

TWO CONJUGATE CONVECTION BOUNDARY-LAYERS OF COUNTER FORCED FLOW

by

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In this study, the conjugate heat transfer problem of two laminar forced convection boundary-layers of counter flow on the opposite sides of a conductive wall is analyzed by employing the integral method. The analysis is conducted in a dimensionless framework to generalize the solution. The dimensionless parameters affecting the thermal interaction between the two convection layers are deduced from the analysis. These parameters give a measure of the relative importance of interactive heat transfer modes. Mean Nusselt number data are obtained for a wide range of the main affecting parameters.

Key words: *conjugate heat transfer, laminar forced flow, analytical heat convection*

Introduction

Solving a convection heat transfer problem as a conjugate problem yields physically more accurate results than that as a direct problem. This is because convection results depend mainly on applied boundary conditions, and in the conjugate solution, no solid-fluid interface conditions are prescribed in the analysis, but they are determined from the solution like other unknown variables [1]. Therefore, the subject of conjugate problems in convection heat transfer has received a special attention in the research work during the past few decades. This attention reflects in a lot number of studies reported in the literature. Dorfman [2] presented a broad review on conjugate problems of convection heat transfer.

Some studies have been reported on thermal communication between two free convection systems via heat conduction across a vertical wall separating two fluid-fluid [3], porous-porous [4, 5], or porous-fluid [6] reservoirs. Other studies have been conducted on conjugated free convection and forced convection [7-9]. Sparrow and Faghri [7] treated numerically forced flow inside a vertical tube of negligible thermal resistance coupled with surrounding air convection. Shu and Pop [8] used the singular perturbation method to solve the same problem treated by Sparrow and Faghri, however, for a vertical wall of considerable thermal resistance.

Some studies have also been conducted on the conjugate conduction-convection problem of laminar forced flow over a solid plate with the backside maintained at uniform temperature [10-17]. In the earlier popular study of Luikov [10], polynomial velocity and temperature profiles were assumed in the thermal boundary-layer, and the wall conduction was considered only in the cross-wise direction. Later, the same problem has been treated under the approxi-

mation of 1-D wall conduction by employing the integral technique [11, 12], the superposition principle [15] or the Lighthill method [16]. The effect of 2-D wall conduction was modeled in the numerical solution of Chida [17]. Other authors [18, 19] treated the same problem when the back plate side is heated uniformly. Trevino and Linan [18] employed the perturbation technique, while Hajmohammadi and Nourazar [19] used the differential transform method (DTM) to solve the integro-differential equation resulted from the analysis.

Some authors [20-23] investigated the conjugate heat transfer problem of two fluid currents of forced flow on the sides of a moving wall. The topic of thermal interaction between two fluid currents of forced flow on fixed wall sides has received a special interest in the recent research. This is due to its significance for the design and operation of many heat transfer equipment, such as heat exchangers and electronic cooling systems. Several studies dealing with parallel-flow [24, 25] and counter-flow [26-29] arrangements have recently been reported. These studies indicated that the theoretical treatment of the counter-flow pattern yields a more complex mathematical problem compared to that of the parallel flow pattern, even with neglecting the longitudinal conduction in both fluid and solid domains. This mathematical complexity is owing to the elliptical nature of the governing equations of the counter-flow pattern.

Viskanta and Abrams [28] employed the method of superposition to solve the conjugate heat transfer problem of two fluid currents of counter forced flow on the opposite sides of a solid plate. Unfortunately, they used known empirical and analytical expressions of forced flow on isothermal surfaces to predict the convection heat transfer coefficient on the plate sides. Later, Medina *et al.* [29] developed a numerical model for the same problem treated by Viskanta and Abrams by using the lighthill method, which is appropriate only for high Prandtl numbers. Therefore, they assumed a linear velocity profile in the thermal boundary-layer.

In the present work, the conjugate problem of two convection boundary-layers of counter forced flow on the opposite sides of a conductive solid plate is analyzed, however, without introducing such oversimplifications adopted in the previous studies of Viskanta and Abrams [28] and Medina *et al.* [29]. This study is considered of theoretical and practical interest for the design and operation of plate heat exchangers among other thermal equipment. As a brief summary of the analysis presented next, each boundary-layer flow is analyzed separately by employing the well-known integral method. Then, the two analyses are coupled by applying the interfacial conditions of the temperature and heat flux continuity at the plate. In this analysis, neither the temperature nor the heat flux at the plate sides is prescribed in the analysis, but they are determined from the solution. The analysis is conducted in a dimensionless framework to generalize the solution. To overcome the singularity problem encountered in solving the resultant governing equations due to their elliptical nature, an efficient iterative numerical procedure is applied. The main advantage of such a semi-analytical model is that the role of the derived dimensionless parameters controlling the conjugate heat transfer process becomes more evident than in a numerical model.

