ANALYTICAL STUDY OF UNSTEADY SEDIMENTATION ANALYSIS OF SPHERICAL PARTICLE IN NEWTONIAN FLUID MEDIA

by

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Unsteady settling behavior of solid spherical particles falling in water as a Newtonian fluid is investigated in this research. Least square method (LSM), Galerkin method, LSM-Padé, and numerical model are applied to analyze the characteristics of the particles motion. The influence of physical parameters on terminal velocity is discussed and it is showed that LSM and Galerkin method are efficient techniques for solving the governing equation. Among these methods, LSM-Padé demonstrates the best agreement with numerical results. The novelty of this work is to introduce new analytical methods for solving the non-linear equation of sedimentation applicable in many industrial and chemical applications.

Key words: *spherical particles, acceleration motion, LSM, Galerkin method, LSM-Padé*

Introduction

Description of the motion of immersed bodies in fluids has long been a subject of great interest due to its wide range of applications in industry *e. g.* sediment transport, deposition in pipelines, alluvial channels, *etc.* [1, 2]. The settling mechanism of solid particle, bubble, or drop, both in Newtonian and non-Newtonian fluids, is reported by Clift *et al.* [3] and Chhabra [4] and several types of drag coefficients for spherical and non-spherical particles were presented by Haider and Levenspiel [5]. Guo [6] and Mohazzabi [7] have studied the behavior of spheres and objects falling into fluids.

A particle falling vertically in a stationary fluid under the influence of gravity accelerates until the gravitational body force is balanced by the resistance forces, including buoyancy and drag forces. At the equilibrium, particle reaches a constant velocity so-called *terminal velocity* or *settling velocity*. The knowledge of the terminal velocity of solids and particles falling in liquids is required in many industrial applications such as mineral processing, hydraulic transport, fluidized bed reactors and so on [3].

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Recently, several attempts have been made to develop analytical tools to solve the motion equation of falling objects in fluids. Domairry Ganji [8] employed variational iteration method (VIM) and derived a semi-exact solution for the instantaneous velocity of the particle over time, setting in incompressible fluid. Yaghoobi and Torabi [9] investigated the acceleration motion of a vertically falling on-spherical particle in incompressible Newtonian media by VIM. They developed their model for the combined VIM with Padé approximation for modeling the non-spherical particle equation of motion [10]. More applications of VIM for solving the non-linear differential equations are presented by Mohyud-Din *et al.* [11].

Jalaal *et al.* [12] used homotopy analysis method and obtained the solution of the 1-D non-linear particle equation. The same group [13] applied He's homotopy perturbation method (HPM) [14, 15] to solve the acceleration motion of a vertically falling spherical particle in incompressible Newtonian media. Mohyud-Din *et al.* [16] used HPM for solving a wide class of non-linear problems and they suggest that this method is capable to cope with the versatility of the physical problems such as sedimentation process, the system of linear PDE for waves [17] and MHD flow over non-linear stretching sheeting [18]. Torabi and Yaghoobi [19] combined HPM with Padé approximation for increasing the solution accuracy of the particle equation of motion. The motion of a spherical particle rolling down an inclined plane in Newtonian media was studied by Jalaal and Domairry Ganji [20, 21].

There are some simple and accurate approximation techniques for solving non-linear differential equations called the weighted residuals methods (WRM). Collocation method (CM), Galerkin method (GM) and LSM are examples of the WRM which are introduced by Ozisik [22] for using in heat transfer problem. Stern and Rasmussen [23] used collocation method for solving a third order linear differential equation. Vaferi *et al.* [24] have studied the feasibility of applying of orthogonal collocation method to solve diffusivity equation in the radial transient flow system. Recently Hatami *et al.* [25] used DTM, CM, and LSM for predicting the temperature distribution in a porous fins with temperature dependent internal heat generation. Moreover, Nouri *et al.* [26] and Hatami and Domairry Ganji [27-29] studied the motion of a spherical particle using different analytical methods: DQM, DTM, and DTM-Padé. The application of analytical methods in solving the non-linear problems in MHD and nanofluid flow has also been reported in [30-38].

In this paper, after following the recent progress and the outcome made by [26-29] in the analytical methods, LSM-Padé method is employed to solve the unsteady particle spherical motion equation and its obtained results have been compared with the ordinary LSM, GM, and numerical methods. This is concluded that the proposed method has got a better agreement with the numerical model.

