RELAMINARIZATION OF WALL TURBULENCE BY HIGH-PRESSURE RAMPS AT LOW REYNOLDS NUMBERS

by

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Reverse transition from the turbulent towards the laminar flow regime was investigated experimentally by progressively increasing the pressure up to 400 MPa in a fully developed pipe flow operated with silicone oil as the working fluid. Using hot-wire anemometry, it is shown indirectly that at low Reynolds numbers a rapid increase in pressure modifies the turbulence dynamics owing to the processes which induce the effects caused by fluid compressibility in the region very close to the wall. The experimental results confirm that under such circumstances, the traditional mechanism responsible for self-maintenance of turbulence in wall-bounded flows is altered in such a way as to lead towards a state in which turbulence cannot persist any longer.

Key words: *relaminarization, wall turbulence, high pressure ramp*

Introduction

For incompressible fluids, it can be shown that a Taylor series expansion near the wall ($x_2 = 0$) of the instantaneous velocity fluctuations, u_i, $i = 1, 2, 3$, which satisfy non-slip boundary conditions:

$$\left.\begin{aligned} u_1 &= a_1 x_2 + a_2 x_2^2 + \ldots \\ u_2 &= b_1 x_2 + b_2 x_2^2 + \ldots \\ u_3 &= c_1 x_2 + c_2 x_2^2 + \ldots \end{aligned}\right\} \quad \text{as} \quad x_2 \to 0 \tag{1}$$

leads to the following expressions for the non-vanishing components of the turbulent stresses, $\overline{u_i u_j}$, in simple wall-bounded turbulent flows:

$$\left.\begin{aligned} \overline{u_1^2} &= \overline{a_1^2} x_2^2 + \ldots \\ \overline{u_2^2} &= \overline{b_1^2} x_2^4 + \ldots \\ \overline{u_3^2} &= \overline{c_1^2} x_2^2 + \ldots \\ \overline{u_1 u_2} &= \overline{a_1 b_2} x_2^3 + \ldots \end{aligned}\right\} \quad \text{as} \quad x_2 \to 0 \tag{2}$$

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The asymptotic behaviour of $\overline{u_i u_j}$ implies that the velocity fluctuations inside the region of viscous sublayer $0 \le x_2^+ \le 5(x_2^+ = x_2 u_\tau / \nu)$ can be considered, to a fair degree of approximation, as two component since $b_1 = 0$. Here $u_\tau = (\tau_w/\rho)^{1/2}$ and $\tau_w = \mu(\partial \bar{U}_1/\partial x_2)_w$ are the friction velocity and the wall shear stress, respectively.

For compressible fluids, fluctuations in the fluid density, ρ', vary in a similar manner as the velocity field and therefore may be represented in a form equivalent to (1):

$$\rho' = d_0 + d_1 x_2 + d_2 x_2^2 + \dots, \qquad \text{as} \qquad x_2 \to 0 \tag{3}$$

Imposing the mass conservation on (1) with (3):

$$\left.\begin{array}{l} \dfrac{\partial d_0}{\partial t} + \overline{\rho_w} b_1 + d_0 b_1 - \overline{d_0 b_1} = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{array}\right\} \tag{4}$$

yields $b_1 \neq 0$ due to non-vanishing of the density fluctuations at the wall, d_0, where $\overline{\rho_w}$ denotes the mean density of the fluid evaluated at the wall. With this result, the asymptotic behaviour of the turbulent stresses $\overline{u_i u_j}$ becomes:

$$\left.\begin{array}{r} \overline{u_1^2} = \overline{a_1^2} x_2^2 + \dots \\ \overline{u_2^2} = \overline{b_1^2} x_2^2 + \dots \\ \overline{u_3^2} = \overline{c_1^2} x_2^2 + \dots \\ \overline{u_1 u_2} = \overline{a_1 b_1} x_2^2 + \dots \end{array}\right\} \qquad \text{as} \qquad x_2 \to 0 \tag{5}$$

and suggests that for compressible fluids turbulence in the viscous sublayer cannot be regarded as two component and that the compressibility effect is responsible for modification of the turbulence anisotropy.