Analysis

The physical model is sketched in fig. 1. A hot fluid at free temperature, T_{ho} , flows with free velocity, u_{ho} , on the upper surface of a solid wall, while on the back surface, a cold fluid at free temperature $T_{co} < T_{ho}$ flows with free velocity, u_{co} , in the counter direction. To reduce the conjugate analysis complexity, the wall conduction is assumed 1-D in the cross-wise direction, and the two fluids are considered of a Prandtl number of order unity. For clarity in the model presentation, subscripts “c, h, and w” are used to designate cold fluid, hot fluid and wall, respectively, and temperature symbol, T , is used for both fluid and solid media.

For steady forced flow of an incompressible fluid with constant properties on a flat plate (with the x-axis placed on the plane of the plate in the flow direction as in fig. 1), the laminar boundary-layer equations of mass, momentum and energy can be expressed, respectively, by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1a)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dp}{dx} + \nu \frac{\partial^2 u}{\partial y^2} \quad (1b)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (1c)$$

In this case, it is assumed that the viscous dissipation, axial conduction, and buoyancy forces are negligible.

For zero pressure gradient (*i. e.*, $dp/dx = 0$), integrating momentum eq. (1b) across the velocity boundary-layer of the hot plate side yields the integral momentum relation:

$$\frac{d}{dX_h} \int_0^{\Delta_h} U_h (U_h - 1) dY_h = -\frac{\partial U_h}{\partial Y_h} \Big|_{Y_h=0} \quad (2a)$$

Similarity, integrating energy eq. (1c) across the thermal boundary-layer of the hot plate side yields the integral energy relation:

$$\frac{d}{dX_h} \int_0^{\Delta_{th}} U_h (\theta - 1) dY_h = -\frac{1}{Pr_h} \frac{\partial \theta}{\partial Y_h} \Big|_{Y_h=0} \quad (2b)$$

The dimensionless variables previously introduced are:

$$X_h = \frac{x_h}{L}, \quad Y_h = \frac{y_h}{L} \sqrt{Re_h}, \quad \Delta_h = \frac{\delta_h}{L} \sqrt{Re_h}, \quad \Delta_{th} = \frac{\delta_{th}}{L} \sqrt{Re_h}, \quad U_h = \frac{u_h}{u_{h\infty}}, \quad \theta = \frac{T - T_{c\infty}}{T_{h\infty} - T_{c\infty}} \quad (3)$$

The boundary conditions are:

$$\begin{aligned} X_h = 0, \quad U_h = 0, \quad \theta = 1 \\ Y_h = 0, \quad U_h = 0, \quad \frac{\partial^2 U_h}{\partial Y_h^2} = 0, \quad \theta = \theta_{wh}, \quad \frac{\partial^2 \theta}{\partial Y_h^2} = 0 \\ Y_h = \Delta_h, \quad U_h = 1, \quad \frac{\partial U_h}{\partial Y_h} = 0 \quad \text{and} \quad Y_h = \Delta_{th}, \quad \theta = 1, \quad \frac{\partial \theta}{\partial Y_h} = 0 \end{aligned} \quad (4)$$

wherein $Re_h = u_{h\infty} L / \nu_h$ refers to Reynolds number, and Pr_h stands for Prandtl number. While Δ_h and Δ_{th} are velocity and thermal layer thicknesses, and U_h and θ are velocity and temperature, respectively. The symbol θ_{wh} refers to the dimensionless temperature of the wall side facing hot

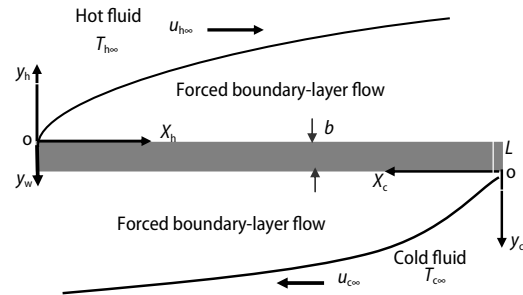


Figure 1. Physical model

fluid, which is assumed an unknown function of X -co-ordinate to be determined from the solution.