Problem description

The particle sediment phenomenon is modelled using gravity, buoyancy, drag forces, and added mass. According to the Basset-Boussinesq-Ossen equation for the unsteady motion of the particle in a fluid, for a dense particle falling in light fluids assuming $\rho \ll \rho_s$ Basset history force is negligible. So by rewriting force balance for the particle, the equation of motion is gained as follows [19]:

$$m\frac{\mathrm{d}u}{\mathrm{d}t} = mg\left(1-\frac{\rho}{\rho_s}\right) - \frac{1}{8}\pi D^2 \rho C_D u^2 - \frac{1}{12}\pi D^3 \rho \frac{\mathrm{d}u}{\mathrm{d}t} \tag{1}$$

where C_D is the drag coefficient. In the right hand side of the eq. (1), the first term represents the buoyancy effect, the second term corresponds to drag resistance, and the last term is the added mass effect due to acceleration of fluid around the particle. The main difficulty of solving eq. (1) is the non-linear terms due to the non-linearity nature of the drag coefficient C_D. The suggested correlation for C_D of spherical particles which has a good agreement with the experimental data in a wide range of Reynolds number $0 \le \text{Re} \le 105$ is formulated by:

$$C_D = \frac{24}{\text{Re}}\left(1 + \frac{1}{48}\text{Re}\right) \tag{2}$$

Jalaal *et al.* [13] have shown that eq. (2) represents a more accurate resistance of the particle in comparison with the pervious equations presented by others. Based on the mass of particle formulated by:

$$m = \frac{1}{6}\pi D^3 \rho_s \tag{3}$$

Equation (1) can be rewritten:

$$a\frac{\text{d}u}{\text{d}t} + bu + cu^2 - d = 0, \quad u(0) = 0 \tag{4}$$

where

$$a = \frac{1}{12}\pi D^3\left(2\rho_s + \rho\right), \quad b = 3\pi D\mu, \quad c = \frac{1}{16}\pi D^2\rho, \quad d = \frac{1}{6}\pi D^3 \text{g}\left(\rho_s - \rho\right) \tag{5}$$

In the present study, we choose three different materials for solid particle, Aluminum, Copper and Lead with three different diameters (1, 3, and 5 mm). A schematic of the described problem is shown in fig. 1. Physical properties of the selected material as well as the resulted coefficients used in eq. (5) are summarized in tabs. 1 and 2, respectively. It is needed to note that the more complicated problem addressing this observation has been reported by several researchers. However, the application of new analytical methods has been introduced in this research as a new concept.

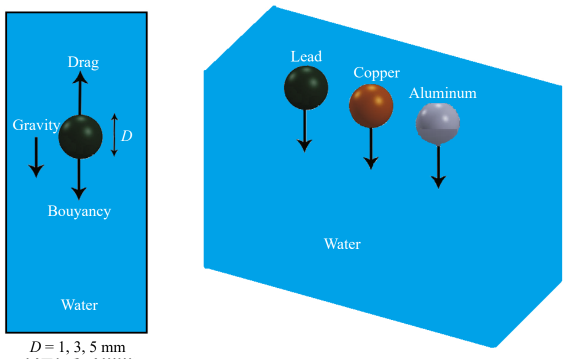

Figure 1. Schematic picture of free falling spherical particles in a Newtonian fluid [26]

Table 1. Properties of the selected materials

Material	Density [kgm^{-3}]	Viscosity [kgm^{-1}s^{-1}]
Aluminum	2702	–
Copper	8940	–
Lead	11340	–
Water	996.51	0.001

Analytical methods

The LSM

As Sheikholeslami *et al.* [34] and Hatami and Domairry Ganji [28] mentioned LSM is one of the weighted residual methods which are constructed on minimizing the residuals of the trial function introduced to the non-linear differential equation. The principle of LSM is based on a differential operator, D, acting on a function u to produce a function p:

Table 2. Coefficient in equation (4)

