A FRACTIONAL MODEL FOR INSULATION CLOTHINGS WITH COCOON-LIKE POROUS STRUCTURE

by

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Both silkworm cocoons and wild silkworm cocoons have excellent mechanical properties, as a protective barrier against environmental damage and attack by natural predators. In particular, this multilayer porous structure can be exceptionally tough to enhance the chance of survival for pupas while supporting their metabolic activity. Here, a fractional derivative is defined through the variational iteration method, and its application to explaining the outstanding thermal protection of insulation clothings with cocoon-like porous structure is elucidated. The fractal hierarchic structure of insulation clothings makes human body mathematically adapted for extreme temperature environment.

Key words: *fractional model, insulation clothings, variational iteration method*

Introduction

There evolved over thousands of years' natural selection, silkworm cocoons are of hierarchical and multi-functional structure. Their biological functions, *e. g.* defense against natural enemies, thermal regulation and anti-bacterial function, are essential for the survival of silkworms residing inside [1]. In addition, wild silkworms are reared in the open environment and conceivably need much greater protection from environmental, biotic, and physical hazards. The research has shown that wild silkworm cocoons produced in winter are more robust than their summer counterparts [2]. Wild silkworm cocoons have also shown higher tensile strength than domesticated ones [3]. The silkworm cocoon has a multi-layer structure and is called to be *breathing*, which has a superior heat and moisture transport capability comparing with other natural or man-made fibers [4-6]. The fractal model for heat transfer in hierarchic cocoons has been analyzed with assistance of the fractal derivative model [7, 8].

In this paper, a new fractional derivative is introduced through the variational iteration method [9], and adopted for explaining the excellent thermal protection of insulation clothings with cocoon-like porous structure. There will be helpful for developing new multi-functional shoes and clothings.

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Definition on fractional derivative through the variational iteration method

There are many definitions on fractional derivatives. The variational iteration method was first used to solve fractional differential equations in 1998 [9], and it has been shown to solve a large class of non-linear differential problems effectively, easily, and accurately with the approximations converging rapidly to accurate solutions, and now it has matured into a relatively fledged theory for various non-linear problems, especially for fractional calculus [9-16].

We consider the following linear equation of n^{th} order:

$$u_m^{(n)}(t) \quad f_m(t) \quad 0 \tag{1}$$

By the variational iteration method [9], we have the following variational iteration algorithm:

$$u_{m\ 1}(t) \quad u_m(t) \quad (\ 1)^n \ {}_{t_0}^{t} \frac{1}{(n\ \ 1)!}(s\ \ t)^{n\ 1}[u_m^{(n)}(s) \quad f_m(s)]\mathrm{d}s \tag{2}$$

We introduce an integration operator I^n defined by He [17]:

$$I^n f \quad {}_{t_0}^{t} \frac{1}{(n\ \ 1)!}(s\ \ t)^{n\ 1}[u_0^{(n)}(s) \quad f(s)]\mathrm{d}s \quad \frac{1}{\Gamma(n)}\ {}_{t_0}^{t}(s\ \ t)^{n\ 1}[f_0(s) \quad f(s)]\mathrm{d}s \tag{3}$$

where $f_0(t) \quad u_0^{(n)}(t)$.

We can define a fractional derivative in the form:

$$\mathrm{D}_t^{\alpha} f \quad \mathrm{D}_t^{\alpha}\frac{\mathrm{d}^n}{\mathrm{d}t^n}(I^n f) \quad \frac{\mathrm{d}^n}{\mathrm{d}t^n}(I^{n\ \alpha} f) \quad \frac{1}{\Gamma(n\ \ \alpha)}\frac{\mathrm{d}^n}{\mathrm{d}t^n}\ {}_{t_0}^{t}(s\ \ t)^{n\ \alpha\ 1}[f_0(s) \quad f(s)]\mathrm{d}s \tag{4}$$

where $f_0(t)$ is a known function, its physical explanation will be given in the next section.

An application

As an application of the new fractional derivative, we consider insulation clothings with cocoon-like porous structure. The insulation clothings are superior properties such as the excellent thermal protection. Its cocoon-like hierarchy plays an important role.

Using Fourier's law of thermal conduction in fractal porous hierarchy, we obtain the following fractional differential equation:

$$\frac{\partial^{\alpha}}{\partial x^{\alpha}} \ k\frac{\partial^{\alpha} T}{\partial x^{\alpha}} \quad 0 \tag{5}$$

with boundary conditions:

$$T(0) \quad T_0, \quad T(L) \quad T_L \tag{6}$$

where T is the temperature, k – the thermal conductivity of heat flux in the fractal medium, α – the fractional dimensions of the fractal medium, $^{\alpha} \quad x^{\alpha}$ – the fractional derivative defined as [17] from eq. (4):

$$\frac{\partial^{\alpha} T}{\partial x^{\alpha}} \quad \frac{1}{\Gamma(n\ \ \alpha)}\frac{\mathrm{d}^n}{\mathrm{d}x^n}\ {}_{t_0}^{t}(s\ \ x)^{n\ \alpha\ 1}[T_0(s) \quad T(s)]\mathrm{d}s \tag{7}$$

where $T_0(x)$ can be the solution of its continuous partner of the problem with the same boundary/initial conditions of the fractal partner.