The cubic temperature and velocity profiles satisfying boundary conditions (4) are found, respectively, by:

$$\frac{\theta - 1}{\theta_{wh} - 1} = 1 - 1.5 \frac{Y_h}{\Delta_{th}} + 0.5 \left(\frac{Y_h}{\Delta_{th}} \right)^3, \quad 0 \leq Y_h \leq \Delta_{th} \quad (5)$$

$$U_h = 1.5 \frac{Y_h}{\Delta_{th}} - 0.5 \left(\frac{Y_h}{\Delta_{th}} \right)^3, \quad 0 \leq Y_h \leq \Delta_{th} \quad (6)$$

Solving eqs. (2a) and (2b) for the temperature and velocity profiles yields, respectively,

$$\Delta_{th} = \sqrt{280 \frac{X_h}{13}} \quad (7)$$

$$\frac{d}{dX_h} [A_{th} (\theta_{wh} - 1)] = \frac{10(\theta_{wh} - 1)}{\phi_h \text{Pr}_h A_{th}}, \quad \text{for } \phi_h = \frac{A_{th}}{\Delta_{th}} \quad (8)$$

By following a similar analysis procedure for the cold-side convection layer, one gets the following corresponding results:

$$U_c = 1.5 \frac{Y_c}{\Delta_c} - 0.5 \left(\frac{Y_c}{\Delta_c} \right)^3, \quad 0 \leq Y_c \leq \Delta_c \quad (9)$$

$$\frac{\theta}{\theta_{wc}} = 1 - 1.5 \frac{Y_c}{\Delta_{tc}} + 0.5 \left(\frac{Y_c}{\Delta_{tc}} \right)^3, \quad 0 \leq Y_c \leq \Delta_{tc} \quad (10)$$

$$\Delta_c = \sqrt{280 \frac{X_c}{13}} \quad (11)$$

$$\frac{d}{dX_c} [A_{tc} \theta_{wc}] = \frac{10\theta_{wc}}{\phi_c \text{Pr}_c A_{tc}}, \quad \text{for } \phi_c = \frac{A_{tc}}{\Delta_c} \quad (12)$$

The symbol θ_{wc} refers to the dimensionless temperature of the wall side facing the cold fluid, which is also an unknown function of X -co-ordinate to be found from the solution.

Considering the thickness-to-length ratio of solid wall, b/L , is much less than one, the wall conduction can be assumed significant only in the cross-wise direction. Consequently, the temperature distribution across the solid wall is determined:

$$\theta = \theta_{wh} - (\theta_{wh} - \theta_{wc}) Y_w, \quad \text{for } 0 \leq Y_w \leq 1 \quad (13)$$

wherein $Y_w = y_w/b$, $\theta_{wh} = (T_{wh} - T_{c\infty})/(T_{h\infty} - T_{c\infty})$, and $\theta_{wc} = (T_{wc} - T_{c\infty})/(T_{h\infty} - T_{c\infty})$.

Matching conditions

Coupling between previous conduction solution (13) and convection results (8) and (12) can be accomplished by applying the interfacial conditions of the temperature and heat flux continuity at the wall sides. This yields the following two relations:

$$\omega \left(\frac{\partial \theta}{\partial Y_h} \right)_{Y_h=0} = - \left(\frac{\partial \theta}{\partial Y_w} \right)_{Y_w=0} \quad (14)$$

$$\eta \left(\frac{\partial \theta}{\partial Y_c} \right)_{Y_c=0} = - \left(\frac{\partial \theta}{\partial Y_h} \right)_{Y_h=0} \quad (15)$$

The two dimensionless variables ω and η are defined:

$$\eta = \frac{k_c}{k_h} \sqrt{\frac{\text{Re}_c}{\text{Re}_h}} \quad \text{and} \quad \omega = \frac{bk_h}{Lk_w} \sqrt{\text{Re}_h} \quad (16)$$

The variable η represents the thermal resistance ratio of hot-side to cold-side convection layer. Thus, it can be used to determine whether the conjugate problem is controlled mainly by forced convection of the hot side or that of the cold side. While w -parameter relates the thermal resistance of the solid wall to that of hot-side convection layer.

Calculating the temperature derivative terms in eqs. (14) and (15) by eqs. (5), (10), and (13) yields:

$$\theta_{wh} = 1 - \frac{\Delta_{th}}{\Omega}, \quad \Omega = \Delta_{th} + \frac{\Delta_{tc}}{\eta} + 1.5\omega \quad (17)$$

$$\theta_{wc} = \frac{\Delta_{tc}}{\eta \Omega} \quad (18)$$

Inserting θ_{wh} and θ_{wc} from the two previous equations into eqs. (8) and (12), respectively, with replacing X_c by $(1 - X_h)$ this gives after some mathematical manipulations and variables separation the following two differential equations:

