MODELING OF DRYING RATE OF NON-HYGROSCOPIC PRODUCTS
Application to the Pozzolana Drying

by

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Polynomial model adopted for the moisture content influence to the drying rate of non-hygroscopic products like the pozzolana presented some limits. This model was not universal and has diverged whether moisture content different to the appropriate experiment values has been used. Thus, a new exponential model based on the inverse of moisture content and respecting to the imposed boundary conditions has been proposed. Model parameters were determined using the least square approximation method of Newton-Marquart. The predicted temporal variations of moisture content were provided from the Runge-Kutta fourth order integration scheme computed on the new model of drying rate. Moreover, two-way ANOVA method was performed on data analysis, at the parameters statistical significance and in residual analysis for the model fit quality. Model validations were done by comparisons among theoretical and experimental values and between predicted and polynomial models. Comparisons showed a good agreement with highest R^2 and lowest reduced chi-square, MBE, RMSE, and MAPE.

Key words: *universal model, pozzolana, non-hygroscopic products, drying, drying rate*

Introduction

Drying rate modeling of food and/or industrial products has been an interesting domain for economic, sanitary, and environmental reasons. Several researchers have already established drying rate models for the hygroscopic products such as pineapple and vanilla [1], peel of grenadine [2], banana [3], rice paddy [4], fish [5], whey [6], wood [7], shrimp [8], corn [9], cooked rice and sewage sludge from wastewater treatments [10], tomato [11]. These models were generalized using the Henderson relationship for hygroscopic products [12].

However, in the case of the non-hygroscopic products, Lambert *et al.* [13], Benkaddour *et al.* [14], Thikare *et al.* [15], Bottoni [16], and Billong *et al.* [17] have investigated the

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physico-chemical properties about the mixture of non-hygroscopic product with other materials and have analyzed the pozzolana importance in the industrial use. Yet, these authors did not study the drying kinetics of these materials. Indeed, established model for the drying kinetics of non-hygroscopic products is rare and non-universal, namely the pozzolana drying rate as seen into the work of Ramamonjisoa [18]. On this model, drying rate was expressed on several parameters that influenced the drying process like the product thickness, the temperature and the moisture content. In the literature, a polynomial model has been used to describe the influence of product moisture content in drying rate [18]. The polynomial model has had some limits in conditions of use that were different from those of the experiment. The model was diverged at moisture content values outside of the experimental values.

To palliate this problem, moisture content effect into the drying rate was rephrased to better describe the drying rate variations with the moisture content of non-hygroscopic products. Therefore, a new model of exponential type that was converged for a large range at the moisture content values is predicted. This new model will be validated by using the corresponding experimental data on the pozzolana drying and by comparison to the polynomial model [18].

Methodology

Modeling of f(N_S) function

The general form of drying rate was given:

$$\frac{\mathrm{d}N_\mathrm{S}}{\mathrm{d}t} = -mf(N_\mathrm{S}) \tag{1}$$

wherein

$$m = \exp\left(-\frac{2.28\cdot 10^3}{T_p} + \frac{1.89}{e} - 1.89\right)$$

for the pozzolana was established in [18].

The boundary conditions were as $f(N_\mathrm{S})$ must be converged toward a finite value for the large moisture content values and was tended toward zero for small value of this moisture content.

In this study, new model of exponential form was proposed in which the expression was presented:

$$f(N_S) = \alpha \exp\left(-\frac{a}{N_S}\right) \tag{2}$$

The parameters α and a were determined through the experimental data for the non-hygroscopic products drying like the pozzolana. Determination of $f(N_\mathrm{S})$ function has required the numerical approximation methods. The experimental values of pozzolana drying rate corresponding to the moisture content variations were computed via the finite differences method [19] at a stationary time interval Δt. Obtained values were the experimental values of drying rate variations on the drying time. Therefore, experimental values for $f(N_\mathrm{S})$ were calculated by using the general definition of drying rate. By taking the absolute values of drying rate in eqs. (1) and (2) was led to eq. (3) as:

$$f(N_S) = \frac{1}{m}\left|\frac{\mathrm{d}N_\mathrm{S}}{\mathrm{d}t}\right| \tag{3}$$

The eqs. (1)-(3) allowed the estimation of α and a. Determination of these parameters was carried out by the least square method of Newton-Marquart [20, 21]. In fact, eq. (2) can be written:

$$\ln[f(N_\mathrm{S})] = -\frac{a}{N_\mathrm{S}} + b, \quad \text{with} \quad b = \ln\alpha \tag{4}$$

While combining eqs. (3) and (4), eq. (5) was obtained like :

$$\ln\left[\frac{1}{m}\left|\frac{\mathrm{d}N_\mathrm{S}}{\mathrm{d}t}\right|\right] = -\frac{a}{N_\mathrm{S}} + b \tag{5}$$

Profiles of $\ln[f(N_S)]$ on the inverse of product moisture content have provided from the eq. (5).

