STUDY OF DECAY CHARACTERISTICS OF RECTANGULAR AND ELLIPTICAL SUPERSONIC JETS

by

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Experiments were carried out to study the decay characteristics of non-circular supersonic jets issued from rectangular and elliptical nozzles, and results were compared with circular case. Numerical simulations and Schlieren image study were carried out to validate the experimental results obtained from total pressure data. The supersonic core lengths of the jets were found to be different for different exit shaped geometries and area ratio for those nozzles was same. To avoid the losses in divergent section, the shape of cross-section of throat was maintained same that of as the exit. The exit shape geometry played the important role to enhance the mixing of the jet with ambient air, without requiring secondary method to increase the mixing characteristics. The mixing with ambient air in case of non-circular jets was more intense when compared to that of circular jets, which resulted in reduction of supersonic core length. The behavior of supersonic core length had identical signature for both under-expanded and over-expanded operation. The supersonic core length was characterized by exit shape factor. In literature, the supersonic jet characterization and the related experimental correlation are available for optimum expansion conditions whereas for other expansion (under and over) conditions the experimental correlations are barely available. While investigating experimentally, a new empirical relation was obtained which was the improved form of earlier correlations for supersonic core length. The current results obtained from three different methods (total pressure data, schlieren image and numerical simulation) had shown the reasonable agreement with the experimentally obtained relation.

Key words: *non-circular jet, exit-geometry, decay characteristics, jet bifurcation, geometric shape factor, supersonic core length*

Introduction

In last two decades, the high-speed gas-dynamics researchers have shown special interest to understand the supersonic free jet characteristics. Due to complex supersonic jet structure, most of the studies were limited to qualitative conclusion. A very few empirical relations are built for quantitative results. For certain high speed gas-dynamic applications require to reduce the supersonic core length. Earlier research scholars had used several techniques for controlling of supersonic core length. The supersonic core length was controlled by enhancing the mixing characteristics. The flow mixing augmentation was achieved by introducing the tab, wire, attaching chevron at the exit of the nozzle. Bradbury and Khadem [1] were the first to study the effect of mechanical tabs on subsonic jet flows.

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Eggers and Torrence [2] had carried out an investigation in a coaxial configuration on the mixing of parallel, circular, compressible-air with outer jet Mach number 1.3 and a near-sonic central jet. They have suggested the relationship to determine the supersonic core length by using the relation of exit density and velocity given in eq. (1):

$$\frac{x_l}{D} = 11.0\sqrt{\frac{p_j u_j}{p_e u_e}} \tag{1}$$

They had extrapolated and suggested the relation $x_l/D = 9.4$ for computation of supersonic core length of primary jet issuing from a circular shaped nozzle. By using the relation to current study the supersonic core length was obtained as $x_l = 110$ mm for circular shaped nozzle with optimum pressure expansion. The obtained result was coherent with current study of circular and optimum expansion supersonic jet. However, they did not suggest any relation or correction for non-circular supersonic jets and off-design pressure (under and over expansion) operation conditions.

Seubold and Shirie [3] attempted to correlate the results of their experimental study with the results of Anderson and Johns [4] who had plotted x_s/d_e against M_e and obtain good results. Where, x_s, d_e, and M_e *are* the supersonic core length, nozzle exit diameter, and Mach number, respectively. They attempted to incorporate additional data for suitable modifications. They have defined the supersonic core length as *the axial point farthest downstream at which there exists a flow Mach number of unity*. Further they had investigated by using additional data, that x_s/d_e at constant p_e/p_a was well described by an exponential function involving the ratio of specific heats (γ) and Mach number (M_e). They provided the relation to find supersonic core length as $(x_s/d_e)\mathrm{e}^{-11}\gamma_e M_e^2$, accompanied by a lookup table for hot and cold jet. However, in their discussions did not include the non-circular supersonic jet relation.

Pao and Abdol-Hamid [5] defined the potential core length as *the distance where the value of the centreline jet velocity has dropped to 0.99 times the jet exit velocity*. They suggested the equation to find the potential core length as function of Mach number, which was given by Lau *et al.* [6] in eq. (2).

$$\frac{L_c}{R} = 8.4 + 2.2M^2 \tag{2}$$

where R is jet exit radius or area equivalent radius.