$$\frac{d\Delta_{th}}{dX_h} = \left[\frac{2}{\phi_h \text{Pr}_h \Delta_{th}} - \frac{1}{\phi_c \text{Pr}_c \left(\Omega - \frac{0.5\Delta_{tc}}{\eta} \right) \eta} \right] \frac{10\eta \Omega \left(\Omega - \frac{0.5\Delta_{tc}}{\eta} \right)}{2\eta(2\Omega - \Delta_{th}) \left(\Omega - \frac{0.5\Delta_{tc}}{\eta} \right) - \Delta_{tc} \Delta_{th}} \quad (19)$$

$$\frac{d\Delta_{tc}}{dX_h} = \left[\frac{2\Delta_{tc}}{2\phi_h \text{Pr}_h (2\Omega - \Delta_{th}) \Delta_{th}} - \frac{1}{\phi_c \text{Pr}_c \Delta_{tc}} \right] \frac{10\eta \Omega (2\Omega - \Delta_{th})}{2\eta(2\Omega - \Delta_{th}) \left(\Omega - \frac{0.5\Delta_{tc}}{\eta} \right) - \Delta_{tc} \Delta_{th}} \quad (20)$$

Equations (19) and (20) are considered the main results of analysis, whose solution will provide the distributions of Δ_{th} , Δ_{tc} , θ_{wh} , and θ_{wc} along the wall as functions of η and ω parameters. However, it may be considered of a design interest to calculate the mean conjugate Nusselt number, which can be defined in terms of hot-side properties:

$$\text{Nu} = \frac{\bar{q}k_h}{T_{h\infty} - T_{c\infty}} \quad (21)$$

wherein $\bar{q}$ is the mean wall heat flux calculated by integrating the local wall heat flux from $X_h = 0$ to 1 by using eq. (5). Substituting this integration result in eq. (21) gives the Nusselt number relation:

$$\frac{Nu}{Re_h^{1/2} Pr_h^{1/3}} = 0.332 \int_0^1 \frac{1 - \theta_{wh}}{\sqrt{X_h}} dX_h \quad (22)$$

Similarly, by defining the mean conjugate Nusselt number based on cold-side parameters, one gets the alternative Nusselt number relation:

$$\frac{Nu}{Re_c^{1/2} Pr_c^{1/3}} = -0.332 \int_0^1 \frac{\theta_{wc}}{\sqrt{1 - X_h}} dX_h \quad (23)$$

Solution

Asymptotic results

For the special problem case of zero or negligible wall resistance, asymptotic results can be deduced from the general analysis derived in the previous section. In this case, the solid wall acts a partition of zero thermal resistance, whose temperature is a function of X -direction only. For this case of $\omega \equiv 0$, eqs. (17) and (18) show, respectively, that $\theta_{wh} \rightarrow 0$ and $\theta_{wc} \rightarrow 0$ as $\eta \rightarrow \infty$. This means that on the $\eta \rightarrow \infty$ limit, both wall sides assume the minimum cold-side temperature of a zero dimensionless value. Hence, forced convection layer on the cold side disappears, and consequently, the conjugate problem reduces to the classical problem of forced convection on an isothermal surface. This behavior is expected when considering the physical significance of η defined by eq. (16).

Now, solving eq. (22) for $\theta_{wh} = 0$ gives:

$$\frac{Nu}{Re_h^{1/2} Pr_h^{1/3}} = 0.664 \quad (24)$$

The previous result is the same exact one of forced convection on an isothermal flat surface [30].

On the other $\eta \rightarrow 0$ limit, eqs. (17) and (18) show for this case of $\omega = 0$ that $\theta_{wh} \rightarrow 1$ and $\theta_{wc} \rightarrow 1$, respectively. This means that both wall sides take the maximum hot-side temperature of 1-D value. Hence, the hot-side convection layer collapses, and consequently, the conjugate problem dimensions to that of forced convection on an isothermal surface of one dimensionless temperature. Solving eq. (23) for $\theta_{wc} = 1$ gives:

$$\frac{Nu}{Re_c^{1/2} Pr_c^{1/3}} = 0.664 \quad (25)$$

The previous result is also the same exact one of forced convection on an isothermal plane surface. Here, it is important to state that asymptotic results (24) and (25) prove the model's validity.