If these profiles were arisen likely in straight line, parameters a, b, and α could be determined across different values at other drying parameters as the temperature and the thickness of product, fig. 1.

Indeed, the predicted drying rate integration allowed calculating the values of theoretical moisture content. This integration was carried out of Runge-Kutta fourth order integration scheme. The model validations were performed by comparing the experimental moisture content values and those obtained about the predicted model as well as between the predicted and the polynomial $f(N_S)$ function. The model validations were performed by comparing the experimental moisture content values and those obtained about the predicted model as well as between the predicted and the polynomial $f(N_S)$ function. Thus, two-way ANOVA method was applied to perform each parameter statistical significance and the residual analysis for the model fitting [22-24].

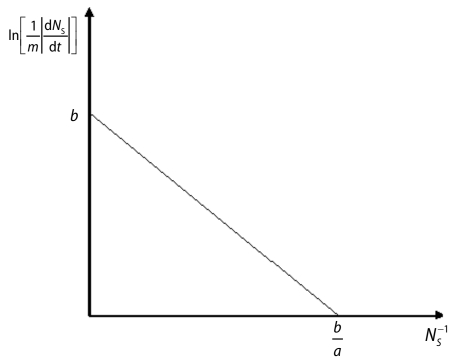

Figure 1. The $\ln[f(N_\mathrm{S})]$ profile *vs.* $1/N_\mathrm{S}$

Uncertainties calculation and residual analysis

The committed errors to the parameters assessment were already included into the least square method [21]. But the relative uncertainties on some parameters such as b could be analytically calculated. Indeed, as $b = \ln\alpha$ then the uncertainty to the measure of b was shown:

$$\Delta b = \frac{\Delta\alpha}{\alpha} \tag{6}$$

The uncertainty into the determination of $f(N_\mathrm{S})$ function was given by minimizing the mean quadratic errors on the parameters estimation based in the least square method [20]. Propagation of committed relative error to the drying rate was written [25]:

$$\frac{\Delta V_\mathrm{S}}{V_\mathrm{S}} = \frac{\Delta m}{m} + \frac{\Delta f(N_S)}{f(N_S)} \tag{7}$$

where $V_S = |\mathrm{d}N_\mathrm{S}/\mathrm{d}t|$ was the drying rate absolute value.

The Δm has been already determined in [18].

The theory of two-way ANOVA was developed in the literatures [26, 27] and goodness of fit for the reduced chi-square, χ^2_reduced, [28], coefficient of determination, R^2, [29], root mean square error (*RMSE*) [23, 30], mean bias error (*MBE*) [21], and mean absolute percentage error (*MAPE*) [31, 32] were computed across the following eqs. (8)-(12). Indeed, moisture ratio (*MR*) at time *t* was simplified as: $MR = N_S / N_{S0}$, [8, 18, 31], where N_{S0} was the product initial moisture content. Then, the statistical parameters for the model fitting were written:

$$\chi^2_{\text{reduced}} = \frac{\sum_{i=1}^{N}\left(MR_{\text{ei}} - MR_{\text{pi}}\right)^2}{N - z} \tag{8}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{N}\left(MR_{\text{ei}} - MR_{\text{pi}}\right)^2}{\sum_{i=1}^{N}\left(MR_{\text{ei}} - \overline{MR}_{\text{pi}}\right)^2} \tag{9}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}\left(MR_{\text{ei}} - MR_{\text{pi}}\right)^2}{N}} \tag{10}$$

$$MBE = \frac{1}{N}\sum_{i=1}^{N}\left(MR_{\text{ei}} - MR_{\text{pi}}\right) \tag{11}$$