Hileman and Samimy [7] were studying the noise source in supersonic jets issued from circular nozzles. They have investigated with three different Mach numbers (0.9, 1.3, and 2.9). The respective supersonic core length values were computed by using the relation $x/D = 7$, 8, and 11. The supersonic core length was interpolated linearly for current investigation, had yield the supersonic core length approximately $x/D = 9.6$. The obtained result was well in agreement with current investigation for circular shaped nozzle. However, the computed value for noncircular supersonic jets did not provide any satisfactory results.

Rathakrishnan and Shantanu [8] have studied experimentally the supersonic core decay characteristics of square jet. They have used the relation $x/D = 8$ to compute supersonic core length for uncontrolled jet with Mach number 1.86 and optimum pressure operation. The method of finding D was not available in the script. By using this relation in current study, assuming D was the equivalent hydraulic diameter, it produced supersonic core length $x = 84$ mm. The obtained supersonic core length issued from a square shaped nozzle had difference from the current results.

Murakami and Papamoschou [9] using Mach number 1.5 studied compressible flow development of co-axial and eccentric nozzles. They have studied the various flow characteristics including potential core length. They have found that the potential core length of single jet was $L_p/D_p = 9.2$. The result obtained from this relation for circular nozzle with optimum pressure expansion case was within the limit of agreeable acceptance.

Supersonic core is composed of shock waves, expansion waves, Mach disc, slip line, expansion and compression of shock cell, barrel shock as shown in fig.1. Axis switching phenomena in non-circular jet, the supersonic and subsonic region make the jet structure very complex to apply standard formulae. To study such heterogeneous jet structure both experimental and numerical simulations were carried out.

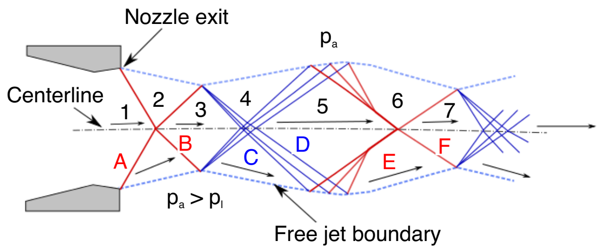


Figure 1. Supersonic jet structures

Investigation approach

The supersonic experimental set-up was comprises with total pressure reading facility and Schlieren set-up. The equipment set-up details are depicted in fig. 2. Pressure scanner (NetScanner 9116) used to carry out the total pressure data acquisition.

Nozzle geometric similarity

The inlet nozzle areas of three nozzles were designed with circular shape and same dimension. The throat and exit area geometry was maintained identical. For all nozzles the convergent length and divergent length was kept equal dimension. To obtain same exit Mach number the area ratio between nozzle exit to throat was kept same.

Nozzle exit geometric configuration factors

The nozzle exit geometry factors like hydraulic diameter (D_h) and shape factor (ζ) were governed by exit geometry shape. The aspect ratio of characteristic major to minor dimensions was 2:1 for rectangular and elliptical shaped nozzles. The nozzle exit geometry shapes are shown in fig. 3.

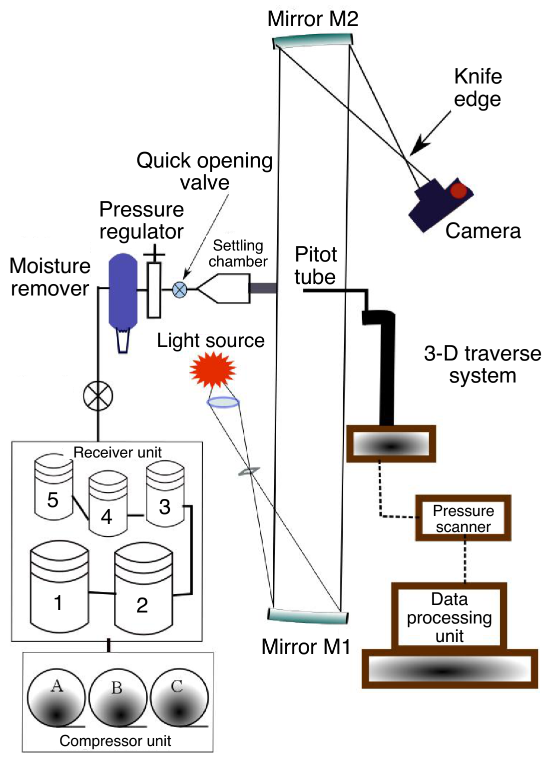


Figure 2. Experimental set-up

Two governing geometrical parameters are defined as below and the computed values are tabulated in tab. 1:

– the hydraulic diameter $D_h = \dfrac{4 \times \text{area}}{\text{wetted perimeter}}$

– shape factor $\xi = \dfrac{c_e}{c_{nc}} = \dfrac{\textit{Circular}\ \text{nozzle exit perimeter}}{\textit{Non-circular}\ \text{nozzle exit perimeter}}$

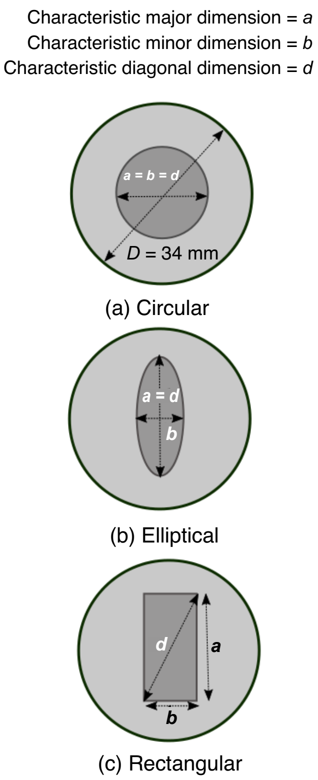


Figure 3. Nozzle dimension

Table 1. Nozzle exit shape, dimensions and geometric configuration parameters

Exit shape and dimensions, [mm]	Hydraulic diameter (D_h), [mm]	Shape factor [ζ]
Circular D = 11.73	$D_h = D = 11.73$	$\zeta = 1.0$
Elliptical Semi-major axis $a_1 = 8.3$ Semi-minor axis $b_1 = 4.15$	$D_h = \dfrac{2\sqrt{2}(a_1 b_1)}{(a_1^2 + b_1^2)^{1/2}} = 10.45$	$\zeta = 0.94$
Rectangular Width $a = 14.7$ Height $b = 7.35$	$D_h = \dfrac{2ab}{a+b} = 9.8$	$\zeta = 0.84$

Exit geometry shape

Figure 3(a) – circle, fig. 3(b) – ellipse, fig. 3(c) – rectangle, represent nozzle exit shape geometry. For circular nozzle the characteristic major, minor, and diagonal dimensions were equal in length and equivalent to exit characteristic diameter D. In elliptical exit shape, the characteristic major axis is the longest distance and equal to characteristic diagonal that runs through the center and both foci. The characteristic minor axis is the shortest diameter of an ellipse, which bisects the characteristics major dimension. In rectangular nozzle, the characteristic major axis is equal to width and characteristic minor axis is equal height of the rectangle. The characteristic diagonal is the distance between opposite angle. In fig. 3, exit geometry are depicted as a major axis, b minor axis, d diagonal, and R (17 mm) outer radius of nozzle. The nozzle lip perimeter (C_{lip}) was computed as $2\pi R$ = 106.81 mm.