Numerical results

The two main governing eqs. (19) and (20) are dependent, non-linear, ODE, which should be solved simultaneously to determine the distributions of Δ_{th} , Δ_{tc} , θ_{wh} , and θ_{wc} along the wall as functions of η and ω parameters. This solution could be conducted numerically by employing the well-known fourth-order Runge-Kutta integral technique. The numerical integration begins at $X_h = 0$, *i. e.*, at the start point of the hot-side convection layer (cf., fig. 1). However, because of the singularity problem encountered in the solution for using $\Delta_{th} = 0$ at $X_h = 0$, an approximate start value of Δ_{th} very close to zero, which is calculated by eqs. (7) at

small $X_h = 10^{-6}$, was used to overcome this problem. Another problem encountered in the solution is that the maximum Δ_{lc} value at the solution start point of $X_h = 0$ is unknown. To solve this problem, this maximum value of Δ_{lc} is assumed at the solution start. Then, the solution procedure advances in small steps of ΔX until $X_h = 1$. When the predicted Δ_{lc} at $X_h = 1$ is found different from zero, the solution trial is repeated by using a new adjusted maximum value of Δ_{lc} until, eventually, the predicted Δ_{lc} at $X_h = 1$ is found very close to zero, actually less than 0.00001. Hence, the solution trials are stopped. In preliminary solution tests, asymptotic results (24) and (25) were used as a reference to check the correctness of numerical results as well to adjust its accuracy. It has been found that the solution with step size $\Delta X = 0.005$ gives stable and reliable results. Once the distributions of Δ_{th} and Δ_{lc} along the wall have been obtained for certain η and ω parameters, the corresponding distributions of wall-side temperatures θ_{wh} and θ_{wc} can be calculated by eqs. (17) and (18), respectively. Results have been obtained for $0.01 \leq \eta \leq 100$ and $0 \leq \omega \leq 3$. The numerical solution was found stable for these ranges of controlling parameters ω and η . Figures 2-9 demonstrate obtained results.

At first, numerical results obtained for the special problem case of negligible wall resistance are discussed. In this case, the wall acts a partition of zero thermal resistance, whose temperature θ_w is a function of the X -co-ordinate only. The variation of convection layer thickness along the wall on both sides is depicted in fig. 2, for different convection conjugation parameter η . The results show that for a certain η -value, the layer thickness increases on both sides with distance from the start point of $X = 0$. However, for a higher η -value, the convection layer becomes thicker on the hot side while it gets thinner on the cold side. Figure 3 displays the velocity profile across the two convection layers for different η -parameter. Figure 4 demonstrates the effect of η -parameter on the temperature distribution across two fluid media at the wall midpoint of $X = 0$. It is clear that for $\eta = 1$, the temperature drop across hot-side convection layer is equal to that across the cold-side layer. This means that the heat transfer effectiveness of the hot-side convection layer is equivalent to that of the cold-side layer. However, for a higher η , the temperature drop across convection layer gets higher on the hot side while it becomes lower on the cold side. Figure 5 shows the dependence of wall midpoint temperature θ_w on η -parameter. It is observed that θ_w decreases with increasing η to assume finally the minimum cold-side temperature of zero dimensionless zero as η goes to infinity. While θ_w increases with decreasing η to take finally the maximum hot-side temperature of one dimensionless value as η approaches zero. This means that $\theta_w \rightarrow 0$ as $\eta \rightarrow \infty$, while $\theta_w \rightarrow 1$ as $\eta \rightarrow 0$. This behavior can be explained: as η goes to infinity, the cold-side convection layer disappears, and consequently, the wall assumes the minimum temperature of the cold side. On the opposite limit of $\eta \rightarrow 0$, the

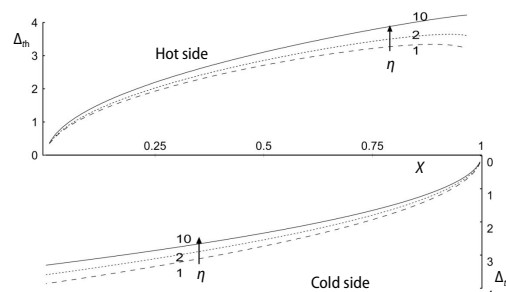


Figure 2. Variation of convection layer thickness on wall sides at different η -parameter for $\omega = 0$

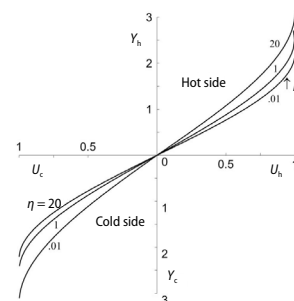


Figure 3. Velocity profile across two fluid media at $x = 0$ for $\omega = 0$ at different η

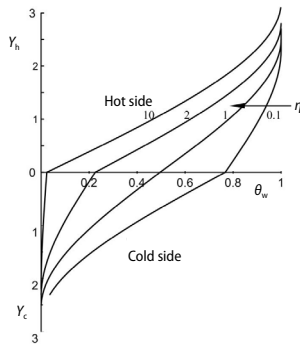


Figure 4. Temperature profile across two fluid media at $X = 0.5$ for $\omega = 0$ at different η