To provide further understanding of the compressibility effects, it is logical to proceed by projecting the dynamics of turbulence across the functional space formed by two scalar invariants $\mathrm{II}_a = a_{ij}a_{ji}$ and $\mathrm{III}_a = a_{ij}a_{jk}a_{ki}$ of the anisotropy tensor $\mathbf{a}_{ij} = \overline{u_i u_j}/\overline{u_s u_s} - 1/3\delta_{ij}$. In contrast to the real space where observations usually take place, turbulence can appear in the invariant space, known as the anisotropy-invariant map, only within a bounded domain which is quite narrow [1]. Figure 1 shows trajectories of the joint variation of invariants, (II_a, III_a), for incompressible turbulent channel flow as representative of wall-bounded flows. In these flows, trajectories of joint variation of invariants lie along the two-component state, $\mathrm{II} = 2/9 + 2\mathrm{III}_a$, and very close to the axisymmetric state, $\mathrm{II}_a = 3/2(4/3|\mathrm{III}_a|)^{2/3}$. In the region of the viscous sublayer, the data follow the two-component state. Starting from the wall ($x_2^+ = 0$), the anisotropy first increases towards the right corner point of the map which characterizes one-component turbulence, (2/9, 2/3). At the edge of the viscous sublayer ($x_2^+ \simeq 8$), the invariants reach their maximum values and thereafter follow the right boundary of the map which characterizes axisymmetric turbulence. At the channel centreline, anisotropy almost vanishes, indicating the tendency towards isotropy, (0, 0).

If the compressibility effect precludes turbulence in the viscous sublayer from residing in the two-component state, the results in fig. 1 (top-left), which correspond to low Reynolds numbers, imply that turbulence will tend towards the axisymmetric state in order to stay within the anisotropy map and satisfy realizability. The tendency towards an axisymmetric

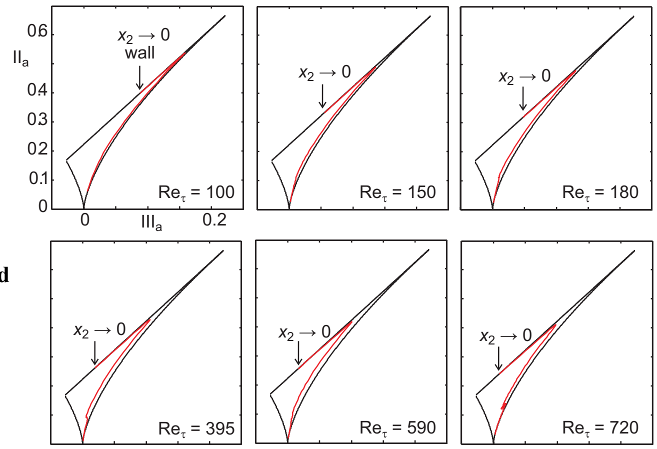

Figure 1. Anisotropy-invariant mapping of turbulence in an incompressible turbulent channel flow compiled from existing numerical databases $Re_\tau = 180$ [2]; $Re_\tau = 395$ [3]; $Re_\tau = 100, 150$ [4]; $Re_\tau = 590$ [5]; $Re_\tau = 720$ [6]} by [7]; data which correspond to low Reynolds numbers Re_τ (top-left), based on the channel half-width and the wall friction velocity, show that the sublayer region is constrained between two possible states: the two-component state and the axisymmetric state

state is followed by a continuous reduction in the turbulent shear stress $\overline{u_1 u_2}$ (since in axisymmetric turbulence $\overline{u_1 u_2} = 0$), and a subsequent decrease in the turbulent energy production, leading to a state in which turbulence cannot be self-sustained and must undergo reverse transition to the laminar state, [8] and [9].

We may support this fundamental deduction more systematically by examination of the equations for the mean flow in simple wall-bounded flows by demanding statistical axisymmetry in the turbulent stress tensor: it is straightforward to show that for such a stress configuration the equations transform from the unclosed to the closed form and lead to solutions which coincide with corresponding solutions for laminar flows. This holds for incompressible and compressible fluids with the difference, however, that in the latter case fluctuations around mass-weight mean flow have to be considered for the treatment of the terms involved in nonlinear turbulence interactions in the momentum and the continuity equations assuming that the molecular diffusion is not influenced by the viscosity fluctuations.