$$MAPE = \frac{1}{N}\sum_{i=1}^{N} 100\left|\frac{y_{\text{i}} - f_{\text{i}}}{y_{\text{i}}}\right| \tag{12}$$

where MR_{ei} and MR_{pi} are, respectively, the i^{th} experimental and predicted dimensionless moisture ratios, but $\overline{MR}_{\text{pi}}$ was the i^{th} mean of predicted dimensionless moisture ratio at time *t*. The y_{i} and f_{i} were the i^{th} experimental and predicted model values at time *t*. The sum square error (*SSE*) was obtained by the results of ANOVA method. Moreover, eqs. (8)-(11) could be extended to compute another model fittings as the $f(N_S)$ function and drying rate by changing *MR* to the corresponding dimensionless ratios. Goodness of fit and trueness criteria of model was obtained like the highest values of R^2 and the lowest values for χ^2_{reduced}, *RMSE, MBE,* and *MAPE* [8, 23, 33].

Hypothesis statistic test

The modeling formulation was often attached some assumed hypotheses [22] in which null hypothesis, H_0, *vs.* alternative hypothesis, H_1, have been to test. Indeed, null hypothesis, H_0, to test is that samples have the same mean (no significance effect of parameters) *vs.* the alternative hypothesis, H_1, as follows, one sample mean was at least different of another sample means (significance effect of parameters). The statistic test was the F-test of Fisher-Snedecor. Main effects of model parameters were determined through the calculated ratios values $F_{\text{obs.}}$ (as F_1 and F_2). Threshold values of F-statistic have given by Fisher-Snedecor table with the corresponding degree of freedom at the critical significance likelihood p-value of 0.05 [26], as chance to reject the null hypothesis, H_0.

Furthermore, the null hypothesis, H'_0, for interaction parameters is that the parameters are independent. Moreover, reduced chi-square test was likely performed to test normality and homoscedasticity [22, 34]. All tests were performed to the one sided test.

Experimental conditions

The pozzolana granular arose a round shape at porous structure. They had a gray color and presented a little rough surface. Pozzolana physical characteristics at dry state, elementary chemical composition, mineralogical, and granulometric analyses were found [18]. The pozzolana sample was come directly from Ravine Blanche career to Saint Pierre (La Réunion Island-France) without undergoing previous treatments. Drying process was discontinuous and performed in thin-layer [18]. The sample was locked into the flexible metallic grid with very fine mesh of 80 μm of diameter, basis surface of 16×18 cm^2 and height of e (sample thickness). The air flux was ascending during all the operation. For every measure, sample thickness was ranged from 1 to 2 cm. This value was big enough compared to the average diameter of pozzolana particles that was of 0.13 mm. Furthermore, drying air velocity measured by the wind gauge was of 2 m/s that was likewise the common value for all measures. The initial moisture content value was calculated from the dry matter mass contained into the pozzolana sample and that value was of 0.156 kg/kg, dry basis.

Results and discussion

Variations of pozzolana moisture content via experimental measures

The experimental values for the pozzolana moisture content at different temperatures allowed to describe the moisture content variations versus the drying time as shown in the fig. 2.

In fact, pozzolana moisture contents were decreased and had tended toward zero at the prolonged drying period. Besides, reduced chi-square test was assumed to test the normality into the experimental values [28, 32]. The following results were obtained by ANOVA analysis applied onto the data set for experimental values of pozzolana moisture content. In degree of freedom of 87, chi-sqaure was 87 and reduced chi-sqaure was of 1. Obtained results allowed to conclude that all samples are coming from a normal distribution that has the same variance [35].

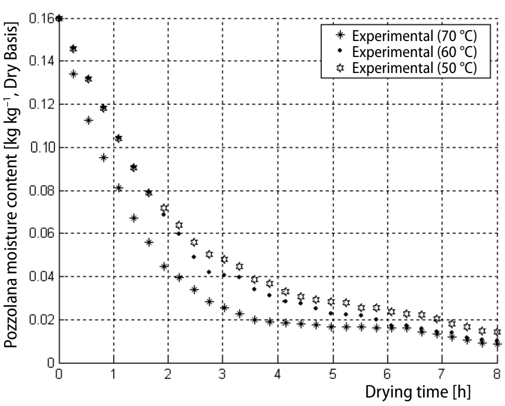

Figure 2. Variations of experimental pozzolana moisture contents on the time

Table 1. Effects of temperature and drying time in the pozzolana moisture content