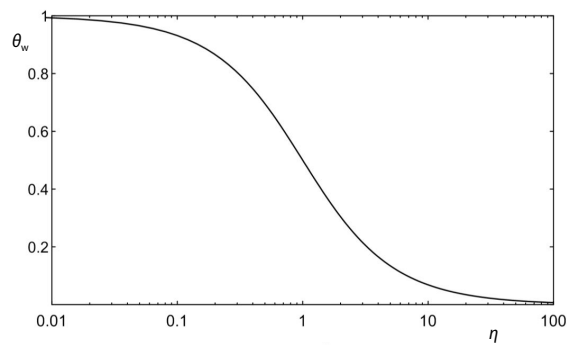


Figure 5. Variation of local wall temperature at $x = 0$ with η -parameter for $\omega = 0$

hot-side convection layer disappears, hence, the wall takes on the extreme temperature of the hot side. The variation of mean conjugate Nusselt number with η parameter is presented in fig.

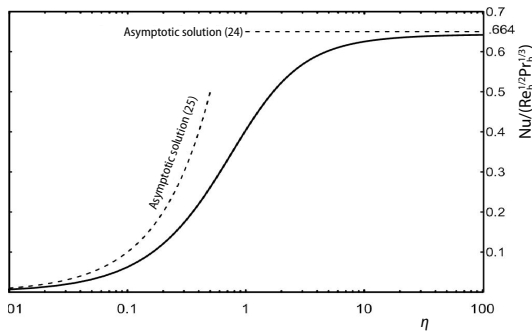


Figure 6. Mean Nusselt number as a function of η -parameter for $\omega = 0$

θ_{wh} on η -parameter is demonstrated in fig. 7: for $\omega = 2$. It is noted that for η -value ≤ 0.01 , both wall-side temperatures θ_{wc} and θ_{wh} assume nearly a constant value very close to the extreme hot-side temperature of the 1-D value. However, for η -value > 20 , only the wall side facing the cold fluid takes a temperature close to the minimum cold fluid temperature of zero dimensionless value. The remarkable difference between any two corresponding local wall-side temperatures, *i. e.*, the local temperature drop across the solid wall, is attributed to the effect of wall resistance. This effect of wall resistance parameter ω on the temperature drop across the solid wall is more clearly displayed in fig. 8, for $\eta = 1$. It is noted that the temperature drop rises with an increase in ω -value. In fact, the wall works as a thermal damper between the two interactive fluid media. The dependence of mean conjugate Nusselt number on convection conjugation parameters η and wall resistance parameter ω is displayed in fig. 9, for $0 \leq \omega \leq 3$ and $0.01 \leq \eta < 100$. In the graph, the upper curve of $\omega = 0$ is limited by the two dashed lines representing two exact results (24) and (25) of forced convection on isothermal surfaces. It is clear that Nusselt number is higher for a higher η -parameter, while it is lower for a higher ω -parameter. These results indicate also that ω and η values, Nusselt number increases as Pr and/or Re increases.

6. It is noted that Nusselt number increases with an increase in η -parameter to approach asymptotic result (24) as $\eta \rightarrow \infty$. While Nusselt decreases with a decrease in η -parameter to approach finally asymptotic result (25) as η goes to zero. This behavior is expected when considering the physical meaning of η -parameter defined by eq. (16). For more details, the reader can return to the discussion cited in the previous subsection of asymptotic results.

Next, numerical results obtained for the case of $\omega > 0$ are discussed. The dependence of wall-side temperature profiles: θ_{wc} and

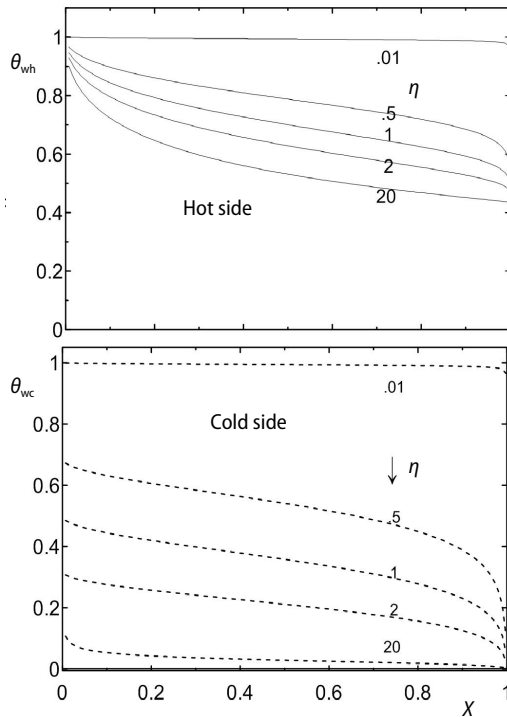


Figure 7. Dependence of wall-side temperature profiles on η -parameter for $\omega = 2$