The present paper is therefore centred around the following fundamental question: Is it possible to influence the dynamics of wall turbulence by a sudden increase in pressure which induces the effects associated with compressibility in such a way as to force initially fully developed turbulence to return back to the stable laminar state at $Re > Re_{crit.}$ and in this way contribute towards a better understanding of the process of reverse transition studied extensively in [10].

Test section, fluid properties, and instrumentation

High-pressure test section

To perform detailed investigations of the effects induced by sudden pressure variations in a fully developed turbulent pipe flow, a special set-up was designed and built. Its major components are shown in fig. 2. It consisted of a high-pressure hydraulic pump, a container with a pressure-transmitting liquid, a hydraulic pressure intensifier for increasing the working pressure up to 14 times that produced by the pump, and a high-pressure vessel capable of withstanding pressures up to 500 MPa.

The high-pressure vessel was manufactured from high-quality DIN 1.4542 steel block.

Internally the vessel had a cylindrical form 0.080 m in diameter and 0.3 m in length, providing an effective volume of $1.5\cdot10^{-3}$ m^3. Figure 3 shows an exploded view, with the in-

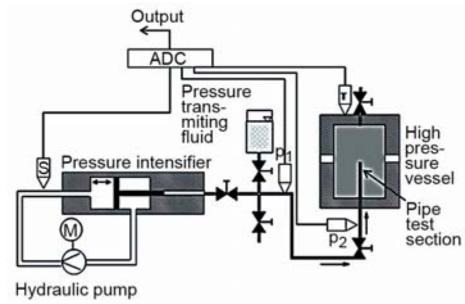

Figure 2. Experimental set-up for the investigation of reverse transition in pipe flow during rapid pressurization

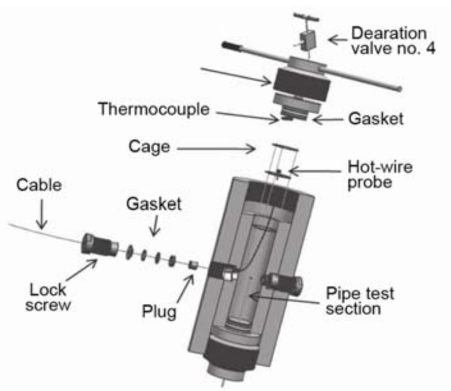

Figure 3. Exploded view of a high-pressure vessel and sealing rings capable of maintaining a maximum pressure up to 500 MPa

corporation of a large screw which closes the vessel entrance and the assembly of three round windows with an optical diameter of 0.012 m that allow optical access for potential application of laser-Doppler anemometry. The temperature of the working fluid was measured with three thermocouples located at different positions in the vessel. Pressure measurements were carried out with transducers which were located close to the vessel entrance. Temperature and pressure signals together with the piston position of the pressure intensifier were recorded by the computer-controlled data acquisition system for determination of the volume flow rate.

The vessel was pressurized by transmitting liquid through the centrally mounted pipe of $1.6 \cdot 10^{-3}$ m internal diameter, which extended 0.14 m into the vessel from the inlet. During the pressurization process, a fully developed flow was established in the pipe and maintained by the computer-controlled regulation system in such a way as to produce the desired increase in pressure with minimum disturbances produced by the regulation system itself.

Fluid properties

For preliminary experiments, glycerol was chosen as the pressure-transmitting fluid since it is a very low electrically conductive medium and therefore permits easy application of hot-wire techniques. Owing to the high viscosity of glycerol, $\nu = 1.19 \cdot 10^{-3}$ m^2/s at the working temperature of 298 K, only a laminar flow condition was possible, corresponding to very low Reynolds numbers, $\mathrm{Re} = U_\mathrm{B} d/\nu \leq 10$, based on the pipe diameter (d), the bulk velocity (U_B), and the fluid viscosity (ν).

For measurements in the turbulent regime, silicone oil was chosen as the working fluid. It is an electrically non-conductive medium and has a relatively low viscosity of $\nu = 0.65 \cdot 10^{-6}$ m^2/s at the working temperature of 298 K. Figure 4 shows variations of the physical properties of the silicone oil with pressure and temperature based on interpolation of the experimental data. Viscosity was measured with the use of the rolling ball viscosimeter according to the Hoppler viscosimeter, for details see [11]. Using silicone oil, it was possible to reach Reynolds numbers in the range $2.6 \cdot 10^3 < \mathrm{Re} < 10 \cdot 10^3$.