Source of variations	Degree of freedom	Sum squared	Mean squared	Root mean square	$F_{obs.}$	$F_{0.95,\,(n1,n2)}$
Within group (temperature effect)	2	SSF_1 = 0.0033	MS_1 = 0.0017		F_1 = 75.3811	$F_{0.95\,,(2\,;58)}$ = 3.16
Between groups (drying time effect)	29	SSF_2 = 0.1487	MS_2 = 0.0051		F_2 = 233.4420	$F_{0.95\,,(29\,;58)}$ = 1.67
Residuals (errors)	58	*SSE, SSR* = 0.0013	*MSE* = 2.1966e–005	*RMSE* = 0.0047	F_{12} = 1	$F_{0.95,(58\,;58)}$ = 1.55
Total	89	*TSS* = 0.1533				

Two-way ANOVA analysis was employed to determine the effects of temperature and drying time into the pozzolana moisture content. Corresponding results were shown in tab. 1.

Compared to the corresponding $F_{0.95,(2;58)}$ and $F_{0.95,(29;58)}$ threshold values obtained by Fisher-Snedecor table, F_1 and F_2 values have been greater than these threshold values. Then, null hypothesis, H_0, was rejected. Hence, temperature and drying time effects were statistically significant in the data set from the moisture content of pozzolana drying. In terms of interaction between the temperature and drying time, F_{12} calculated value is lower than the $F_{0.95,(58;58)}$ threshold value. In doing so, H'_0 is not rejected. The both parameters were significantly independents.

Parameters determination of f(N)

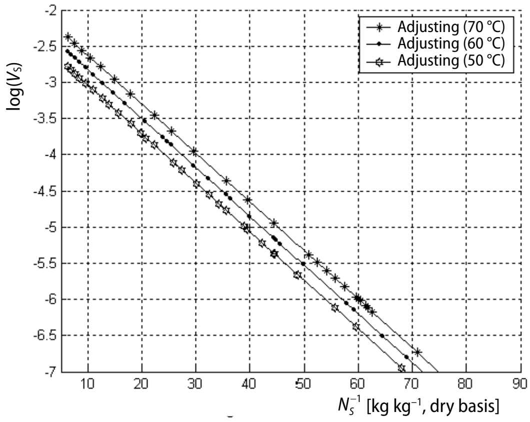

Figure 3. Variations of log(V_S) *vs.* inverse of moisture content

The profiles of log(V_S) *vs.* inverse of moisture content were shown on fig. 3.

About the calculation of V_S experimental values, these values were computed with the finite differences method by taking $\Delta t = 0.25\,h$.

All the variations of log(V_S) on the moisture content have occurred in straight lines. Then, parameters α, a and b for the $f(N_S)$ function could be determined by least square method of Newton-Marquart. The corresponding results were presented to the tab. 2. Reference thickness was of 1 cm.

An inspection on these results was led that the parameters were determined in acceptable reliability at 10^{-4}. The average values were, respectively: $a = (0.067 \pm 0.009)$ kg/kg, dry basis, $\alpha = (1.1 \pm 0.2)10^2$ kg/kgh, dry basis, and $b = 4.7 \pm 0.2$.

Table 2. Parameters values of $f(N_S)$ function at different temperatures

T [°C]	50	60	70
a [kgkg^{-1}, dry basis]	0.0765	0.0594	0.0663
Δa [kgkg^{-1}, dry basis]	0.0240	0.0224	0.0151
α [kgkg^{-1}h^{-1}, dry basis]	122.3028	87.2040	117.4596
$\Delta\alpha$ [kgkg^{-1}h^{-1}, dry basis]	34.7359	25.6014	21.7994
b [–]	4.8	4.5	4.8
Δb [–]	0.3	0.3	0.2
D (quadratic errors) [–]	9.0367e–004	12e–004	7.2968e–004

Hence, $f(N_S)$ function was expressed as shown in eq. (13):

$$f(N_S) = 1.110^2 \exp\left(-\frac{0.067}{N_S}\right) \tag{13}$$

Profiles of f(N_S) function and drying rate

At different temperatures, the profiles of $f(N_S)$ function and corresponding drying rate on the product moisture content were shown respectively to the figs. 4 and 5. Predicted $f(N_S)$

