PARTIAL SLIP AND THERMAL RADIATION EFFECTS ON HYDROMAGNETIC FLOW OVER AN EXPONENTIALLY STRETCHING SURFACE WITH SUCTION OR BLOWING

by

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This paper is devoted to analyze computational simulation to study the partial slip and thermal radiation effects on the flow of a viscous incompressible electrically conducting fluid through an exponentially stretching surface with suction or blowing in presence of magnetic field. Using suitable similarity variables, the non-linear boundary-layer PDE are converted to ODE and solved numerically by Runge-Kutta fourth order method in association with shooting technique. Effects of suction or blowing parameter, velocity slip parameter, magnetic parameter, thermal slip parameter, thermal radiation parameter, Prandtl number, and Eckert number are demonstrated graphically on velocity and temperature profiles while skin friction coefficient and surface heat transfer rate are presented numerically. Moreover, comparison of numerical results for non-magnetic case is made with previously published work under limiting cases.

Key words: *partial slip, thermal radiation, hydromagnetic flow, exponentially stretching surface, suction or blowing*

Introduction

The phenomenon of laminar flow and heat transfer of a viscous incompressible fluid driven by a linearly stretching surface has received great appreciation due to its applications in several technological processes. These applications involve paper production, hot rolling, annealing of Cu wires and glass blowing. It is also important in geothermal areas because the shallow surface layers are being stretched with a small velocity. It is worth mentioning that the hydromagnetic flows over a moving surface have been extensively studied in the past few decades, because of its increasing applications in various manufacturing processes, such as the enhanced recovery of petroleum resources, spinning of metals and extrusion of plastic sheets. In all of this engineering processes, to get the desired thickness the mixture issued from a slit is subsequently stretched. Crane [1] was the first to study an analytical solution of the steady 2-D flow over linearly stretching surface in a quiescent incompressible fluid. Later, many researchers such as Chakrabarti and Gupta [2], Carragher and Crane [3], Kumaran and Ramanaiah [4], Ishak *et al.* [5] Liu and Andersson [6], Jat and Chaudhary [7], Sahoo and Do [8], Mahapatra *et al.* [9] and Makinde *et al.* [10] have presented various aspects of linear stretching surface problem for non-magnetic and magnetic cases.

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From practical point of view, a continuous surface stretched with a linear velocity is not appropriate for the problem of filaments from a die and continuous extrusion of polymer sheet. Therefore, the stretching velocity is expected to be non-linear. The boundary-layer flow over exponential stretching surface under different situations were studied by Elbashbeshy [11], Parhta *et al.* [12], Sanjayanand and Khan [13], Sajid and Hayat [14], Bidin and Nazar [15], Nadeem *et al.* [16], Mukhopadhyay and Gorla [17], and Raju *et al.* [18].

The central tenet of the boundary-layer problems are no-slip boundary conditions. In this case the fluid velocity is zero at the surface. But in the existence of slip flow the fluid velocity is non-zero at the solid-fluid interface. In various technological processes, the assumption of no-slip is not applicable and must be substituted by partial slip boundary conditions. Such flow situations are encountered in a wide variety of industrial processes like foams and polymer solutions, polishing of artificial heart valves and internal cavities, and emulsion suspensions. Pursuing the pioneering studies of Hasimoto [19], the flow with partial slip boundary condition has been investigated by Wang [20], Ariel [21], Hron *et al.* [22], and Fang *et al.* [23]. Recently, Sajid *et al.* [24] and Das [25] studied about flow and heat transfer with different conditions and slip effects.

Forecasting of heat transfer characteristics of viscous incompressible flow with suction or blowing is very important in engineering and physics namely thermal oil recovery, design of radial diffusers and thrust bearings, prevent corrosion or scaling, reducing the drag and transition to turbulence. In chemical processes, suction can be used to remove reactants while to add reactants, blowing is used. The low energy fluid from the system is removed by suction, whereas blowing reduces the wall shear stress and hence the frictions drag. The boundary-layer flow with suction or blowing was first presented by Gupta and Gupta [26]. Further, Chen and Char [27], Ali [28], Seddeek [29], Pantokratoras [30], and Cortell [31] studied the various aspects of the flow problems with suction or blowing.