The Reynolds number was calculated by:

$$\mathrm{Re}(p,T) = \mathrm{Re}_0 \frac{\rho(p,T)}{\rho_0} \frac{\eta_0}{\eta(p,T)} \tag{6}$$

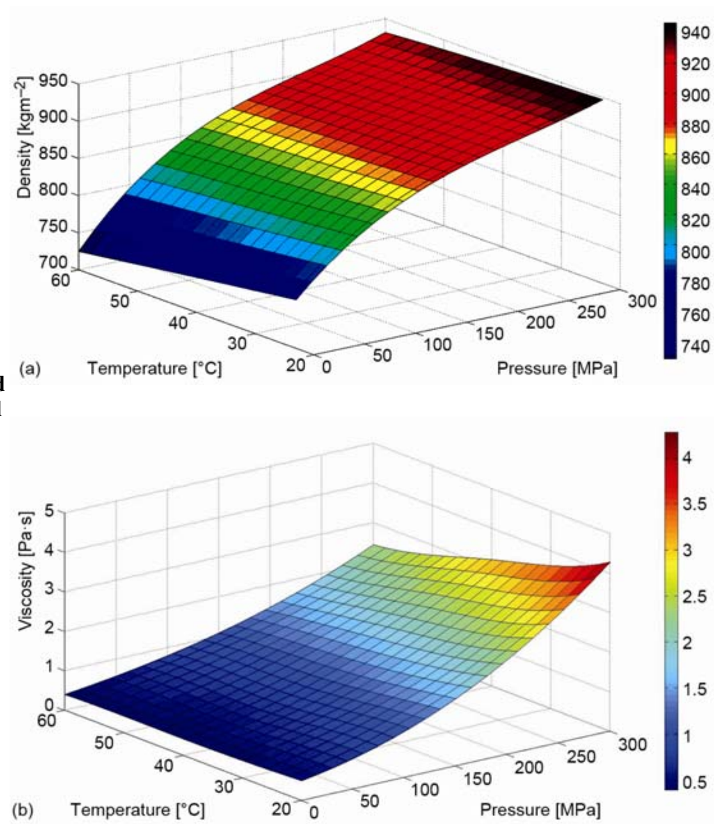

Figure 4. Variations of the physical properties of silicone oil with pressure and temperature: density (a) and viscosity (b)

where Re_0 is the initial Reynolds number calculated from the initial density ρ_0, dynamic viscosity η_0 and the bulk velocity U_B before pressurization. The value of the bulk velocity was determined from measurements of the instantaneous volume flow rate monitored by the data acquisition system. During rapid compression from 10 to 300 MPa, the temperature of the oil increases by about $\Delta T \approx 30$ K, inducing a modest increase in density of the order of 20%. The temperature increase is in agreement with expectation based on the assumption that the process is isentropic:

$$\frac{\partial T}{\partial p} = \frac{\beta T}{\rho c_p} \tag{7}$$

where β and c_p are thermal expansion coefficient and thermal capacity at constant pressure, respectively. However, the viscosity of the silicone oil in the same range of parameters increases nearly eightfold.

Analysis of the conservation equations of fluid flow employing normalization of all variables with characteristic scales of the process involved and proceeding with an order of magnitude estimate for each individual term in these equations leads to the conclusion that these equations are identical with those describing viscous incompressible flow with the exception of the continuity equation [12]. Owing to density variation of the fluid with pressure and temperature during the compression process, the continuity equation has to account for the compressibility effect on the working fluid:

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho U_i}{\partial x_i}=\frac{\mathrm{D}\rho}{\mathrm{D}t}+\rho\frac{\partial U_i}{\partial x_i}=0 \tag{8}$$

From this equation, it emerges that relative variations in density follow pressure and temperature variations according to the physical properties of the fluid shown in fig. 4:

$$\frac{1}{\rho}\frac{\mathrm{D}\rho}{\mathrm{D}t}=\underbrace{\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p}_{-\beta}\frac{\mathrm{D}T}{\mathrm{D}t}+\underbrace{\frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_T}_{\alpha}\frac{\mathrm{D}p}{\mathrm{D}t} \tag{9}$$