By the fractional complex transform [18-20]:

$$\frac{\partial^\alpha T}{\partial x^\alpha} \quad \frac{\partial T}{\partial s}\frac{\partial^\alpha s}{\partial x^\alpha} \quad c\frac{\partial T}{\partial s} \tag{8}$$

$$s \quad \frac{x^\alpha}{\Gamma(1 \quad \alpha)} \tag{9}$$

Equation (5) is converted to a partial differential equation, which reads:

$$\frac{\partial}{\partial s} \; k\frac{\partial}{\partial s} \quad 0 \tag{10}$$

Equation (10) has the solution:

$$T \quad a \quad bs \quad a \quad \frac{bx^\alpha}{\Gamma(1 \quad \alpha)} \tag{11}$$

After incorporating the boundary conditions of eq. (6), we have:

$$T \quad T_0 \quad \frac{T_L \quad T_0}{L^\alpha}x^\alpha \tag{12}$$

It is obvious that the solution has the following remarkable property:

$$\frac{\mathrm{d}T}{\mathrm{d}x}(x \quad 0) \quad \begin{matrix} 0, \alpha \quad 1 \\ \dfrac{T_L \quad T_0}{L}, \; \alpha \quad 1 \\ \infty, \; \alpha \quad 1 \end{matrix} \tag{13}$$

As shown in fig. 1, the cocoons have the hierarchical porous structure and fractal heat transfer property.

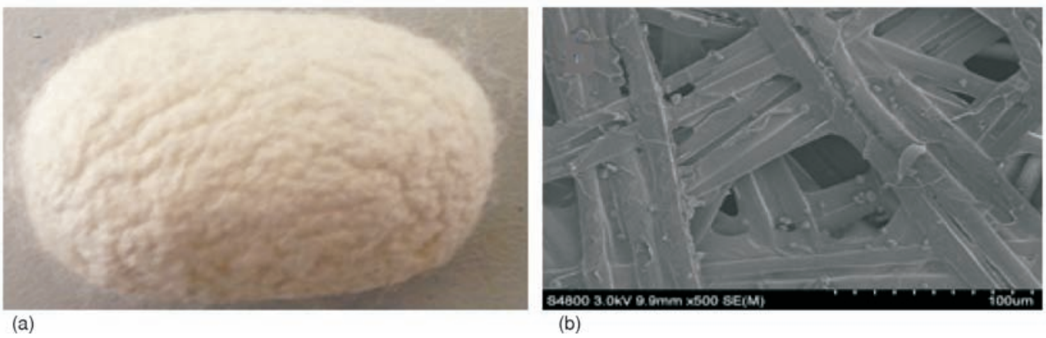

Figure 1. The photographs of Bombyx-mori cocoon; real photo (a) and scanning electron micrographs (b)

According to eq. (12), the slope at $x = 0$ depends strongly upon the value of the fractal dimensions, α [13, 21, 22]. Figure 2 shows that the diagram of heat transfer in the clothings with cocoon-like porous structure. The temperature of the body surface for the human should be changed as smooth as possible, so it requires $\alpha > 1$. Insulation clothings with cocoon-like porous structure can guarantee $\alpha > 1$, whether the environment temperature was as high as 80 °C or low to –45 °C (fig. 2, a, b, c, and d). In the end, the environment temperature approached the body temperature through the heat transfer of clothings.

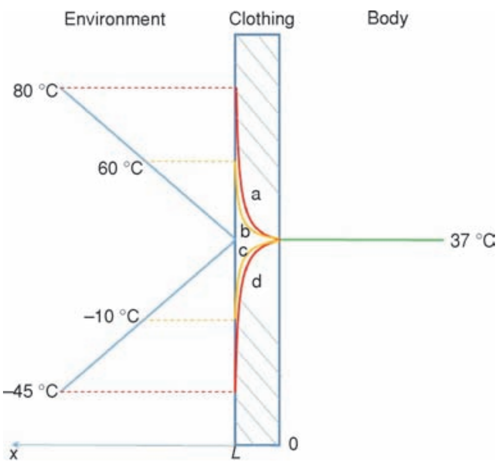

Figure 2. The diagram of heat transfer in the clothings with cocoon-like porous structure

In order to verify previous model, the internal temperature variation of cocoons is performed. The testing set-up is shown in fig. 3. At the certain constant test conditions, five independent cocoons in the chamber were measured at the same time.