Meanwhile, in most of the investigations, the thermal radiation effects on the flow and heat transfer have not been taken into the account. Boundary-layer flow and heat transfer with radiation have a great importance in high temperature processes and space technology. It also plays an important role in many applications in engineering areas which occur at high temperature, like various propulsion devices or aircraft, design of reliable equipment, high temperature plasmas, liquid metal fluids, gas turbines, satellites, missiles, and space vehicles. When the difference between the surface temperature and the ambient temperature is very large then thermal radiation effects become more important besides the convective heat transfer. The radiative heat flux is described by using the Rosseland approximations in the energy equations. The thermal radiation effects on the flow with and without a magnetic field with several cases were presented by Bestman and Adjepong [32], Naroua *et al.* [33], Ouaf [34], Makinde and Ogulu [35], Pal and Mondal [36], and Jat and Chaudhary [37]. Most recently, Elbashbeshy and Emam [38], Khan *et al.* [39], Chaudhary *et al.* [40], and Sandeep *et al.* [41] analyzed the radiation effects over viscous incompressible and MHD flow.

Keeping the aforementioned literature in view, and inspired by the research paper of Mukhopadhyay and Gorla [17], the main motive of this article is to describe the partial slip effects as well as the effects of thermal radiation on hydromagnetic fluid over an exponentially stretching surface with suction or blowing. The present study of the boundary-layer flow will be highly beneficial in various engineering and technological processes such as MHD flight, foodstuff processing, MHD-power generators and in the field of planetary magnetosphere. It is hoped that the current work will be extensively used over previous contents.

Formulation of the problem

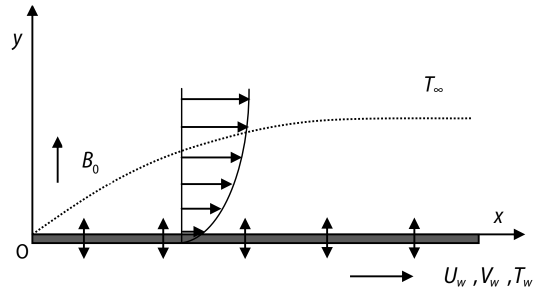

Figure 1. Schematic diagram of the problem

Figure 1 describes the geometrical structure of 2-D flow of a viscous incompressible electrically conducting fluid past an exponentially stretching sheet with thermal radiation and partial slip boundary conditions. In order to study the considered problem, the following assumptions are made:

– The exponentially stretching sheet is placed along the x-axis with the slot as the origin and is stretched along both ends of the sheet with the velocity $U_w(x) = U_0 e^{x/L}$ where U_0 is the reference velocity, x – the co-ordinate measured along the exponentially stretching sheet, and L – the reference length.
– The flow is confined in half plane $y > 0$ and velocity components are u and v in the directions of x- and y-axes, respectively.
– A uniform magnetic field of strength, B_0, is assumed to be applied normal to the stretching surface.
– The magnetic Reynolds number is taken very small than unity so the induced magnetic field is negligible in comparison with the applied magnetic field.
– Surface temperature along the exponentially stretching sheet is $T_w(x) = T_\infty + T_0 e^{x/2L}$ where T_∞ is the free stream temperature and T_0 – the reference temperature.
– The time independent suction or blowing at the surface is also considered.
– All the fluid properties are constant throughout the motion.