Introducing splitting of density, pressure, and temperature into the mean and fluctuating parts in eq. (9) and taking average of the resulting equation yields the following relations:

$$\frac{\mathrm{D}\overline{\rho}}{\mathrm{D}t}=-\beta\overline{\rho}\frac{\mathrm{D}\overline{T}}{\mathrm{D}t}+\alpha\overline{\rho}\frac{\mathrm{D}\overline{T}}{\mathrm{D}t}-\beta\overline{\rho'\frac{\mathrm{D}T'}{\mathrm{D}t}}+\alpha\overline{\rho'\frac{\mathrm{D}p'}{\mathrm{D}t}} \tag{10}$$

$$\frac{\mathrm{D}\rho'}{\mathrm{D}t}=-\beta\overline{\rho}\frac{\mathrm{D}T'}{\mathrm{D}t}+\alpha\overline{\rho}\frac{\mathrm{D}p'}{\mathrm{D}t}-\beta\rho'\frac{\mathrm{D}T'}{\mathrm{D}t}+\alpha\rho'\frac{\mathrm{D}\overline{p}}{\mathrm{D}t}+\alpha\rho'\frac{\mathrm{D}p'}{\mathrm{D}t}+\beta\overline{\rho'\frac{\mathrm{D}T'}{\mathrm{D}t}}-\alpha\overline{\rho'\frac{\mathrm{D}p'}{\mathrm{D}t}} \tag{11}$$

The first term on the right-hand side of eq. (11) indicates that density fluctuations exist and closely follow the temperature fluctuations. In view of the results presented in figs. 7-9, which show that the levels of the temperature and flow fluctuations are comparable in magnitude, we may conclude that inference previously mentioned obtained from eq. (11) justify the analytic considerations (1)-(5) outlined in *Introduction* if and only if the trajectory in the anisotropy map deviates marginally from the axisymmetric state, which is the case with turbulence in pipe flows at very low Reynolds numbers.

Instrumentation

To gain more qualitative than quantitative information on the behaviour of turbulence in pipe flow undergoing rapid pressure variations, hot-wire anemometry was employed as shown in fig. 5. By varying the pressure ramps and by measuring the volume flow rate, pressure and temperature during pressurization, well-defined conditions were ensured to study the effects caused by a rapid pressure increase on shear flow turbulence.

Sub-miniature laboratory-made hot-wire probes were installed at the pipe centreline and very close to its exit, $x/d \approx 1$, for measurements of turbulence and detection of reverse transition. At this location, the measured data reflect essential features of a fully developed state inside the pipe, as can be concluded from flow visualizations [13], and from the experimental results obtained using the hot-wire technique [14].

Probes were made of stainless-steel prongs which were 3 mm apart. Tungsten wires 5 in diameter were suspended between two prongs to which they were welded. Hot-wire probes were operated with a DANTEC 55M constant-temperature anemometer at an overheat ratio of 0.22. Figure 5 shows the hot-wire probe directly above the pipe exit.

The output signals from the anemometer were digitized at a rate of 1 kHz and stored on a personal computer for further processing. No attempts were made during the present investigations to calibrate the hot-wire probes or to account for the effect of spatial integration of the measured signals over the hot-wire length, since the prime interest was to detect qualitative features of turbulence development.

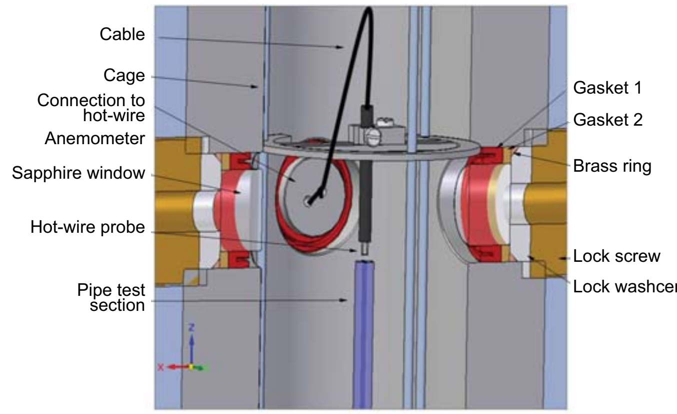

Figure 5. Layout of the hot-wire probe for measurements of turbulence at the exit of the pipe and assembly of sapphire windows permitting optical access for potential application of laser-Doppler anemometry