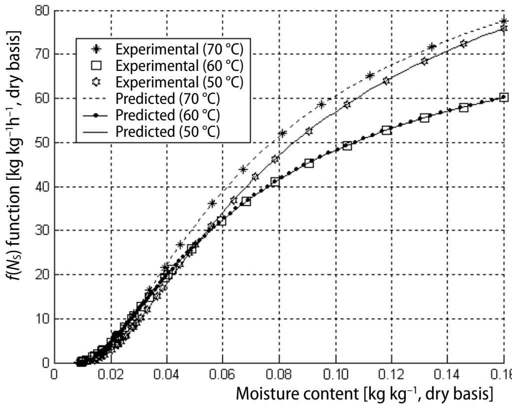

Figure 4. The $f(N_S)$ profiles from predicted and experimental values *vs.* pozzolana moisture content at different temperatures

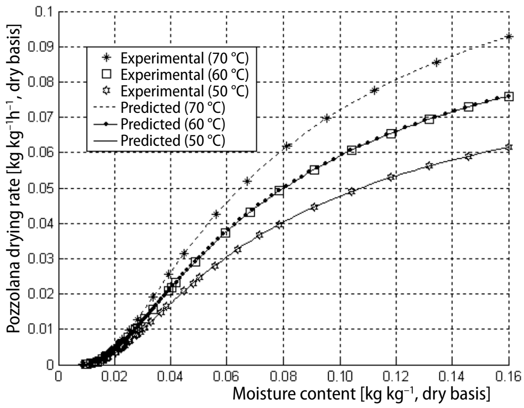

Figure 5. Variations of predicted and experimental values for the pozzolana drying rate with moisture content at different temperature

values were computed through eq. (13) by using the outcome moisture content values of Runge-Kutta fourth order integration scheme. As the drying rate profiles, all variations of $f(N_S)$ function were tended toward zero at lower values of moisture content and were, respectively, converged to the finite appropriate values at higher moisture content values. Then, model boundaries conditions were well validated. Moreover, the comparisons between predicted and experimental values were revealed the weak discrepancies. However, parameters statistical significances and residual analysis could be deducted from the tabs. 3-6. Indeed, Two-way ANOVA was performed for these results. According to the tab. 3, comparisons were likely to test the independence assumption among these $f(N_S)$ parameters through the obtained results. As these ratios F_1 and F_{12} corresponding to the temperature effect and their interaction with moisture content, these values are lower than those given by threshold values indicated to the Fisher-Snedecor table. Then, null hypothesis are not rejected.

Table 3. Two-way ANOVA table for parameters statistical significance on $f(N_S)$ variations

Source of variations	Degree of freedom	Sum squared	Mean squared	$F_{obs.}$	$F_{0.95,(n1,n2)}$
Within group (temperature effect)	2	$SSF_1 = 0.0094$	$MS_1 = 0.0047$	$F_1 = 1.7527$	$F_{0.95,(2;90)} = 3.10$
Between groups (moisture content effect)	29	$SSF_2 = 5.1687$	$MS_2 = 0.1782$	$F_2 = 66.3282$	$F_{0.95,(29;90)} = 1.60$
Interaction	58	$SSI = 0.1981$	$MSI = 0.0034$	$F_{12} = 1.2713$	$F_{0.95,(58;90)} = 1.49$
Residuals	90	$SSE = 0.2418$	$MSE = 0.0027$		
Total	179	$TSS = 5.6181$			

Therefore, temperature effect is not statistically significant about the $f(N_S)$ variations. Hence, hypothesis of independence between predicted model of $f(N_S)$ function and temperature was justified. Besides, moisture content and temperature had not interaction between them. Moisture content effect was performed into the comparison through the F_2 calculated value and those predicted by the Fisher Snedecor table at the 0.05 chance to reject the null hypothesis. Calculated value is greater than the F-statistic threshold value. As a result, pozzolana moisture content is statistically significant in the model.

Dependence between the moisture content and model of $f(N_S)$ function is revealed. Hence, the assumed departure hypothesis that moisture content has been the single explanatory variable into the model formulation of $f(N_S)$ function was justified by this dependence.