Under the aforementioned assumptions, the governing boundary-layer equations are given:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_e B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma_e B_0^2}{\rho C_p}u^2 \tag{3}$$

with the appropriate boundary conditions:

$$\begin{aligned} y = 0 \quad &: \quad u = U_w + N(x)\nu\frac{\partial u}{\partial y}, \quad v = -V_w(x), \quad T = T_w + D(x)\frac{\partial T}{\partial y} \\ y \to \infty \quad &: \quad u \to 0, \quad T \to T_\infty \end{aligned} \tag{4}$$

where y is the co-ordinate measured along normal to the exponentially stretching sheet, $\nu = \mu/\rho$ – the kinematic viscosity, μ – the coefficient of fluid viscosity, ρ – the fluid density, σ_e – the electrical conductivity, T – the temperature of the fluid, κ – the thermal conductivity, C_p – the specific heat at constant pressure, q_r – the radiative heat flux, $N(x) = N_1 e^{-x/2L}$ – the velocity slip factor, N_1 – the initial value of velocity slip factor, $V_w(x) = V_0 e^{x/2L}$ – the velocity of suction and blowing at the surface when $V_w(x) > 0$ and $V_w(x) < 0$, respectively, V_0 – the initial strength of suction, $D(x) = D_1 \mathrm{e}^{-x/2L}$ – the thermal slip factor, and D_1 – the initial value of thermal slip factor.

The radiative heat flux, q_r, can be written by using the Rosseland approximation for radiation [42]:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the absorption coefficient, respectively.

Considering that the differences of temperature within the flow is such that the term T^4 can be expanded in a Taylor series about T_∞ and neglecting higher-order terms to yield:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

In view of eqs. (5) and (6), eq. (3) becomes:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3k^* \rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma_e B_0^2}{\rho C_p}u^2 \tag{7}$$

Analysis

In order to investigate the heat transfer on exponentially stretching surface the following dimensionless similarity variables [17] are introduced:

$$\psi(x,y) = \sqrt{2\nu L U_0}\,\mathrm{e}^{x/2L} f(\eta) \tag{8}$$

$$\eta = \sqrt{\frac{U_0}{2\nu L}}\,\mathrm{e}^{x/2L} y \tag{9}$$

$$T = T_\infty + T_0\,\mathrm{e}^{x/2L}\,\theta(\eta) \tag{10}$$

where $\psi(x,y)$ is the stream function defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ which automatically satisfy the continuity eq. (1), $f(\eta)$ – the dimensionless stream function, η – the similarity variable, and $\theta(\eta)$ – the dimensionless temperature. Finally the momentum and energy eqs. (2) and (7) subject to the boundary conditions (4), can be transformed to non-linear ODE:

$$f''' + ff'' - 2f'^2 - Mf' = 0 \tag{11}$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + \Pr\left[f\theta' - f'\theta + \mathrm{Ec}\left(f''^2 + Mf'^2\right)\right] = 0 \tag{12}$$

with the transformed boundary conditions:

$$\begin{aligned} &\eta = 0 \;\;: \quad f = S, \quad f' = 1 + \lambda f'', \quad \theta = 1 + \delta\theta' \\ &\eta \to \infty: \quad f' \to 0, \quad \theta \to 0 \end{aligned} \tag{13}$$

where the prime denotes differentiation with respect to η, $M = 2L\sigma_e B_o{}^2/\rho U_w$ – the magnetic parameter, $R = 4\sigma^* T_\infty{}^3/\kappa k^*$ – the thermal radiation parameter, $\Pr = \mu C_p/\kappa$ – the Prandtl number, $\mathrm{Ec} = U_w{}^2/C_p T_0 \mathrm{e}^{x/2L}$ – the Eckert number, $S = V_0(2L/\nu U_0)^{1/2}$ – the suction or blowing parameter, $\lambda = N_1(\nu U_0/2L)^{1/2}$ – the velocity slip parameter, and $\delta = D_1(U_0/2\nu L)^{1/2}$ – the thermal slip parameter.