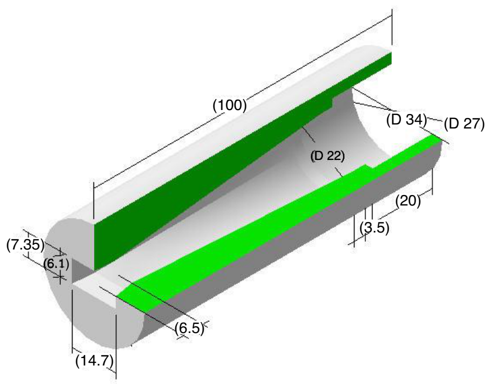


Figure 4 . Rectangular nozzle cut section

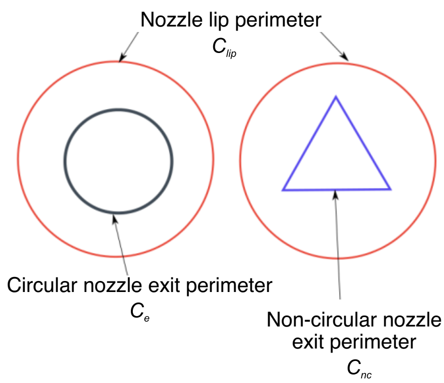


Figure 5. Typical exit geometry influencing parameters on decay characteristics

A typical cut section of rectangular nozzle is illustrated in fig. 4 which depicts the various nozzle parameters. Figure 5 describes the nozzle exit parameters, which were considered as influencing parameters to compute shape factor (ζ) and supersonic core length.

Investigation methodology

The decay characteristics of supersonic core length measured by total pressure data was validated by numerical simulation and Schlieren image. The data obtained from different methods were analyzed and the results were compared. During the supersonic core decay analysis an experimentally obtained correlation was built. The computed result from the obtained relation was coherent with experimental data, and Schlieren image.

Total pressure measurements

Total centerline pressure measurements were carried out by Pitot pressure probe and data acquisition was done by NetScanner system 9116 [10]. The 3-D traverse system was used to position the Pitot probe. The positioning of Pitot probe, *i. e.* both in linear and angular mounting accuracy were within the agreeable limit of acceptance. The total pressure was measured for over and under expansion cases.

Schlieren image analysis by using MATLAB

The Schlieren images were diagnosed by using MATLAB to obtain the supersonic core length. Mariani and Kontis [11] had used to compute the location of various jet parameters from Schlieren image. Techniques used by Mariani and Kontis were employed in this study to measure the supersonic core length. The pixel intensity of jet profile was measured along the jet centerline axis, *i. e.* parallel to x-axis. A typical pixel intensity *vs.* distance is shown in fig. 6. Initially the dimensions were measured in pixels and subsequently converted into physical length by using the nozzle outer diameter which establishes a pixel-to-millimeter relation. The lip diameter of nozzle was equal to 34 mm.

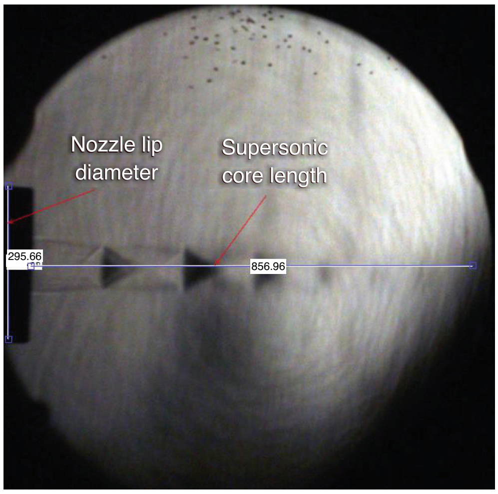


Figure 6. Typical Schlieren image

Schlieren image and numerical result comparison

The results obtained from Schlieren image were validated by numerical simulation. In this section, results obtained from optimum pressure expansion are illustrated in fig. 9. The over expansion and under expansion jet structure were compared between Schlieren and simulation results for circular and noncircular jets. The Schlieren image and density gradient data of numerical simulations were compared for jets issued from various nozzle shape and results were coherent.

Numerical simulation

The numerical simulations were executed for domain and grid independent tests. The mesh density was created along the centre line of jet, near nozzle exit and in the entrainment zone. The simulation domain and type of boundary conditions are shown in fig. 7. For numerical analysis turbulent model was used.