Particle material	Diameter [mm]	*a*	*b*	*c*	*d*
Aluminum	1	0.0000016756496	0.00000942477796	0.000195664281	0.000008760256
	3	0.0000452425392	0.00002827433389	0.001760978529	0.000236526917
	5	0.0002094562001	0.00004712388981	0.004891607024	0.001095032024
Copper	1	0.0000049418587	0.00000942477796	0.000195664281	0.000040801768
	3	0.0001334301866	0.00002827433389	0.001760978529	0.001101647738
	5	0.0006177323456	0.00004712388981	0.004891607024	0.005100221010
Lead	1	0.0000061984958	0.00000942477796	0.000195664281	0.000053129377
	3	0.0001673593872	0.00002827433389	0.001760978529	0.001434493196
	5	0.0007748119783	0.00004712388981	0.004891607024	0.006641172207

$$\mathrm{D}[u(x)] = p(x) \tag{6}$$

It is considered that u is estimated by a function, which is a linear combination of fundamental functions chosen from a linearly independent set. This is:

$$u \cong \tilde{u} = \sum_{i=1}^{n} c_i \varphi_i \tag{7}$$

by substituting eq. (9) into the differential operator, D, the result of the operations generally is not $p(x)$ and a difference will be appeared. Hence an error or residual will exist:

$$R(x) = \mathrm{D}[\tilde{u}(x)] - p(x) \neq 0 \tag{8}$$

The main concept of LSM is to make the residual become zero over the domain. Hence:

$$\int_x R(x) W_i(x) \mathrm{d}x = 0, \quad i = 1, 2, ..., n \tag{9}$$

Where the number of weight functions, W_i, is accurately equal the number of unknown coefficients, c_i. The result is a set of n algebraic equations for the undefined coefficients c_i. If the continuous summation of all the squared residuals is minimized, the rationale behind the LSM name can be seen. In other words, a minimum of:

$$S = \int_x R(x) R(x) \mathrm{d}x = \int_x R^2(x) \mathrm{d}x \tag{10}$$

In order to achieve a minimum of this function eq. (10), the derivatives of S with respect to all the each unknown parameter should be zero:

$$\frac{\partial S}{\partial c_i} = 2 \int_x R(x) \frac{\partial R}{\partial c_i} \mathrm{d}x = 0 \tag{11}$$

Comparing with eq. (11), the weighted functions for LSM will be:

$$W_i = 2 \frac{\partial R}{\partial c_i} \tag{12}$$

By neglecting the constant coefficient, the weighted functions, W_i, for the LSM are the derivatives of the residuals with respect to the unknown constants:

$$W_i = \frac{\partial R}{\partial c_i} \tag{13}$$

Padé approximation

There are some techniques to increase the convergence of series. Among them, the Padé technique is widely applied. Suppose that a function $f(\eta)$ is represented by a power series so that:

$$f(\eta) = \sum_{i=0}^{\infty} c_i \eta^i \tag{14}$$

This expansion is the fundamental starting point of any analysis using Padé approximants. The notation c_i, $i = 0, 1,2, \ldots$ is reserved for the given set of coefficients:

$$P[L,M] = \frac{a_0 + a_1\eta + \ldots + a_L\eta^L}{b_0 + b_1\eta + \ldots + b_M\eta^M} \tag{15}$$

LSM-Padé

It should be noted that the trial solution must satisfy the boundary conditions, so the trial solution can be written:

$$u(t) = c_6 t^6 + c_5 t^5 + c_4 t^4 + c_3 t^3 + c_2 t^2 + c_1 t \tag{16}$$

By introducing this equation into eq. (4) residual function will be found and by substituting the residual function into eq. (16) a set of equations with six equations and six unknown coefficients will be appeared and by solving this system of equations, coefficients c_1-c_6 will be determined. By using LSM when $a = b = c = d =1$, following equation will be determined for velocity of spherical particles in Newtonian fluid media.

$$\begin{aligned} u(t) = 0.03368239895t^6 - 0.180492214t^5 + 0.3593344626t^4 - \\ -0.1842649152t^3 - 0.4978433912t^2 + 0.9999134150t \end{aligned} \tag{17}$$

After obtaining the result of LSM-Padé, approximation is applied for variations of particle velocities. The Padé technique for variations of velocity at $a = b = c = d = 1$ gives:

$$u(t)\left(LSM - pade[3-3]\right) = \frac{0.9999134149t + 1.531206359t^2 + 0.05828486567t^3}{1 + 2.029225450t + 1.252894743t^2 + 0.6383812347t^3} \tag{18}$$

Results and discussion

In the present study, different analytical methods are applied to obtain an explicit analytic solution of the unsteady settling behavior of solid spherical particles falling in water as a Newtonian fluid. Figure 2 illustrates the comparison between the LSM, LSM-Padé, GM with numerical method for $a = b = c = d = 1$. As observed, Padé approximation [3/3] leads to the results closer to the numerical solution. After claiming the compatibility and accuracy of this model, aluminum and copper as well as lead are selected in various sizes submerged in water.