Numerical solution

For computations of the eqs. (11) and (12) along with the boundary conditions (13), the Runge-Kutta fourth order method in the association with shooting technique is applied. First introducing the new set of dependent variables w_1, w_2, w_3, p_1 and p_2, eqs. (11) and (12) with the boundary condition (13) are converted into the following simultaneous linear differential equations of first order:

$$w_1' = w_2 \tag{14}$$

$$w_2' = w_3 \tag{15}$$

$$w_3' = -\left(w_1 w_3 - 2w_2^2 - M w_2\right) \tag{16}$$

and

$$p_1' = p_2 \tag{17}$$

$$p_2' = -\frac{3\,\mathrm{Pr}}{3+4R}\left[w_1 p_2 - w_2 p_1 + \mathrm{Ec}\left(w_3^2 + M w_2^2\right)\right] \tag{18}$$

with the boundary conditions:

$$\begin{aligned} &\eta = 0\ \ :\ w_1 = S,\ \ w_2 = 1+\lambda w_3,\ \ p_1 = 1+\delta p_2 \\ &\eta \to \infty :\ w_2 \to 0,\ \ p_1 \to 0 \end{aligned} \tag{19}$$

where $w_1 = f$, $w_2 = f'$, $w_3 = f''$, $p_1 = \theta$, and $p_2 = \theta'$.

To solve the eqs. (16) and (18) as an initial value problem, one requires a value for $w_3(0)$ and $p_2(0)$ but no such values are given at the boundary. Using shooting technique, the suitable estimated values for $w_3(0)$ and $p_2(0)$ are chosen randomly and the fourth order Runge-Kutta method is applied to obtain the solution. Comparing the calculated values for $w_2(0)$ and $p_1(0)$ for various values of different parameters at the far field boundary condition assuming $\eta \to \infty = 6$ with the given boundary conditions $w_2(6) \to 0$ and $p_1(6) \to 0$, the values of $w_3(0)$ and $p_2(0)$ are adjusted for a better approximation. As the criterion of convergence the step size is taken as $\Delta\eta = 0.001$ and accuracy of the six decimal places is considered.

Rate of shear stress and rate of heat transfer

The physical quantities of primary interest are the skin friction coefficient, C_f, and the local Nusselt number, Nu_x, which are defined by:

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U_w^2} \tag{20}$$

$$\mathrm{Nu}_x = \frac{x q_w}{\kappa\left(T_w - T_\infty\right)} \tag{21}$$

where $\tau_w = \mu(\partial u/\partial y)_{y=0}$ is the wall shear stress and $q_w = -\kappa(\partial T/\partial y)_{y=0}$ is the heat transfer from the sheet. In the present case, the eqs. (20) and (21) can be expressed in the following forms:

$$C_f = \frac{2}{\sqrt{\mathrm{Re}_x}} f''(0) \tag{22}$$

$$\mathrm{Nu}_x = -\sqrt{\mathrm{Re}_x}\,\theta'(0) \tag{23}$$

where $\mathrm{Re}_x = U_w x/\nu$ is the local Reynolds number. The rate of shear stress, $f''(0)$, and the rate of heat transfer, $\theta'(0)$, are proportional to the skin friction coefficient, C_f, and the Nusselt number, Nu_x, respectively.

Validation of the proposed method

In order to validate the numerical method which was proposed in the previous section, the results of heat transfer rate, $\theta'(0)$, for different values of the thermal radiation parameter, R, the Prandtl number and the Eckert number are compared in the absence of the suction or blowing parameter, S, the velocity slip parameter, λ, the magnetic parameter, M, and the thermal slip parameter, δ, with the earlier researchers like Bidin and Nazar [15], Nadeem *et al.* [16], and Mukhopadhyay and Gorla [17] in tab. 1. In this table, the comparison shows that the present results are very close to those researchers. It can also be claimed that the demonstrated results are reliable and efficient.

Table 1. Comparison of $-\theta'(0)$ for several values of R, Pr, and Ec with $S = \lambda = M = \delta = 0$ and $f''(0) = -1.2821307$

R	Pr	Ec	[15]	[16]	[17]	Present results
0.5	1	0.0	0.6765	0.680	0.6765	0.6859730
	2		1.0735	1.073	1.0734	1.0737274
	3		1.3807	1.381	1.3807	1.3805010
	1	0.2	0.6177			0.6270190
	2		0.9654			0.9655080
	3		1.2286			1.2282404
1.0	1	0.0	0.5315	0.534	0.5315	0.5527834
	2		0.8627	0.863	0.8626	0.8653065
	3		1.1214	1.121	1.1213	1.1214546
	1	0.2	0.4877			0.5094000
	2		0.7818			0.7843958
	3		1.0067			1.0066859