Table 4. Residual analysis of $f(N_S)$ function

$\chi^2_{reduced}$	*MBE*	*RMSE*	*MAPE* [%]	R^2
0.9939	2.1971e–004	0.0518	0.0478	0.9570

Residuals analyses were carried out across the tab. 4 to perform normality test, zero mean of errors and model verification. Reduced chi-square test value is very close to 1. Therefore, these $f(N_S)$ values belong to the gaussian normal distribution. In this fact, the distributions had the same within group variances and the homoscedasticity for $f(N_S)$ samples was verified. *MBE*, *RMSE,* and *MAPE* were extremely lower and the R^2 was very close to the unit. In doing so, the data set corresponding to the $f(N_S)$ has been well described by the predicted model. The mean bias error has had the very weak values. Then, the zero mean of errors hypothesis could be accepted to consider that the mean bias error was like the formulation residual. Parameters statistical significance corresponding to the drying rate variations were shown on the tab. 5.

Table 5. Two-way analysis of variances table for statistical significance of temperature and moisture content on the drying rate variations

Source of variations	Degree of freedom	Sum squared	Mean squared	$F_{obs.}$	$F_{0.95,\,(n1,n2)}$
Within group (temperature effect)	2	SSF_1 = 8.3691e–005	MS_1= 4.1845e–005	$F_1 = 1.09$	$F_{0.95,(2\,;90)} = 3.10$
Between groups (moisture content effect)	29	SSF_2= 0.0907	MS_2= 0.0031	F_2= 81.06	$F_{0.95,(29\,;90)} = 1.60$
Interaction	58	*SSI* = 0.0012	*MSI* = 2.1173e–005	$F_{12} = 0.55$	$F_{0.95,(58\,;90)} = 1.49$
Residuals	90	*SSE* = 0.0035	*MSE* = 3.8566e–005		
Total	179	*TSS* = 0.0954			

As seen in tab. 5, like these F_1 and F_{12} values are lower than threshold values of F-statistic given by Fisher-Snedecor table, then temperature effect is not statistically significant and interaction between moisture content and temperature was insignificant. As F_2 value was greater than F-statistic threshold value provided by Fisher-Snedecor table, moisture content effect was statistically significant. These results have been allowed to conclude that independence and non-interaction hypotheses between the temperature and the moisture content were verified to describe drying rate variations.

Moisture content influence in drying rate variations was well described by the new model of $f(N_S)$.

Corresponding results of residual analysis for drying speed variations were presented on the tab. 6.

About the tab. 6, *MBE*, *RMSE,* and *MAPE* have had very small values. Whereas, $\chi_{2reduced}$ and R^2 values were all close to the unit. Furthermore, zero mean of errors was exhibited as well as mean bias error had the weak value like that considered for residuals values. The predicted temporal variations of $f(N_S)$ function and drying rate with their corresponding experimental values were presented on the figs. 6 and 7.

Table 6. Residual analysis of drying rate variations

$\chi^2_{reduced}$	*MBE*	*RMSE*	*MAPE(%)*	R^2
0.8085	8.7160e–007	0.0062	0.0404	0.9636

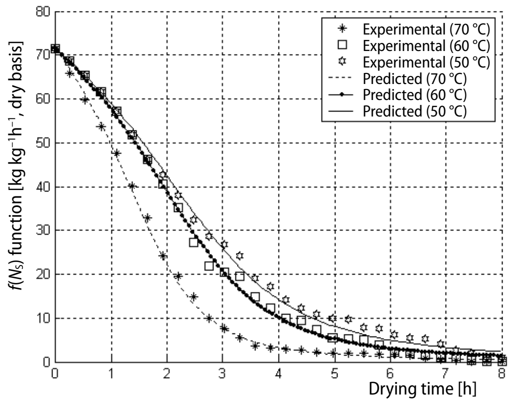

Figure 6. Predicted *vs.* experimental values for *f(N$_S$)* function with drying time

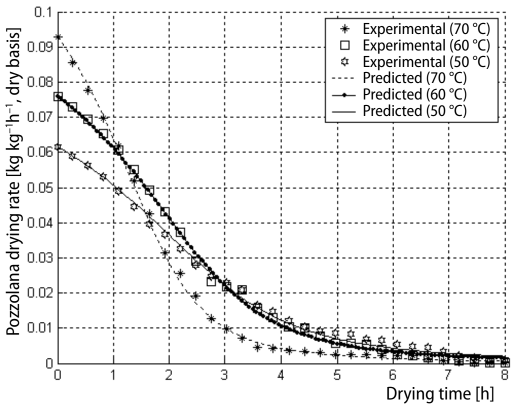

Figure 7. Variations of predicted and experimental drying rates values with drying time

As the $f(N_S)$ function and at different temperatures, drying rate was decreased and tended toward zero by increasing the drying time. The drying rate annulment was come from the absence of heat and mass transfers between the products and drying air so much as this last was not saturated.