Model validity

Here, it is important to point out that in the case of negligible wall resistance of $\omega \rightarrow 0$, the model validity has been proved by showing that the model yields exactly the well-known solution of laminar forced convection on isothermal surfaces, cf., eqs. (24) and (25). However, for the case of $\omega > 0$, there is no experimental data available in the literature, which can be used to do comparison with in order to prove the model validity. Therefore, special model problems have been constructed and solved by the known numerical FLUENT software (V. 14.5). Hence, calculated FLUENT results could be compared with the corresponding model predictions. In these special model problems, hot standard engine oil is assumed to flow on the upper plate side with cold water flowing on the lower side. The free temperatures of engine oil and water are, respectively, assumed 120 °C and 30 °C. The plate is assumed of 0.1 mm thickness and 0.5 m length. The problem was solved by FLUENT software and present model for the six different flow condi-

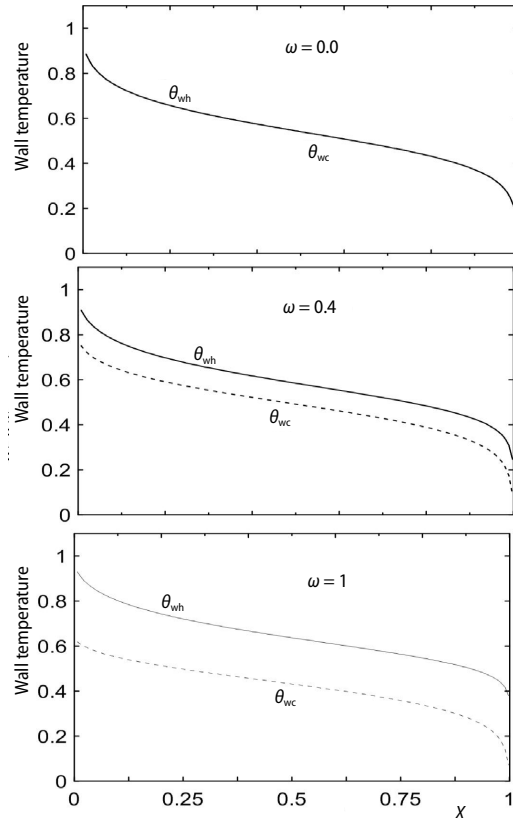


Figure 8. Temperature profiles of wall sides at different wall parameter ω for $\eta = 1$

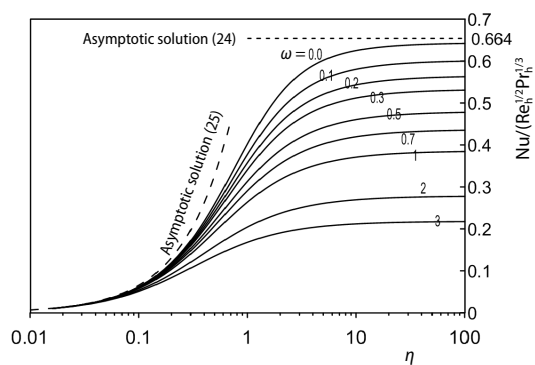
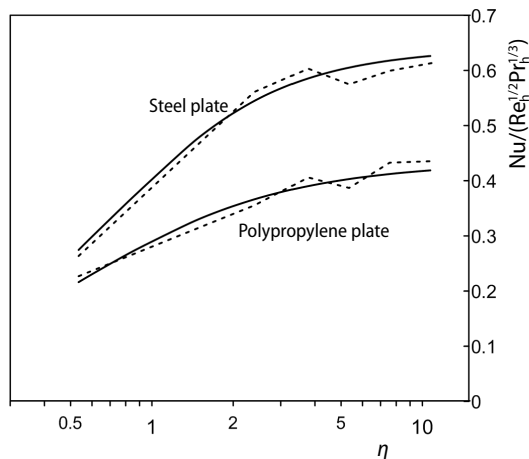