Discussion of the computed results

Figures 2-4 show the influence for the various values of the suction or blowing parameter, S, the velocity slip parameter, λ, and the magnetic parameter, M, on the velocity distribution, $f'(\eta)$, respectively, while the other parameters are constant. From these figures, it is observed that the velocity decreases with the increasing values of the suction or blowing

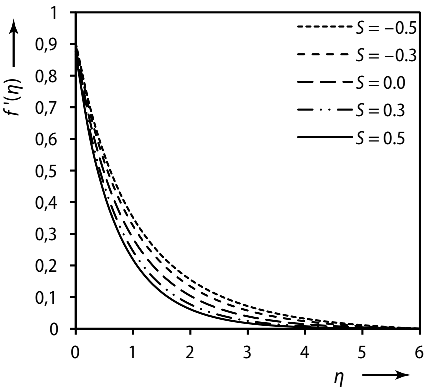

Figure 2. Influence of S on velocity against η for $\lambda = 0.1$ and $M = 0.1$.

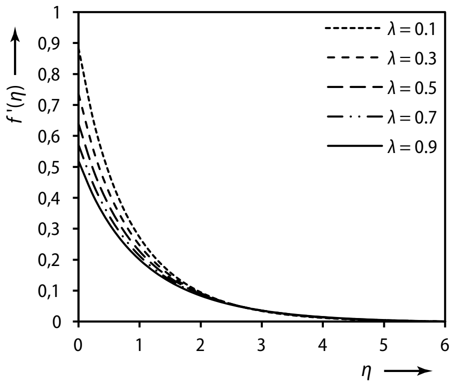

Figure 3. Influence of λ on velocity against η for $S = 0.1$ and $M = 0.1$.

parameter, the velocity slip parameter and the magnetic parameter while an opposite phenomenon occurs for the velocity slip parameter at $\eta > 2.7$ in fig. 3.

The behavior of the temperature profiles, $\theta(\eta)$, for several values of the suction or blowing parameter, the velocity slip parameter, the magnetic parameter, the thermal slip parameter, the thermal radiation parameter, the Prandtl number, and the Eckert number are presented in figs. 5-11, respectively, keeping other parameters constant. It is ascertained from these figures that the temperature decreases with the increasing values of the suction or blowing parameter, the thermal slip parameter, and the Prandtl number but the reverse is true for the velocity slip parameter, the magnetic parameter, the thermal radiation parameter, and the Eckert number. When a uniform magnetic field is applied normal to the flow direction, a force is produced which acts in negative direction of flow. This force is known as Lorentz force. The increasing values of the magnetic parameter make this force stronger, which ultimately slows down the fluid flow and accelerate the temperature.

Table 2 depicts the computations for the skin friction coefficient $f''(0)$ and the Nusselt number $\theta'(0)$ at the surface for different values of the suction or blowing parameter, the velocity slip parameter, the magnetic parameter, the thermal slip parameter, the thermal radiation parameter, the Prandtl number, and the Eckert number. From the table, it is obvious that the local