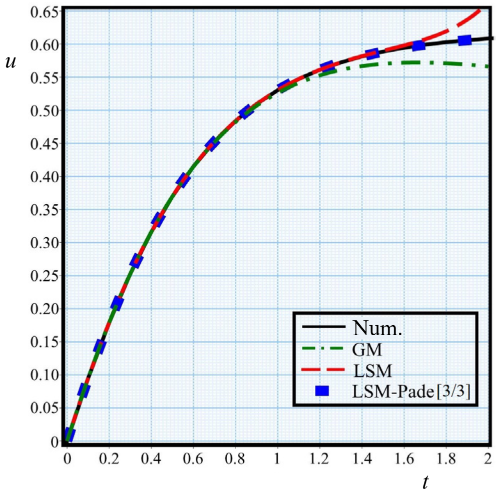

Figure 2. Comparison between LSM, LSM-Padé, GM, and numerical methods for $a = b = c = d = 1$ (*for color image see journal web site*)

Figure 3 depits the variations of velocity for aluminum, copper, and lead for different diameters, respectively. As seen, the velocity enhances by the increase in the particle diameter. In order to better compare the results, the variation of velocity and acceleration with time for different type of particles and the diameter equal to 1 mm is depicted in fig 4. As seen the velocity and acceleration of lead particle is highest which is followed by copper and finally aluminum. Figure 5 represents the position of the particle as well as particle size in sedimentation process for each time step equal to 0.02 second. For all selected particles, terminal velocity is calculated and presented in tab. 3. Results also are compared with HPM [13] and DTM-Padé [27]. It can be concluded from tab. 3

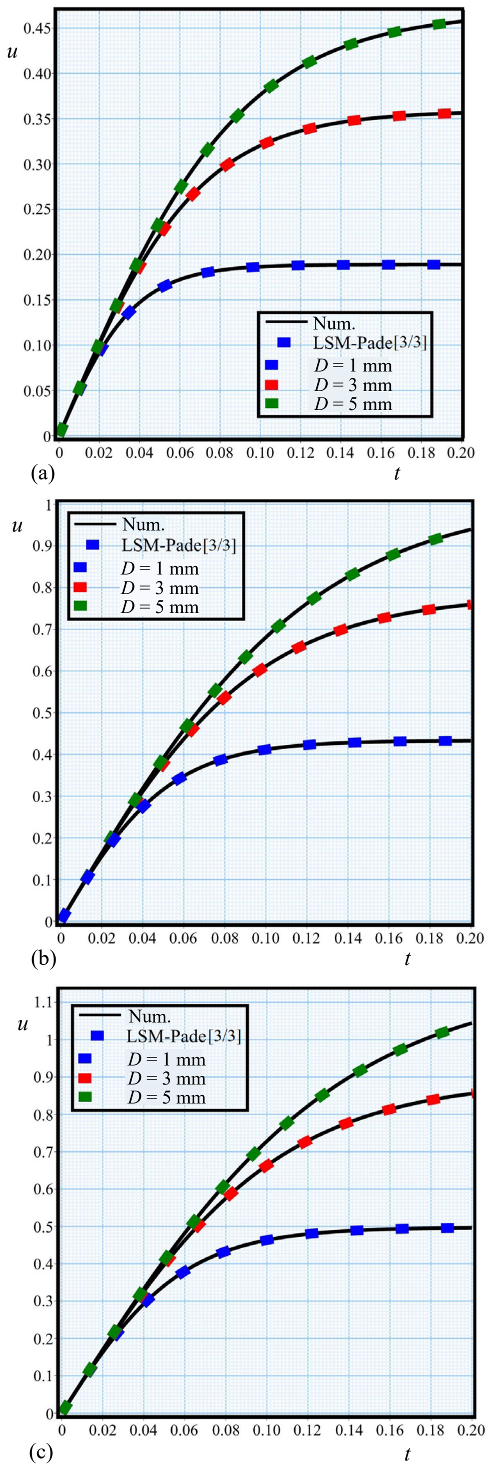

Figure 3. Velocity variation for different particle diameters; (a) Aluminum, (b) copper, (c) lead (*for color image see journal web site*)