Postulate of generalized equation

The relations suggested by earlier researchers [1-9] are limited to optimum pressure expansion and jet issued from circular shaped nozzle. A very few analysis was carried out

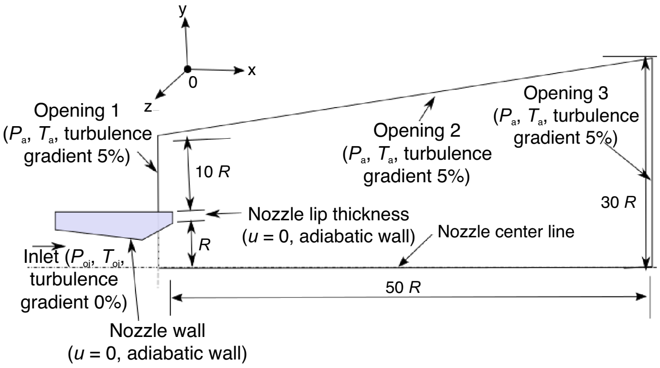


Figure 7. Computational domain and boundary conditions

for non-circular supersonic jets and valid for over and under expansion pressure ranges. Therefore, a less constrained relation was required for predicting supersonic core length with wide range of operating pressure, in terms of p_{oi} and p_a rather than optimum pressure operation (*i. e.* designed Mach number). As a result, the relation is applicable for under expansion and over expansion cases. The governing parameters like exit geometry, pressure expansion ratio, shape factors, and nozzle lip thickness could be taken into account, covering all expansion cases and exit geometric shapes.

The following influencing parameters were considered for developing an empirical relation:

– P_{oi}/P_a, pressure ratio of nozzle inlet pressure to ambient pressure,
– nozzle lip perimeter (C_{Lip}), $2\pi R$ = 106.81 mm,
– circular nozzle exit perimeter (C_e),
– non-circular nozzle exit perimeter (C_{nc}),
– characteristics diagonal – d,
– hydraulic diameter D_h, and
– shape factor $\xi = C_e / C_{nc}$.

With this influencing parameters, the supersonic core length was approximated as:

$$L_c = \sqrt{\frac{P_{oi}}{P_a}} (D_h - \frac{d}{2}) 2l \frac{C_{nc}}{C_{lip}} \frac{1}{\zeta} \tag{3}$$

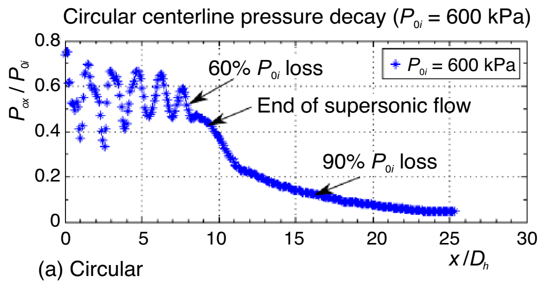
Results and discussion

The supersonic core length obtained from various methods was compared and the findings are discussed below.

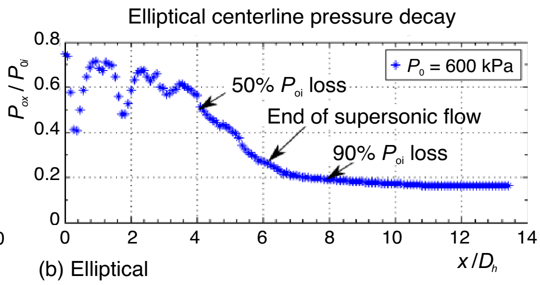
Supersonic core length obtained from total pressure data

The inlet nozzle pressure and exit geometry shape were the major characterizing parameter of supersonic core decay. In fig. 8, the total pressure decay patterns are illustrated. Similar behavior was observed during over and under expansion conditions. The drastic loss of 50% inlet pressure was observed within mid-field. The supersonic core length was approximated at the location from where total pressure drops continuously. The supersonic core length for various nozzles and inlet pressure are tabulated in tab. 3. Pitot tube measured the total pressure behind the normal shock. Apart from the total pressure decay pattern, the supersonic core lengths were obtained from the benchmark results of Schlieren image and eq. (3). The pressure oscillation was observed in the supersonic core region due to jet expansion and compression.