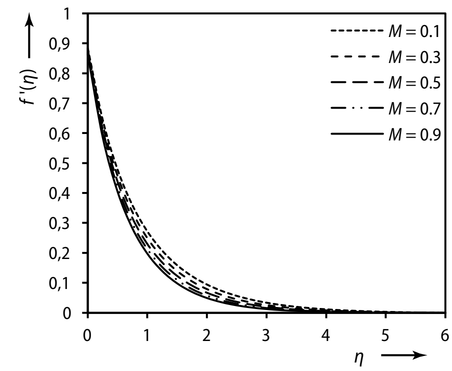

Figure 4. Influence of *M* on velocity against *η* for *S* = 0.1 and *λ* = 0.1

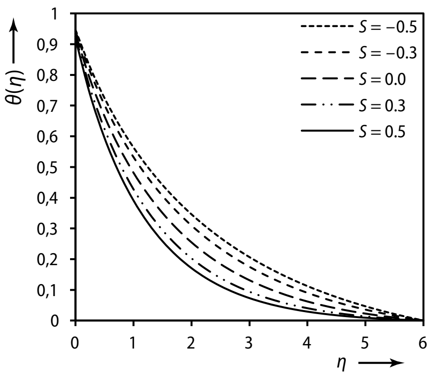

Figure 5. Influence of *S* on temperature against *η* for *λ* = 0.1, *M* = 0.1, *δ* = 0.1, *R* = 10, Pr = 10, and Ec = 0.01

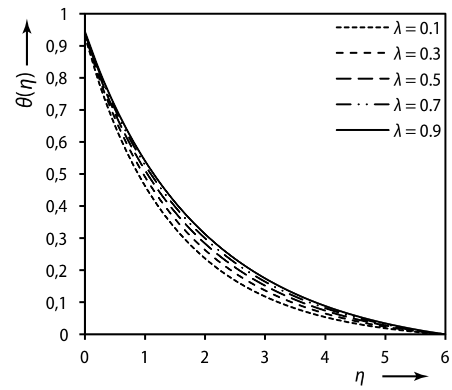

Figure 6. Influence of *λ* on temperature against *η* for *S* = 0.1, *M* = 0.1, *δ* = 0.1, *R* = 10, Pr = 10, and Ec = 0.01

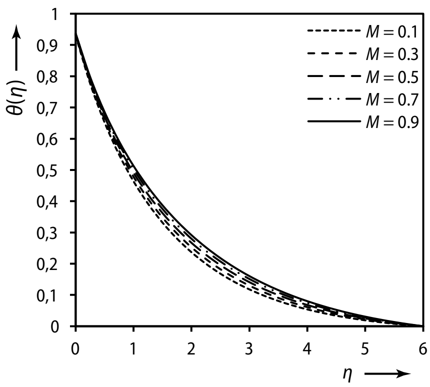

Figure 7. Influence of *M* on temperature against *η* for *S* = 0.1, *λ* = 0.1, *δ* = 0.1, *R* = 10, Pr = 10, and Ec = 0.01

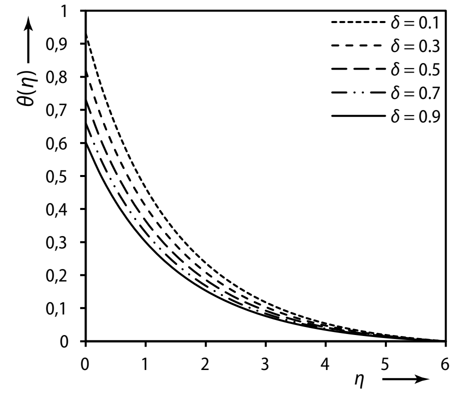

Figure 8. Influence of δ on temperature against η for $S = 0.1$, $\lambda = 0.1$, $M = 0.1$, $R = 10$, Pr = 10, and Ec = 0.01

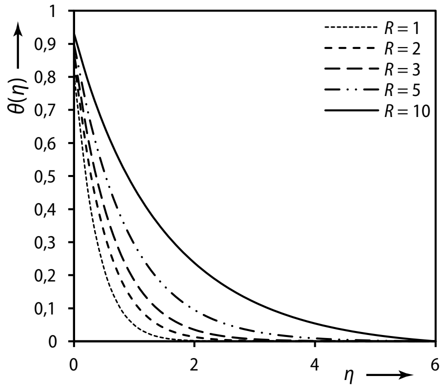

Figure 9. Influence of R on temperature against η for $S = 0.1$, $\lambda = 0.1$, $M = 0.1$, $\delta = 0.1$, Pr = 10, and Ec = 0.01