Experimental results

Measurements in the laminar regime

Results of first exploratory measurements using glycerol as the working fluid are displayed in fig. 6. These measurements correspond to laminar flow at very low Reynolds numbers, Re < 10. The measured signals show that shortly after initial pressurization, the control system required about 20 seconds in order to force the pumping system to produce a monotonic increase in pressure so that only physical effects originating in the flow could be clearly distinguished in the hot-wire signals from other disturbances.

The hot-wire signals reveal that during the period of monotonic increase in pressure 20 s < t < 60 s, the amplitude was almost constant without any noticeable evidence of turbulence, indicating qualitatively that stable laminar flow was maintained during pressurization. By repeating the experiments many times, it was found that hot-wire probes can withstand high-pressure ramps during the pressure build-up, pressure holding and pressure release phases. No damage to the probes occurred during repeated tests carried out at pressures up to 400 MPa. This suggests that hot-wire anemometry can be safely employed for flow investigations at very high pressures.

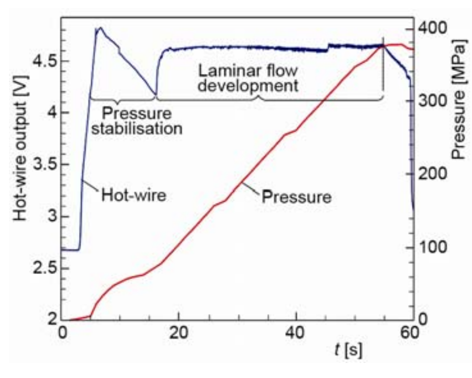

Figure 6. Trace of hot-wire record close to the pipe exit and the pressure signal measured during rapid pressurization of the vessel using glycerol as the working fluid; these experimental results correspond to the laminar flow regime in the pipe

Reverse transition by high-pressure ramps

Figure 7 shows traces obtained from the hot-wire probe and the pressure transducer during rapid pressurization using silicone oil as the working fluid. Owing to the low viscosity of oil during the pressure build-up phase, initially fully developed turbulent flow is established with a bulk Reynolds number of Re $\simeq 1 \cdot 10^4$. As the pressure rises, the turbulence level increases with the appearance of pronounced intermittency in the hot-wire signal at $t \approx 30$ seconds. These trends in hot-wire readings can be explained by alternation of the instantaneous flow between fully turbulent and laminar states. Figure 7 suggests that suppression of turbulence leading to flow relaminarization occurs at $t \approx 40$ seconds, corresponding to a pressure of 150 MPa at a Reynolds number of Re $\approx 4 \cdot 10^3$, which is higher than the $\text{Re}_{\text{crit}} \simeq 2320$ which is commonly reported in the literature for highly disturbed inlet conditions as in the present experiments. These results demonstrate that high-pressure ramps are capable of producing complete relaminarization of a fully developed turbulent pipe flow.

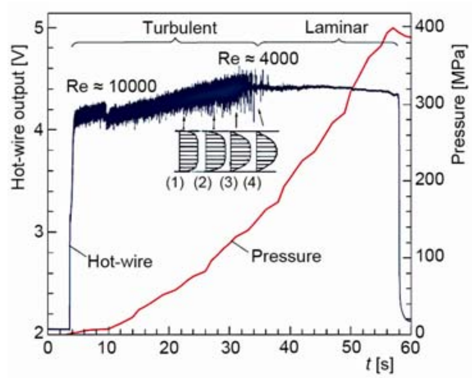

Figure 7. Hot-wire trace recorded close to the pipe exit during rapid high-pressure variation up to $P \approx 400$ MPa; in the recorded signal, it is possible to distinguish periods of turbulent and laminar flow states during the pressure build-up phase