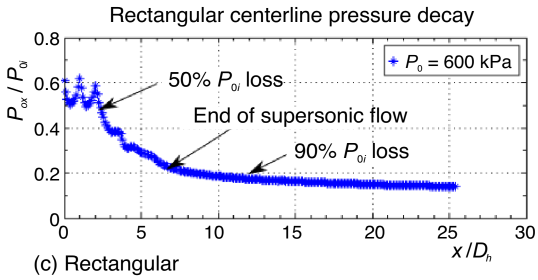
It was observed that jet produced from circular shaped nozzle had the longest supersonic core length compared to elliptical and rectangular. The supersonic core lengths were characterized by shape factor, exit geometry shaped.



(a) Circular

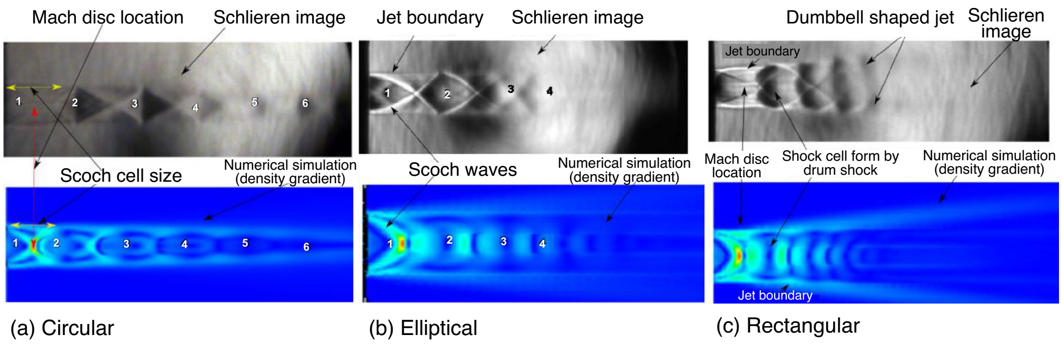


(b) Elliptical



(c) Rectangular

Figure 8. Decay characteristics of total pressure



(a) Circular (b) Elliptical (c) Rectangular

Figure 9. Schlieren image (major dimension) comparison with numerical simulation

Supersonic core length obtained from Schlieren image

The supersonic core lengths computed from Schlieren images and tabulated in tab. 3. The decay characteristics and shock cell structures obtained from Schlieren image and numerical simulations were coherent. It was noticed that jet issued from rectangular nozzle was bifurcated and formed the dumbbell shaped jet. Probably it was one of the reasons to produce shortest supersonic core length.

Table 2. Comparison of supersonic core decay summary

Researcher	Core length [mm]	Deviation from current study [%]
Rathakrishnan and Shantanu [7]	84	22.94%
Depazo and O'Brien [12]	104	5.5%
Seubold and Shirie [3]	100	8.26%
Eggers and Torrence [2]	110	–0.92%
Pao and Abdol-Hamid [5]	91.1	16.42%
Hileman and Samimy [7]	114	–4.59%

Supersonic core length obtained from experimentally developed correlation

The deviation of core length from current stufy is illustrated in fig. 10.

The current data have been compared with the data available in the literature and is tabulated in tab. 2. For comparison jet issued from circular nozzle and optimum pressure expansion were considered and tabulated in tab. 2.

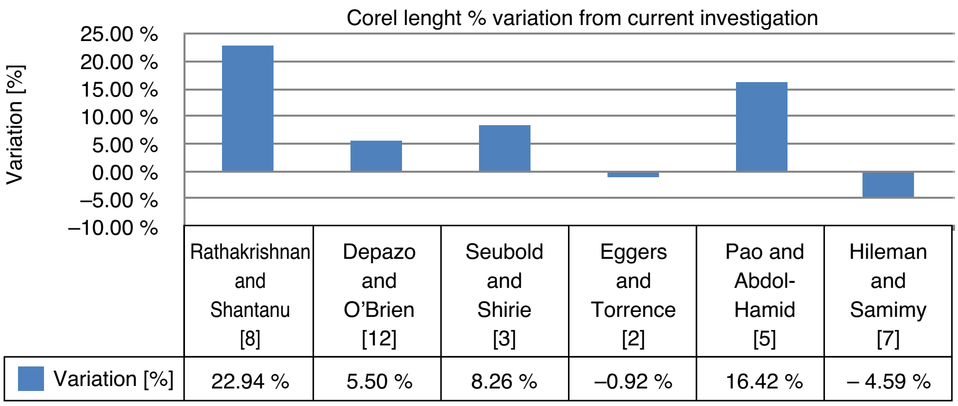


Figure 10. Core length comparison

The computed supersonic core length from various nozzles and inlet pressures are tabulated in tab. 3. The results obtained from other sources of investigations were very in the range of acceptable limit. The small mismatch with experimental data and eq. (3) are probably due to the mass entrainment and need further revision of eq. (3).