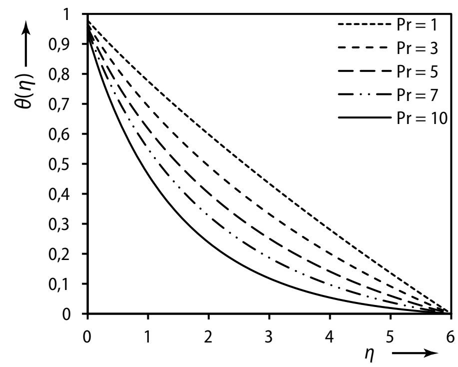

Figure 10. Influence of Prandtl number on temperature against η for $S = 0.1$, $\lambda = 0.1$, $M = 0.1$, $\delta = 0.1$, $R = 10$, and Ec = 0.01

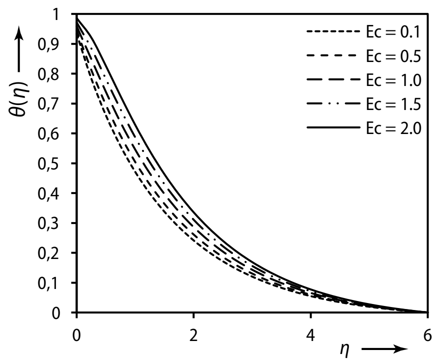

Figure 11. Influence of Eckert number on temperature against η for $S = 0.1$, $\lambda = 0.1$, $M = 0.1$, $\delta = 0.1$, $R = 10$, and Pr = 10

skin-friction coefficient, $f''(0)$, and the local Nusselt number, $\theta'(0)$, decrease with the increasing values of the suction or blowing parameter but a reverse phenomenon occurs for the velocity slip parameter taking other parameters constant, respectively. Further it may be seen that the skin friction coefficient decreases while the local Nusselt number increases with the increasing values of the magnetic parameter. Moreover, it is found that the Nusselt number increases with the increasing values of the thermal slip parameter, the thermal radiation parameter and the Eckert number while an opposite effect occurs for the Prandtl number, when other parameters kept constant, respectively. This table also shows that the skin friction coefficient and the local Nusselt number are always negative for all the values of physical parameters considered. Physically, the negative sign of skin friction coefficient implies that the fluid exerts a drag force from the surface and negative local Nusselt number means there is a heat flow from the surface.

Conclusions

The problem of hydromagnetic flow and heat transfer over an exponentially stretching surface is investigated with the radiation effects and partial slip boundary conditions. From

Table 2. Results of $f''(0)$ and $\theta'(0)$ for several values of S, λ, M, δ, R, Pr, and Ec

S	λ	M	δ	R	Pr	Ec	$-f''(0)$	$-\theta'(0)$
–0.5	0.1	0.1	0.1	10	10	0.01	0.9521657	0.529612
–0.3							1.0119855	0.575976
0.0							1.1112839	0.655864
0.3							1.2223470	0.748100
0.5							1.3027500	0.815985
0.1	0.1	0.1	0.1	10	10	0.01	1.1470008	0.685269
	0.3						0.8788629	0.633248
	0.5						0.7190887	0.596666
	0.7						0.6115220	0.568730
	0.9						0.5335684	0.546309
0.1	0.1	0.3	0.1	10	10	0.01	1.2057009	0.667330
		0.5					1.2604935	0.651047
		0.7					1.3119887	0.636170
		0.9					1.3606528	0.622508
0.1	0.1	0.1	0.3	10	10	0.01		0.602379
			0.5					0.537378
			0.7					0.485038
			0.9					0.441990
0.1	0.1	0.1	0.1	1	10	0.01		1.924327
				2				1.525495
				3				1.288670
				5				1.007521
0.1	0.1	0.1	0.1	10	1	0.01		0.223966
					3			0.339339
					5			0.447530
					7			0.548135
0.1	0.1	0.1	0.1	10	10	0.10		0.660506
						0.50		0.550453
						1.00		0.412886
						1.50		0.275318
						2.00		0.137750

the results of the problem, it can be concluded that the flow field, temperature profiles and the quantities of physical interest are significantly affected by these parameters.