Figure 9. Mean Nusselt number as a function of η and ω parameters

Table 1. Validity test data

Hot fluid –to – cold fluid	Re_h	Re_c	ω		η
			Steel plate	Polypropylene plate	
Engine oil –to – water	$60 \cdot 10^3$	400000	0.01	0.71	10.7
		200000			7.56
		100000			5.34
		50000			3.78
		20000			2.39
		1000			0.54

tions cases listed in tab. 1. The solution was calculated for a stainless steel plate (AISI) of thermal conductivity $k = 14.2 \text{ W/m}^\circ\text{C}$, and repeated for a Polypropylene plate of thermal conductivity $k = 0.1 \text{ W/m}^\circ\text{C}$. The corresponding η and ω values, calculated by eq. (16) and used to calculate the model predictions, are listed in tab. 1. Figure 10 displays the comparison between the FLUENT results (calculated at the input-data points cited in tab. 1) and the corresponding η and ω model predication. The comparison shows an acceptable agreement between the two solutions. The relative deviation between them is within $\pm 2\%$, which may be explained as a numerical calculating error.

**Figure 10. Comparasion of model prediction (solid lines) and FLUENT results (dashed lines)**

the variation in the temperature and velocity in all grid domain was set to be less than 10^{-6} . In the preliminary solution trails, the numerical model was examined against the known exact solution, cf., eq. (24) of laminar forced convection on isothermal surfaces to test the numerical solution validity and accuracy.

Conclusions

In this paper, a semi-analytical model has been developed for the conjugate heat transfer problem of two convection boundary-layers of laminar forced flow on the opposite sides of a conductive solid plate. The resultant main differential equations have been solved numerically by using the well-known, fourth-order Runge-Kutta numerical technique. The analysis proved that the thermal interaction between the two forced-convection layers across the con-

As a brief description for the FLUENT solution, the same approximations adopted in the present semi-analytical model were also applied in the numerical solution. The governing mass, momentum, and energy equations were solved numerically by using a control volume discretization procedure. A second-order upwind expression was used as the discretization scheme for the energy and momentum equations. A segregated solver was employed for the simultaneous solution of the discretized governing equations. A quadrilateral cell with consecutive ratio of 1.025 was used in the domain, except in the solid plate, where equal-space nodes were used. Relaxation factors were used to control the solution convergence. As a convergence condition,

ductive solid plate is controlled mainly by 2-D variables η and ω . The variable η represents the thermal resistance ratio of hot-side convection layer to cold-side convection layer. While ω variable gives a measure of the thermal resistance ratio of the solid wall to hot-side convection layer. Asymptotic results have been derived for the special problem case of negligible wall resistance, which prove the model validity. Mean conjugate Nusselt number data have been obtained for the controlling parameters range: $0 \leq \omega \leq 3$ and $0.01 \leq \eta < 100$. The demonstrated results show that mean conjugate Nusselt number increases with the increase in η -parameter and/or the decrease in wall parameter ω . Comparison of present results with FLUENT results indicates an acceptable agreement.

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Nomenclatures

b	– wall thick ness, [m]
h	– heat transfer coefficient, [$\text{Wm}^{-2}\text{C}^{-1}$]
k	– thermal conductivity, [$\text{Wm}^{-1}\text{C}^{-1}$]
L	– wall length, [m]
Nu	– mean Nusselt number, defined by eqs. (22) and (23), [–]
Pr	– Prandtl number, [–]
$\bar{q}$	– mean heat flux over entire wall length, [Wm^{-2}]
Re	– Reynolds number, ($= u_{\infty}L/\nu$), [–]
T	– temperature, [$^{\circ}\text{C}$]
$T_{\text{h}\infty}$	– free hot-fluid temperature, [$^{\circ}\text{C}$]
$T_{\text{c}\infty}$	– free cold-fluid temperature, [$^{\circ}\text{C}$]
ΔT	– total temperature drop across two fluid media, ($= T_{\text{h}\infty} - T_{\text{c}\infty}$), [$^{\circ}\text{C}$]
U	– dimensionless velocity component in X-direction [–]
u	– velocity component in x-direction, [ms^{-1}]
X, Y	– dimensionless vertical and horizontal co-ordinates, [–]
x, y	– vertical and horizontal co-ordinates, [m]

Greek symbols

Δ	– dimensionless thickness of velocity layer, [–]
Δ_{ic}	– dimensionless thickness of cold-side thermal convection layer, [–]
Δ_{h}	– dimensionless thickness of hot-side thermal convection layer, [–]
δ	– velocity layer thickness, [m]
η	– dimensionless convection conjugation parameter, cf., eq. (16), [–]
θ	– dimensionless temperature, cf., eq. (3), [–]
ω	– dimensionless wall resistance parameter, cf., eq. (16), [–]
ϕ	– thickness ratio of thermal to velocity layer, [–]

Subscripts

c	– cold fluid
h	– hot fluid
w	– wall
wc	– wall surface facing cold medium
wh	– wall surface facing hot medium

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