \Lambda(t)], \quad (a \leq x \leq b) \quad (2a)$$

From eqs. (1a)-(1d) and (2a) we obtain the functions:

$$\Theta[x, \Lambda(t)] = -\frac{\mu}{\eta} \Lambda(x) \quad (2b)$$

$$\Theta[x, \Lambda(t)] = \frac{\Pi(x)}{\eta} - \frac{\mu}{\eta} \Lambda(x) \quad (2c)$$

$$\Theta[x, \Lambda(t)] = -\frac{\mu}{\eta} \Lambda^4(x) \quad (2d)$$

$$\Theta[x, \Lambda(t)] = \frac{\Pi(x)}{\eta} - \frac{\mu}{\eta} \Lambda^4(x) \quad (2e)$$

Equation (2a) can be generalized:

$$\frac{d^{2\alpha} \Lambda(x)}{dx^{2\alpha}} = \sum_{i=0}^4 A_i(x) \Lambda^i(x) \quad (3)$$

where $A_i(x)$ represent characteristic parameters involving the fractal heat-transfer. Equation (2a) is called as the steady generalized heat-transfer equation in fractal media.

There are some cases in the different characteristic parameters involving the fractal heat-transfer:

(M1) For $A_0(x) = A_2(x) = A_3(x) = A_4(x) = 0$, and $A_1(x) = -\frac{\mu}{\eta}$ we obtain eq. (1a)

(M2) For $A_2(x) = A_3(x) = A_4(x) = 0$, $A_0(x) = \frac{\Pi(x)}{\eta}$, and $A_1(x) = -\frac{\mu}{\eta}$ we obtain eq. (1b)

(M3) For $A_0(x) = A_1(x) = A_2(x) = A_3(x) = 0$, and $A_4(x) = -\frac{\mu}{\eta}$ we obtain eq. (1c)

(M4) For $A_1(x) = A_2(x) = A_3(x) = 0$, $A_0(x) = \frac{\Pi(x)}{\eta}$, and $A_4(x) = -\frac{\mu}{\eta}$ we obtain eq. (1d)

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 in fractal heat transfer**

Following the idea [16] and taking the local fractional integration by eq. (2a), eq. (2a) can be written:

$$\Lambda^{(\alpha)}(x) = \Lambda^{(\alpha)}(a) + \frac{1}{\Gamma(1+\alpha)} \int_a^x \Theta[t, \Lambda(x)](dx)^\alpha \tag{4}$$

Equation (4) is called as the local fractional Volterra integro-differential equation in fractal heat transfer.

With the help of eq. (4), we obtain the following case.

(T1) For $\Theta[x, \Lambda(t)] = -\mu/\eta\Lambda(x)$, we have the local fractional Volterra integro-differential equation in fractal heat transfer:

$$\Lambda^{(\alpha)}(x) = \Lambda^{(\alpha)}(a) - \frac{\mu}{\eta} \int_a^x \Lambda(x)(dx)^\alpha \tag{5a}$$

(T2) For $\Theta[x, \Lambda(t)] = [\Pi(x)]/\eta - (\mu/\eta)\Lambda(x)$, we have the local fractional Volterra integro-differential equation in fractal heat transfer:

$$\Lambda^{(\alpha)}(x) = \Phi(x) - \frac{\mu}{\eta} \int_a^x \Lambda(x)(dx)^\alpha \tag{5b}$$

where

$$\Phi(x) = \Lambda^{(\alpha)}(a) + \frac{1}{\Gamma(1+\alpha)} \int_a^x \frac{\Pi(x)}{\eta} (dx)^\alpha$$

- (T3) For $\Theta[x, \Lambda(t)] = -(\mu/\eta)\Lambda^4(x)$, we have the local fractional non-linear Volterra integro-differential equation in fractal heat transfer:

$$\Lambda^{(\alpha)}(x) = \Lambda^{(\alpha)}(a) - \frac{\mu}{\eta} \int_a^x \Lambda^4(x) (dx)^\alpha \quad (5c)$$

- (T4) For $\Theta[x, \Lambda(t)] = [\Pi(x)]/\eta - (\mu/\eta)\Lambda^4(x)$, we have the local fractional non-linear Volterra integro-differential equation in fractal heat transfer:

$$\Lambda^{(\alpha)}(x) = \Phi(x) - \frac{\mu}{\eta} \int_a^x \Lambda^4(x) (dx)^\alpha \quad (5d)$$

where

$$\Phi(x) = \Lambda^{(\alpha)}(a) + \frac{1}{\eta} \int_a^x \Pi(x) (dx)^\alpha$$

Conclusion

In this work, we considered the inverse problems for the steady generalized heat-transfer equation in fractal media. Based on the theory of the local fractional integral equations, we transferred the steady heat-transfer equations into the local fractional linear and non-linear Volterra integro-differential equation in fractal heat transfer. It opens the new direction in fractal heat transfer involving the local fractional calculus. We consider the numerical and analytical solutions for them in the future.

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Nomenclature

x – space co-ordinate, [m]

α – fractal dimension, [-]

$\Lambda(x)$ – temperature, [K]

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