Table 3. Comparison of supersonic core length of various nozzles with different inlet pressure

Supersonic core length L_c [mm]	Nozzle geometry														
	Circular					Elliptical					Rectangular				
	400 [kPa]	500 [kPa]	600 [kPa]	700 [kPa]	800 [kPa]	400 [kPa]	500 [kPa]	600 [kPa]	700 [kPa]	800 [kPa]	400 [kPa]	500 [kPa]	600 [kPa]	700 [kPa]	800 [kPa]
obtained from p_{oi}	93	95	111	120	133	54	63	67	78	80	48	54	61	66	70
obtained from Schlieren image	92	98	109	121	132	52	63	70	78	80	49	54	62	67	71
obtained from eq. (3)	95	106	116	126	135	56	63	69	75	80	51	57	63	68	73

Error propagation and measuring uncertainty

The fixed error contributed by measuring devices, convergence criterion for numerical computation RMS residual value, Pitot tube and nozzle fabrication. For pressure data acquisition NetScanner is used from MEAS, USA has the measuring precision of ±0.05% [10]. The traverse mechanism used to position the Pitot probe has the capacity to move a minimum linear distance 0.025 mm and 0.25° angular accuracy. For schlieren image capturing, 20.1-megapixel CCD camera was used.

The uncertainty errors resulted during experiment were from traverse system was linear. These uncertainties were within the limit of acceptance criteria. Generally, errors creep into the data due to instrument limitation, systematic error, model mounting and set-up errors. Some time these systematic errors are treated as bias. In the current study, errors contributed by experimental setup were minimized by repeatability and those cancelled out during averaging. Averaging, errors and uncertainty computation methods were referred from University of Pennsylvania, USA, [13].

Uncertainties of pressure measurement

In the current study the source of uncertainties and errors are from:
– experimental set-up
– pressure scanner, and
– schlieren set-up.

Conclusions

Supersonic core length

Irrespective of exit geometry shape supersonic core length increased for under expansion case and decreased for over expansion case with respect to optimum expansion. In this investigation, it showed that circular shaped nozzle produced longest supersonic core and shortest in case of rectangular shaped nozzle. This decay pattern was found to be true for over-expanded, under-expanded and correctly expanded cases.

Experimentally obtained correlation for supersonic core length

$$L_c = \sqrt{\frac{P_{oi}}{P_a}}(D_h - \frac{d}{2})2l\frac{C_{nc}}{C_{lip}}\frac{1}{\zeta}$$

Decay characteristics

- Dumbbell shape supersonic core was observed in midfield of supersonic jet issuing from rectangular shaped nozzle causing longitudinal decay faster due to the rapid momentum flux change.
- For optimum expansion condition 90% of total inlet pressure loss in supersonic jet from a circular nozzle occurred at 200 mm and for rectangular case it was 108 mm from nozzle exit.
- For circular jet 50% of total inlet pressure was lost within the mid field and for the rest of the noncircular jets, 50% of total inlet pressure loss occurred in near field.

Nomenclature

A – area of the nozzle
a – major axis
b – minor axis
C_e – circular nozzle exit perimeter
C_{lip} – nozzle lip perimeter
C_{nc} – non-circular nozzle exit perimeter
D – exit diameter of circular nozzle
D_h – hydraulic diameter of the nozzle
d – diagonal of the nozzle exit geometry
L_c – the supersonic core length
l – characteristics diagonal equivalent
M – Mach number
P – pressure
p – outer radius of nozzle

Superscript and subscript notation

0 – total condition
1 – semi-major or semi-minor condition
a – atmospheric condition
e – exit condition

Greek symbol

ζ – shape factor

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