- The velocity boundary-layer thickness decreases with the increasing values of the suction or blowing parameter, the velocity slip parameter and the magnetic parameter but a reverse behavior is noted being eta greater than 2.7 in case of the velocity slip parameter.
- Thermal boundary-layer thickness decreases with the increasing values of the suction or blowing parameter, the thermal slip parameter and the Prandtl number while it increases with the velocity slip parameter, the magnetic parameter, the thermal radiation parameter, and Eckert number.
- The wall shear stress decreases with the increasing values of the suction or blowing parameter and the magnetic parameter although an opposite phenomenon occurs for the velocity slip parameter.

- Finally the rate of heat transfer decreases with the increasing values of the suction or blowing parameter and the Prandtl number however it increases with an increment in the velocity slip parameter, the magnetic parameter, the thermal slip parameter, the thermal radiation parameter, and the Eckert number.

Nomenclature

B_o – uniform magnetic field, [$\mathrm{kgs^{-2}A^{-1}}$]
C_f – local skin friction coefficient ($= 2\tau_w/\rho U_w^2$), [–]
C_p – specific heat at constant pressure, [$\mathrm{Jkg^{-1}K^{-1}}$]
D – thermal slip factor, [–]
D_1 – initial thermal slip factor, [–]
Ec – Eckert number ($= U_w^2/C_p T_0 \mathrm{e}^{x/2L}$), [–]
f – dimensionless stream function, [–]
k^* – absorption coefficient, [$\mathrm{m^{-1}}$]
L – reference length, [m]
M – magnetic parameter ($= 2L\sigma_e B_o^2/\rho U_w$), [–]
N – velocity slip factor, [–]
N_1 – initial velocity slip factor, [–]
Nu_x – local Nusselt number, [$= xq_w/\kappa(T_w - T_\infty)$], [–]
Pr – Prandtl number ($= \mu C_p/\kappa$), [–]
q_r – radiative heat flux, [$\mathrm{kgm^{-2}}$]
q_w – heat transfer from the sheet, [$\mathrm{kgm^{-2}}$]
R – thermal radiation parameter ($= 4\sigma^* T_\infty^3/\kappa k^*$), [–]
Re_x – local Reynolds number ($= U_w x/\nu$), [–]
S – suction or blowing parameter [$= V_0(2L/\nu U_0)^{1/2}$], [–]
T – temperature of the fluid, [K]
T_0 – reference temperature, [K]
T_w – surface temperature, [K]
T_∞ – free stream temperature, [K]
U_0 – reference velocity, [$\mathrm{ms^{-1}}$]
U_w – surface velocity, [$\mathrm{ms^{-1}}$]
u, v – velocity component in the x- and y-direction, respectively, [$\mathrm{ms^{-1}}$]
V_0 – initial strength of suction and blowing, [$\mathrm{ms^{-1}}$]
V_w – surface velocity of suction and blowing, [$\mathrm{ms^{-1}}$]
x – along the exponentially stretching surface distance, [m]
y – normal distance, [m]

Greek symbols

δ – thermal slip parameter [$= D_1(U_0/2\nu L)^{1/2}$], [–]
η – similarity variable, [–]
θ – dimensionless temperature, [–]
κ – thermal conductivity, [$\mathrm{kgms^{-3}K^{-1}}$]
λ – velocity slip parameter [$= N_1(\nu U_0/2L)$], [–]
μ – coefficient of viscosity, [$\mathrm{Nsm^{-2}}$]
ν – kinematic viscosity ($= \mu/\rho$), [$\mathrm{m^2s^{-1}}$]
ρ – fluid density, [$\mathrm{kgm^{-3}}$]
σ_e – electrical conductivity, [$\mathrm{s^3A^2kg^{-1}m^{-3}}$]
σ^* – Stefan-Boltzmann constant, [$\mathrm{kgm^{-2}K^{-4}}$]
τ_w – wall shear stress, [$\mathrm{kgm^{-1}s^{-2}}$]
ψ – stream function, [$\mathrm{m^2s^{-1}}$]

Superscript

' – differentiation with respect to η

Subscripts

w – surface conditions
∞ – conditions for away from the surface

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