Model validation and presentation of pozzolana drying rate

The log[$f(N_S)$] profiles obtained by the polynomial model and those obtained by the exponential model were presented on the fig. 8.

Comparison between the log[$f(N_S)$] values of polynomial model and those from the new model with $1/N_S$ allowed to conclude that the new model was kept always the very linearity at the wide range of moisture content values.

However, the polynomial model was presented some linearity for certain range values at determined moisture content. But for some values of this latter, divergence was occurred after passing through a minimum. This fact showed the limit by using the polynomial model for $f(N_S)$ function.

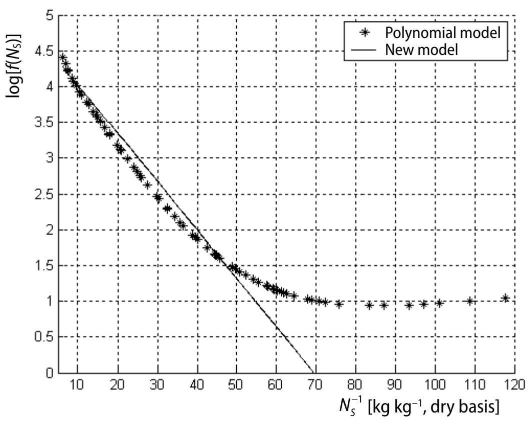

Figure 8. Comparison between the polynomial models and exponential of the function *f(N$_S$)*

The comparison through the profiles of $f(N_S)$ function from the polynomial model and those given by the exponential model with the pozzolana moisture content was shown on fig. 9. Model verification using the polynomial model in five order for $f(N_S)$ function was shown in eq. (14):

$$f(N_S) = b_0 + b_1 N_S + b_2 N_S{}^2 + b_3 N_S{}^3 + b_4 N_S{}^4 + b_5 N_S{}^5,$$
$$\text{with } b_0 = 6.9867,\ b_1 = -829.880,\ b_2 = 4.4830.10^4,\ b_3 = -5.7475\cdot 10^5,\ b_4 = 3.2164\cdot 10^6, \quad (14)$$
$$b_5 = -6.6149\cdot 10^6$$

where N_S is the moisture content of product. Indeed, the polynomial model had a minimum at the lower values and was diverged toward quasi-infinite for the higher values of moisture content. However, new model was converged toward zero for weak values of moisture content and was tended toward a finite value for very large higher values of this last. Moreover, Model validation was performed through the outcome values of residual analysis as presented in the tab. 7. Exponential model for $f(N_S)$ function was compared to the polynomial model.

According to the tab. 7, all corresponding values to the *MBE*, *MSE*, *RMSE,* and *MAPE* for the new model of $f(N_S)$ function were lower than those obtained to the polynomial model. But, new model R^2 and χ^2_{reduced} were greater than those given with this polynomial model.

Besides, these two models were shown a good agreement between them while the experiment conditions were remaining.

Therefore, exponential model of $f(N_S)$ function was the best model to describe moisture content influence into the pozzolana drying rate.

Table 7. New model validation with polynomial model and experimental values for $f(N_S)$

Model	χ^2_{reduced}	*MBE*	*MSE*	*RMSE*	*MAPE* [%]	R^2
Polynomial model	0.9513	0.0289	2.3458e-005	0.0048	5.4271	0.9902
Exponential model	0.9943	0.0053	9.7351e-006	0.0031	0.7235	0.9932
Between the two models	0.9771	0.0044	2.3697e-004	0.0154	0.5230	0.9533

On the fig. 10 was shown the model validation by comparing the experimental and predicted values of pozzolana moisture contents. The residuals analysis allowed to obtain the statistical parameters values on goodness of fit χ^2_{reduced}, *MBE, MSE, RMSE, MAPE,* and R^2 that were, respectively, of 2.5339e-04, 7.0316e-006, 3.2434e-006, 0.0018, 0.45%, and 0.9991. Indeed, high-test value of R^2 and lowest values of χ^2_{reduced}, *MBE, MSE, RMSE,* and *MAPE* were observed. Hence, model showed the good agreement with the moisture content experimental values.