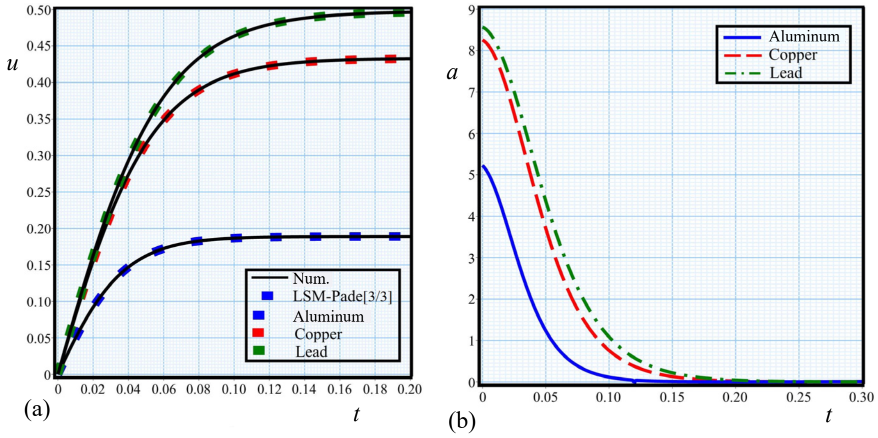

Figure 4. Comparison of (a) velocity and (b) acceleration for different particle materials (*D* = 1 mm) (*for color image see journal web site*)

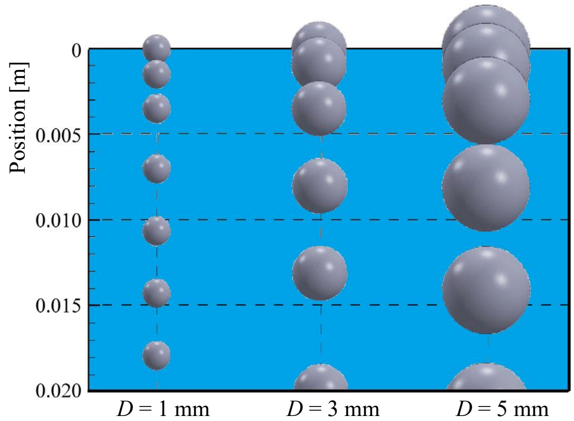

Figure 5. Positions of particles for different size of aluminum particles, time interval = 0.02 second [26]

that LSM-Padé [3/3] estimated the terminal velocity with the acceptable exactness. The results show that the speed values depend on the diameter and density of the particles. Thus, the acceleration period of the smaller particles is shorter. It can be deduced that when the particle is heavier, due to the higher speed has lower positions in the same time step.

Table 3. Terminal velocity [ms^{-1}] for particles calculated by LSM-Padé, HPM [13], DTM-Padé [27], and numerical method

Particle material	Diameter [mm]	Numerical method	LSM-Padé [3/3]	DTM-Padé [8/8]	HPM
Aluminum	1	0.1888759074	0.1888764091	0.1771953458	0.1888743212
	3	0.3585504311	0.3585529684	0.3581483507	0.3585425621
	5	0.4683308257	0.4682026356	0.4682662708	0.4683267958
Copper	1	0.4332008767	0.4332064180	0.4336398188	0.4332014752
	3	0.7829089418	0.7828744656	0.7828929076	0.7828921531
	5	1.0156731422	1.0156759824	1.015681926	1.0156721043
Lead	1	0.4975607949	0.4973452171	0.4837738933	0.4965829147
	3	0.8944267047	0.8944235623	0.8944040156	0.8944159856
	5	1.1589079584	1.1590815413	1.158902486	1.1589059863

Conclusions

In this study, LSM with Padé approximation is applied to solve the problem of the unsteady settling behavior of solid spherical particles falling in water as a Newtonian fluid. The analytical models were presented for particles sedimentation and different particle materials (aluminum, copper, and lead). The following main conclusions can be drawn from the current study:

- The velocity and acceleration of lead particle is higher than copper and aluminum.
- The lower positions in the same time step are observed for the heavier particle.
- The results of LSM with Padé approximation are in excellent agreement with the numerical data.

Nomenclature

a,b,c,d – constants, [–]
C_D – drag coefficient, [–]
D – particle diameter, [m]
$f(\eta)$ – analytic function, [m]
g – gravity, [ms^{-2}]
H – constant value, [–]
m – particle mass, [kg]
P – Padé approximation, [–]
$R(x)$ – residual function, [–]
Re – Reynolds number, [–]
t – time [s]
u – velocity, [ms^{-1}]
W_i – weight function, [–]
$x(t)$ – analytic function, [m^{2}s^{-1}]

Greek symbols

μ – dynamic viscosity [kgms^{-1}]
ρ – density [kgm^{-3}]
ρ_s – particle density [kgm^{-3}]

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