The testing results for cocoons are illustrated in fig. 4. The yelow curve represented environment temperature varying from –45 °C to 80 °C. The internal temperature variation of five independent cocoons was measured and the corresponding curve was also drawn out. We found that the five curves showed almost entirely coincidence. It was suggested that individual difference from sample cocoons was very small. In addition, the internal temperature variation of cocoons became more slowly than that of environment temperature. But the tendency of the curves fell into step. This should be probably attributed to the hierarchical porous structure of cocoons. Finally, when environment temperature changed no longer, the temperature inside cocoons without heat source tended to equilibrium during a certain test time. This is consistent with the previous theory [23, 24].

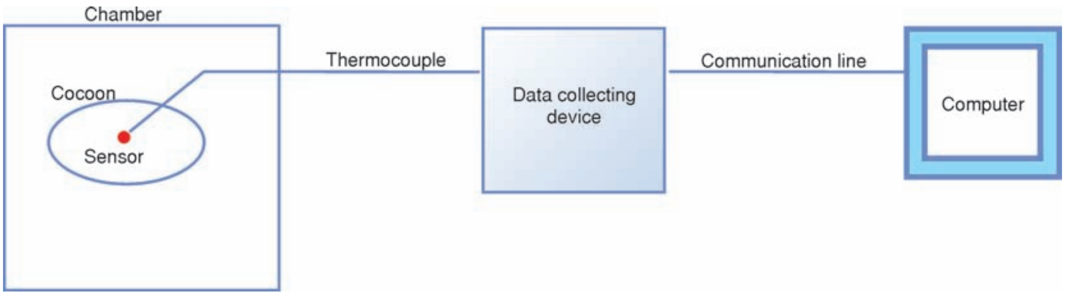

Figure 3. The set-up for internal temperature test of cocoons

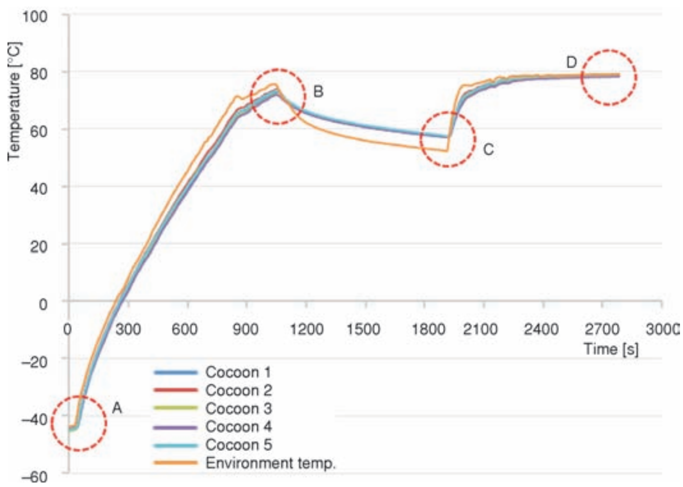

Figure 4. The curves of testing data for cocoons
(for color image see journal web-site)

Figure 5 shows the image magnification of the area A from fig. 4 and schematic di-

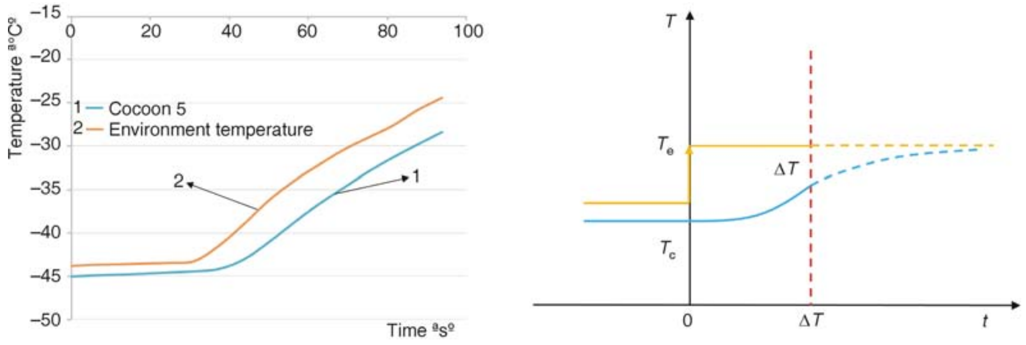

Figure 5. The image magnification (left) of the area A from fig. 4 and schematic diagram of temperature change (right)

agram of temperature change. As shown in fig. 5, the temperature inside the cocoon also raised or lowered as the environment temperature increased or decreased. But the temperature change was always relatively slow.

Conclusions

A more generalized fractional derivative was derived using the variational iteration method. This model was verified by thermal regulation of sample cocoons and has the effectiveness. Therefore, the fractional derivative is an efficient method to handle these complicated heat transfer problems of hierarchic porous media. The slope at the boundary depends strongly upon the fractal structure of insulation clothings with cocoon-like porous structure. The establishment of heat transfer mechanisms for the clothings could be beneficial to the biomimetic design, such as biomaterials, functional textiles, and aviation industry.

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