The new exponential model of pozzolana drying rate was at last expressed by eq. (15):

$$\frac{\mathrm{d}N_S}{\mathrm{d}t} = -16.62.\exp\left(-\frac{0.067}{N_S} - \frac{2.28.10^3}{T_p} + \frac{1.89}{e}\right) \tag{15}$$

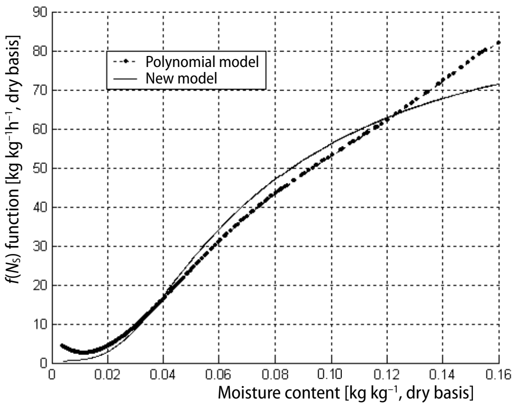

Figure 9. Polynomial *vs.* exponential forms of $f(N_S)$ function with moisture content

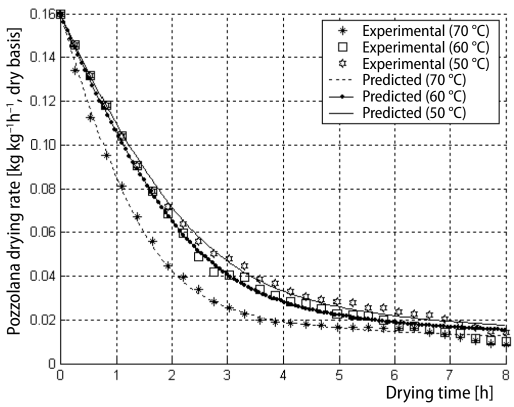

Figure 10. Predicted pozzolana moisture content *vs.* experimental values on the drying time

Conclusions

In this study, a new model of pozzolana drying rate were established at different temperatures. Normality and homoscedasticity hypotheses into the experimental data set and predicted values were verified by using the two-way ANOVA method. The independence hypothesis among the temperature and moisture content was justified. Obtained parameters values of a, α, and b to the exponential model for $f(N_S)$ function were, respectively, ranged from 0.0596 to 0.0765, 87.2040 to 122.3028, and 4.5 to 4.8. The $f(N_S)$ function was always converged for any moisture content values. Moreover, residual analysis of $f(N_S)$ exponential model were revealed a good agreement in which χ^2_{reduced}, *MBE, MSE, RMSE, MAPE,* and R^2 values were, respectively, equal to 0.9943, 0.0053, 0.0031, 0.72%, and 0.9932 by validation with polynomial model and experimental values. A comparison of moisture contents among experimental values and those obtained about the Runge-Kutta 4 method was likely presented the goodness of fit wherein corresponding value to the R_2 was of 0.9991, although χ^2_{reduced}, *RMSE* and *MAPE* were, respectively, of 0.0003, 0.0018, and 0.45%. The new model of pozzolana drying rate was shown the good agreement with the experimental data.

Nomenclature

a – exponential model coefficient, [kgkg^{-1}, dry basis]
b – exponential model parameter as (= log α), [−]
D – quadratic errors between model and experimental values [−].
dN_s/dt – product drying rate, [kgkg^{-1}h^{-1}, dry basis]
e – product or sample thickness, [cm]
$f(N_S)$ – function introducing the moisture content influence on the drying rate model, [kgkg^{-1}h^{-1}, dry basis]
m – coefficient introducing other parameters influences than the product moisture content, [−]
N – total number of data points, [−]
N_S – moisture content, [kgkg^{-1}, dry basis]
R^2 – coefficient of determination, [−]
T_p – superficial temperature of product, [K]
t – drying time, [h]
V – air-flow velocity, [ms^{-1}]
V_S – drying rate absolute value, [kgkg^{-1}h^{-1}, dry basis]
z – number of estimate parameters, [−]

Greek symbol

α – exponential model parameter, [kgkg^{-1}h^{-1}, dry basis]

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