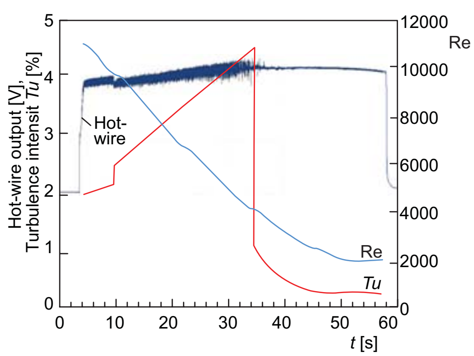

Figure 8. Variations of estimates of the turbulence intensity and bulk Reynolds number during rapid compression leading to flow relaminarization

By performing moving averaging, the statistical features were extracted from hot-wire measurements. Figure 8 shows estimates of the relative turbulence intensity and the bulk Reynolds number during the process of reverse transition. There is great similarity between the present results and those commonly measured during the forward transition if viewed, however, in the reverse order: during relaminarization the turbulence level increases and at the reversion point reaches a maximum which is approximately equal to the difference between the centreline velocities corresponding to laminar and turbulent regimes, respectively, $u' \approx (2U_\text{B} - 1.3\ U_\text{B})/6$, where the factor 6 accounts for the width of the probability density distribution if it is assumed to be Gaussian.

Employing the constant-current mode, low heating currents, i = 3-5 mA, and high signal amplification an attempt was made to complement the results previously presented with measurements of the temperature variations during the relaminarization process. Theoretical considerations by Regulski [15] showed that the natural frequency response of a 5 μm wire was sufficient to resolve instantaneous temperature variations in the flow so that no frequency compensation of the measured signals was necessary. From the measured voltage across the wire, current, gain and temperature coefficient of sensor resistance, it was possible to deduce an increase in the bulk temperature of $\Delta T \approx 30$ K in the vessel due to rapid compression of the liquid.

The development of temperature fluctuations induced by rapid compression is displayed in fig. 9. These results show a similar trend to the measurements presented in figs. 7 and 8. Initially, the temperature fluctuations increase gradually and show a peak value with the appearance of intermittency just prior to reversion to the laminar state. The conditions at the reversion point deduced from hot-wire data and from temperature measurements are almost identical, implying that the scalar field closely follows the velocity field, as can be expected from the Reynolds analogy.

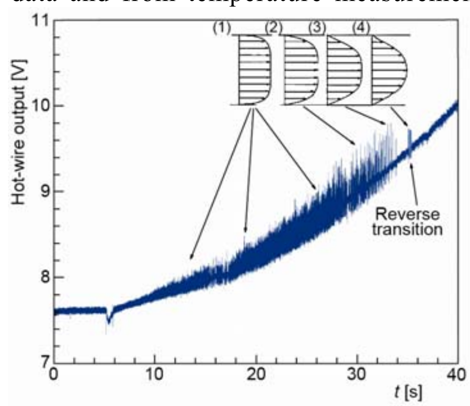

Figure 9. The output of the cold wire showing the variation of temperature during rapid compression leading to relaminarization

Conclusion and final remarks

The reverse transition from fully developed turbulence towards a stable laminar regime under the influence of high-pressure ramps was studied experimentally in a pipe flow. Using glycerol and silicone oil as working fluids, both laminar and turbulent flow conditions were investigated during the compression process at low and moderate Reynolds numbers.

Mass flow and temperature fluctuations measured at the pipe centreline and very close to its exit reveal that by applying pressure ramps and using silicone oil as the working fluid, reversion of the turbulent to the laminar regime occurred at Reynolds numbers for which commonly only the turbulent regime prevails. The reversion process was observed to be accompanied by an increase in turbulence intensity and it is argued that this observation is due to the alternation of the instantaneous velocity profiles between turbulent and laminar flow regimes. The reversion process was completed when the turbulence reached the maximum level, proceeding with intermittent and then continuously stable laminar flow conditions.

The conclusion to be drawn from the presented experimental results is that the observed effect of flow relaminarization is in agreement with the expectation that rapid changes in the fluid density during the compression process reduce the turbulent dissipation and therefore the spectral separation between large- and small-scale motions which finally lead to complete relaminarization [8].

Dedication

This paper we dedicate to Professor Simeon Oka on the occasion of his 80th birthday. We made attempt to provide contribution in the area of near-wall turbulence where Professor Oka was actively involved over decades and made numerous contributions.

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